Quarkonium in nonzero temperature

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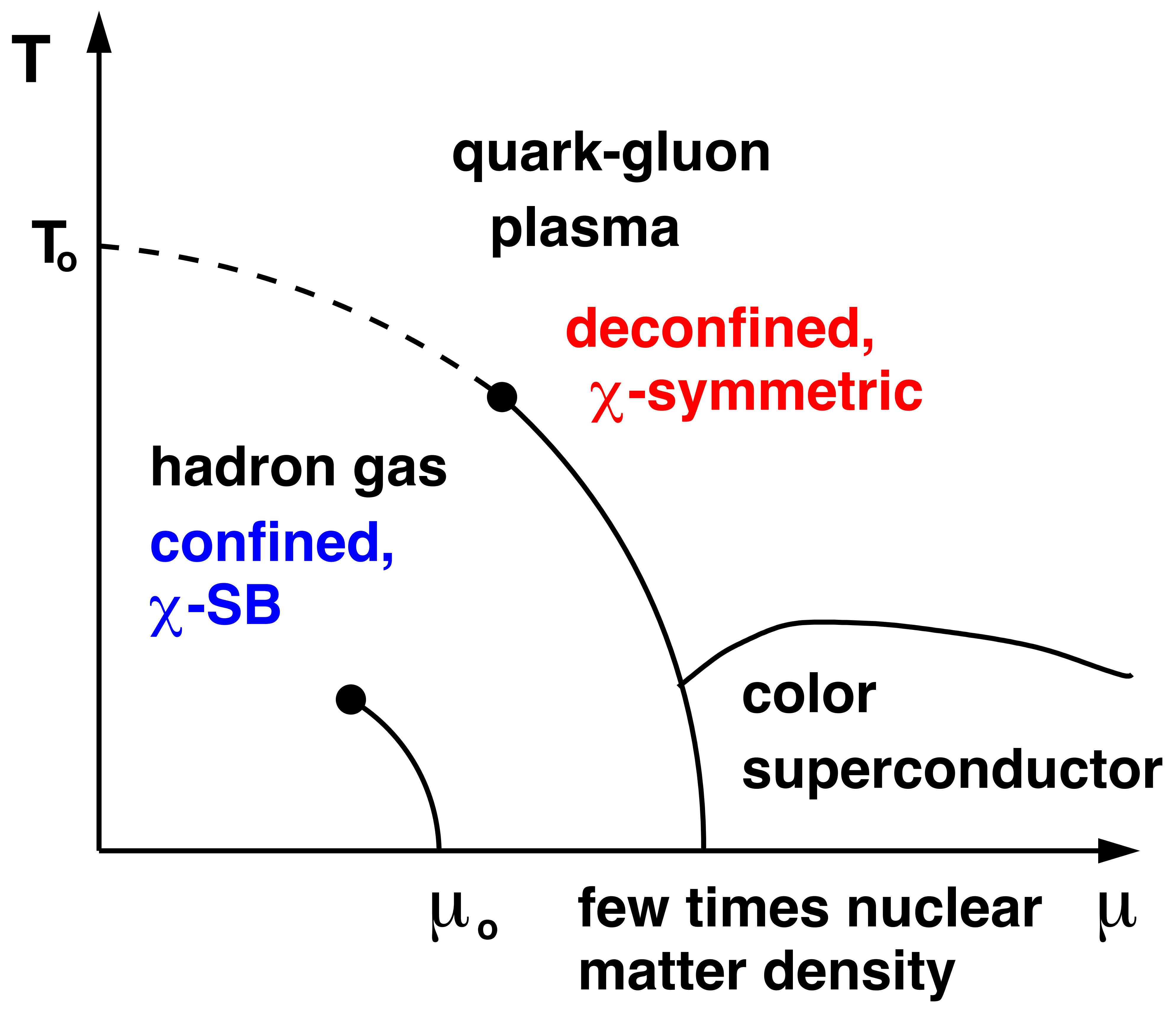
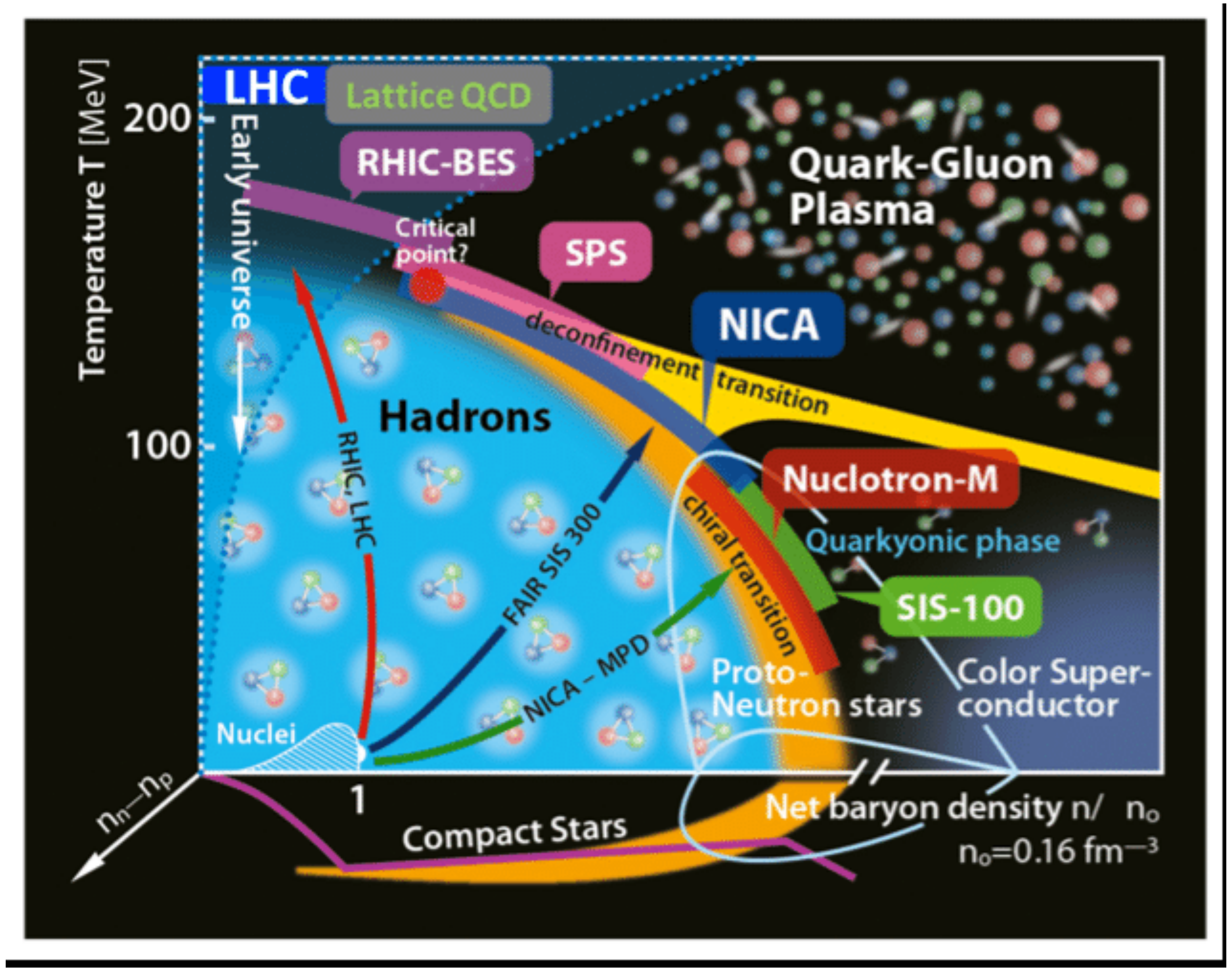
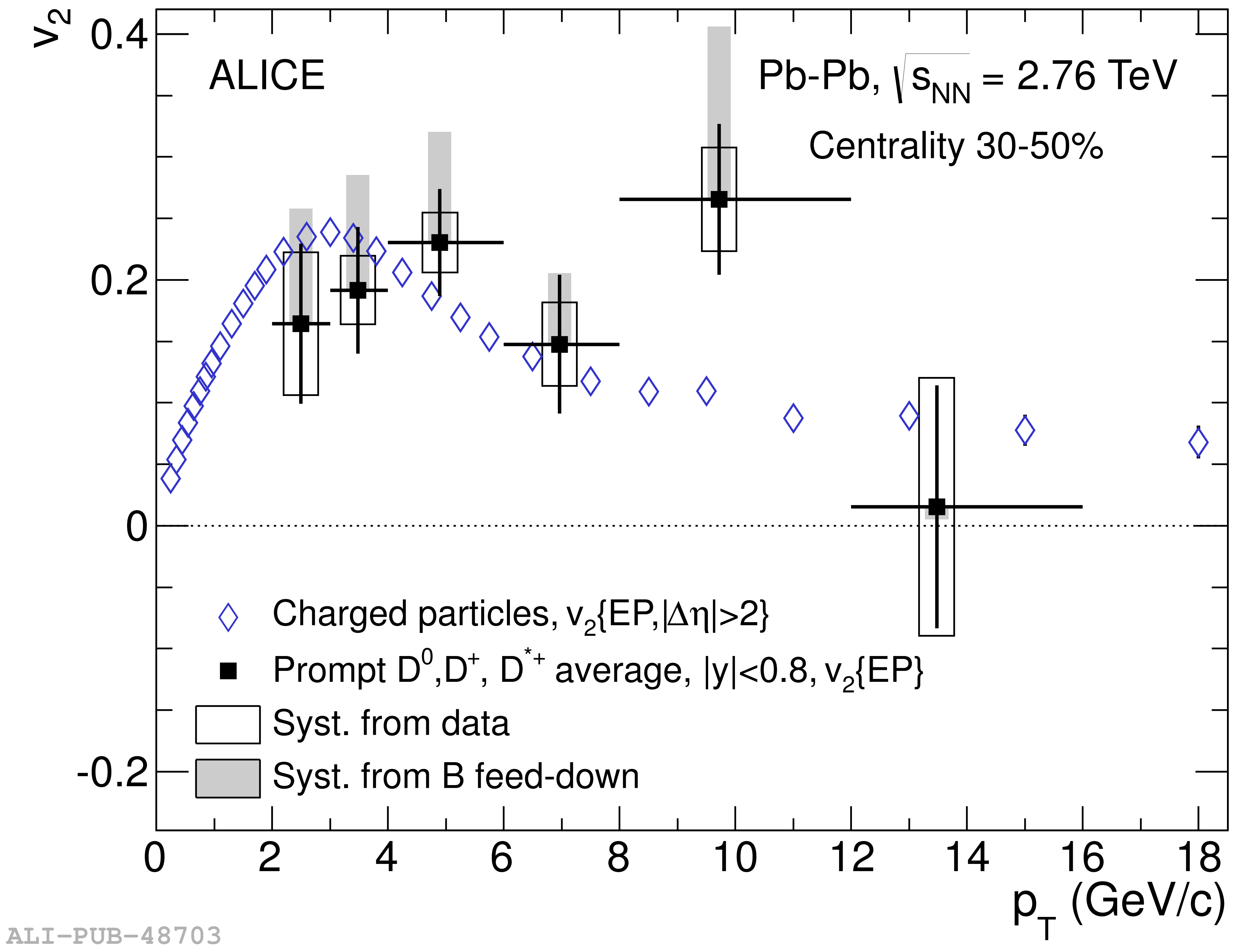
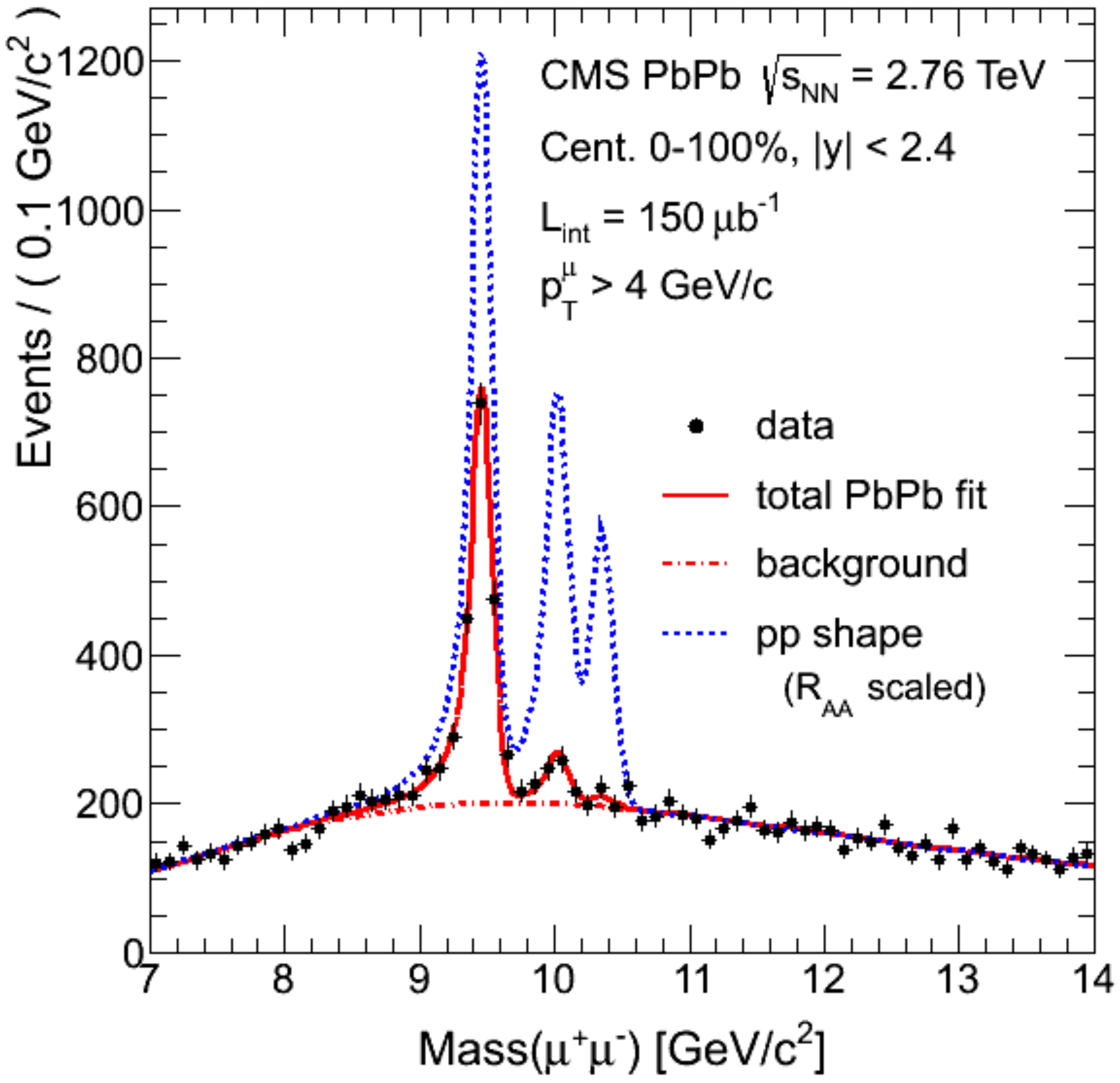
QCD phase diagram





Heavy quarks in $T \neq 0$
(cf. SK, plenary talk at Lat2016, arXiv:1702.02297)





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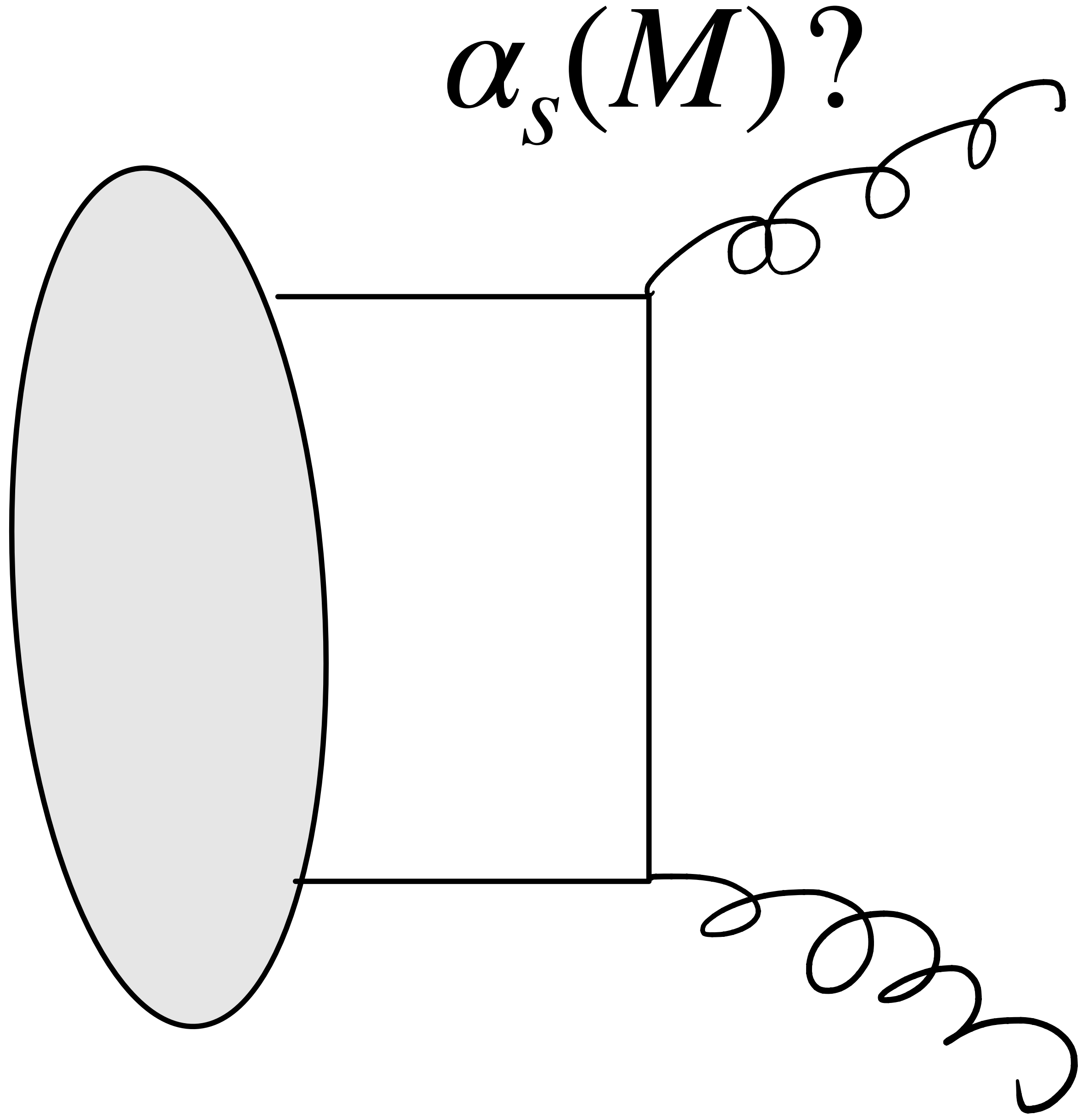


Leading order in α_s doesn't have a scale

But α_s runs if we consider higher order

strong interaction coupling constant (PDG)

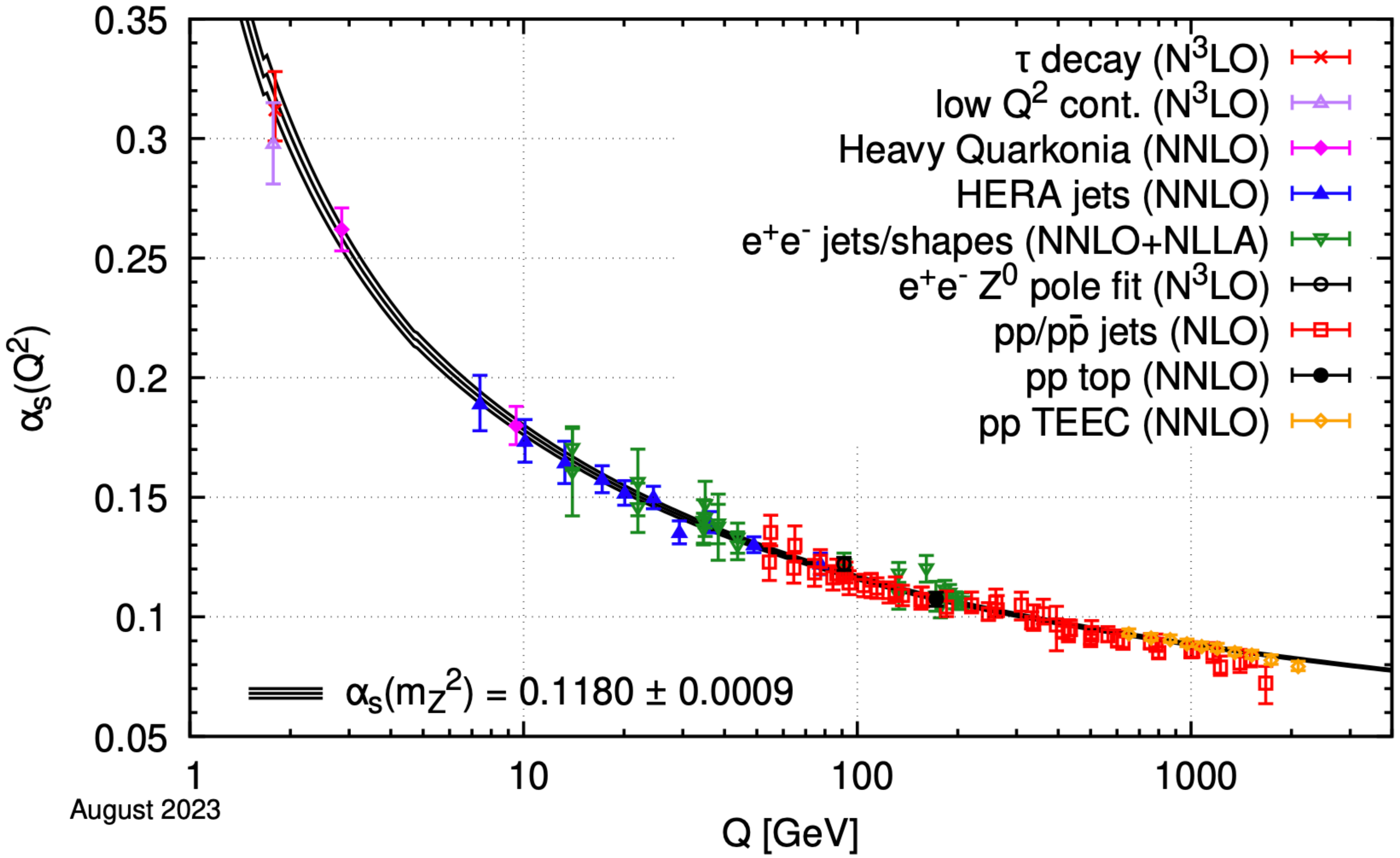


Figure 9.5: Summary of determinations of α_s as a function of the energy scale Q compared to the running of the coupling computed at five loops taking as an input the current PDG average, $\alpha_s(m_Z^2) = 0.1180 \pm 0.0009$. Compared to the previous edition, numerous points have been updated or added.

Heavy quarks are heavy!

- Quarkonium decays are similar to positronium decays
- That is, decay rates is equal to (the probability to produce a heavy quark in quarkonium) × (the annihilation rate of quark into inclusive states)
- Quarkonium is one of the first application of asymptotic freedom (cf. T. Appelquist and H.D. Politzer, PRL 34 (1974) 43
- Quarkonium is "hydrogen atom" in QCD

Heavy quarks are heavy!

- Factorization theorem in decay/production of quarkonium：

  E. Braaten, G.T. Godwin, G.P. Lepage, PRD51 (1995) 1125

- In the rest frame of quarkonium, heavy quark and anti−heavy quark move slowly
- Heavy quark behaves non−relativistically
- $M >> Mv >> Mv^2$, v is the velocity of the heavy quark in the rest frame of quarkonium

Heavy quarks are heavy!

- But $\frac{Mv^2}{r} \sim \frac{\alpha_s}{r^2} \to \alpha_s \sim Mv^2 r \to \alpha_s \sim v$ from $r \sim \frac{1}{Mv}$
- $v^2 \sim 0.3$ for chamornium, $v^2 \sim 0.1$ for bottomonium
- NRQCD is different EFT from Heavy Quark Effective Theory (HQET), a EFT for heavy quark in heavy−light meson
- v is not a parameter of Lagrangian (cf. A. Manohar, PRD56 (1997) 230, M. Luke and A. Manohar, PRD55 (1997) 4129)

Heavy quarks are heavy!

- If the scale M (heavy quark mass) is integrated out, this Effective Field Theory (EFT) is called Non-Relativistic QCD (NRQCD)
- If the scale Mv (heavy quark momentum) is integrated out, this EFT is called potential-Non-Relativistic QCD (pNRQCD) (cf. N. Brambilla et al, Rev.Mod.Phys. 77 (2005) 1423, Nucl.Phys.B566 (2000) 275)

Heavy quarks are heavy!

- Power counting: operator ordering the power of heavy quark velocity, v
- Non-Relativistic QCD or NRQCD: Effective Field Theory of heavy quark in the quarkonium rest frame (caveat: "bound states")
- For example, inclusive (hadronic/electromagnetic) heavy quarkonium decays can be expressed in

$$\Gamma = \sum_n C_n(\Lambda) < H | \mathcal{O}_n(\Lambda) | H > \quad \Lambda \text{ is the factorization scale}$$

- n can be infinite but small n is "useful"
- Quarkonium decay rates are sum of the long distance matrix elements $\times$ short distance "Wilson coefficients"

Non−Relativistic QCD (continuum)

- Foldy−Wouthuysen−Tani transform
- ψ is non−relativistic quark field, χ is non−relativistic anti−quark field
- $\mathscr{L} = \mathscr{L}_0 + \delta\mathscr{L}$
- $\mathscr{L}_0 = \psi^\dagger \left(D_\tau - \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(D_\tau + \frac{\mathbf{D}^2}{2M} \right) \chi,$
- $\delta\mathscr{L} = -\frac{c_1}{8M^3} \left[\psi^\dagger (\mathbf{D}^2)^2 \psi - \chi^\dagger (\mathbf{D}^2)^2 \chi \right] + c_2 \frac{ig}{8M^2} \left[\psi^\dagger (\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D}) \psi + \chi^\dagger (\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D}) \chi \right]$

$-c_3 \frac{g}{8M^2} \left[\psi^\dagger \sigma \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) \psi + \chi^\dagger \sigma \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) \chi \right] + c_4 \frac{g}{2M} \left[\psi^\dagger \sigma \cdot \mathbf{B} \psi - \chi^\dagger \sigma \cdot \mathbf{B} \chi \right]$

2. Quarkonium on a Lattice

Lattice NRQCD: G.P. Lepage at el., PRD46 (1992) 4052

- Additional scale, $a : Ma \sim 1$
- $U_\mu(x) = e^{iA_\mu(x)a}$
- $a\Delta_\mu^+\psi(x) \equiv U_\mu(x)\psi(x + a\hat{\mu}) - \psi(x)$
- $a\Delta_\mu^-\psi(x) \equiv \psi(x) - U_\mu(x)^\dagger\psi(x - a\hat{\mu})$
- $\Delta^\pm \equiv \frac{1}{2}(\Delta^+ + \Delta^-),\ \Delta^2 \equiv \sum_i \Delta_i^+\Delta_i^- = \sum_i \Delta_i^-\Delta_i^+$

Lattice NRQCD:
G.P. Lepage at el., PRD46 (1992) 4052

- $F_{\mu\nu}(x) = -\frac{1}{4a^2}\sum_{P_{\mu\nu}(x)} \mathscr{I}[U_{P_{\mu\nu}}(x)],\ \mathscr{I}[M] \equiv \frac{M - M^\dagger}{2i} - \frac{1}{3}\mathrm{Im}(\mathrm{Tr}M)$

Clover term

- $a\Delta^+_\rho F_{\mu\nu}(x) \equiv U_\rho(x)F_{\mu\nu}(x + a\hat{\rho})U_\rho(x)^\dagger - F_{\mu\nu}(x)$
- $a\Delta^-_\rho F_{\mu\nu}(x) \equiv F_{\mu\nu}(x) - U_\rho(x - a\hat{\rho})^\dagger F_{\mu\nu}(x - a\hat{\rho})U_\rho(x - a\hat{\rho})$

Lattice NRQCD:
G.P. Lepage at el., PRD46 (1992) 4052

$$G(\mathbf{x},\tau_0) = \mathbf{S}(\mathbf{x}),$$

$$G(\mathbf{x},\tau_1) = \left(1 - \frac{H_0}{2n})\right)^n U_4^\dagger(\mathbf{x},\tau_0)\left(1 - \frac{H_0}{2n}\right)^n G(\mathbf{x},\tau_0)$$

$$G(\mathbf{x},\tau_i) = \left(1 - \frac{H_0}{2n})\right)^n U_4^\dagger(\mathbf{x},\tau_{i-1})\left(1 - \frac{H_0}{2n}\right)^n (1-\delta H)G(\mathbf{x},\tau_{i-1})$$

$$H_0 = -\frac{\Delta^2}{2M}, \quad \delta H = -\frac{(\Delta^2)^2}{8M} + \frac{i}{8M}\left(\mathbf{\Delta}^\pm \cdot \mathbf{E} - \mathbf{E}\cdot\mathbf{\Delta}^\pm\right) - \frac{1}{2M}\sigma\cdot\mathbf{B} + \frac{a^2\Delta^4}{24M} - \frac{a(\Delta^2)^2}{16nM}$$

3. Quarkonium at $T \neq 0$ on anisotropic lattices ($T/M < < 1$)

T. Matsui and H. Satz, PLB 178(1986) 416

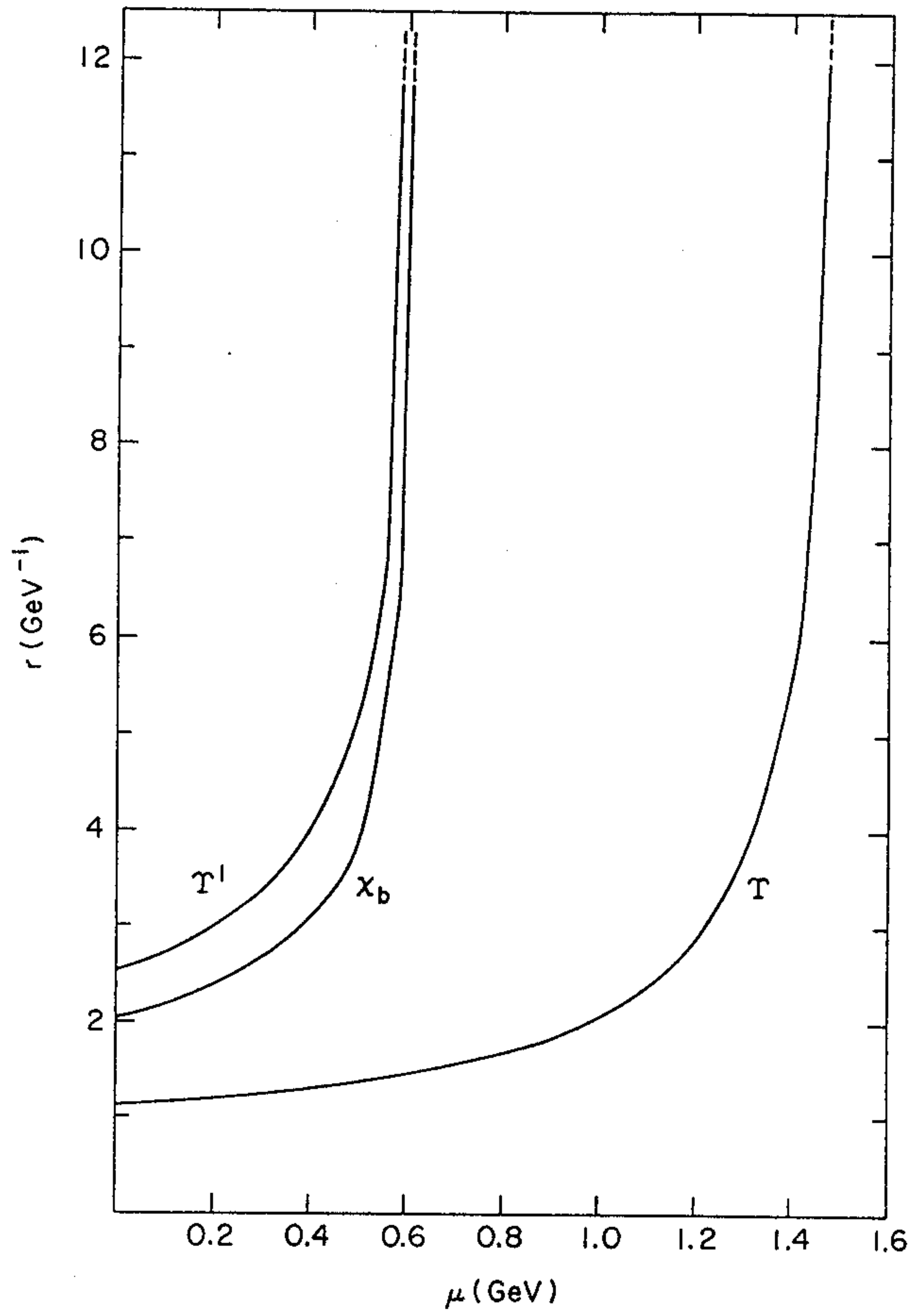
- Schroedinger eq.: $i\frac{\partial}{\partial t}\psi = \mathscr{H}\psi,\ \ \mathscr{H} = 2M - \frac{1}{2M}\nabla^2 + V(r)$

- $T = 0$, Cornel potential: $V(r) = -\frac{\alpha}{r} + \sigma r$

- $T \neq 0$, Debye screening:

$$V(r) = \frac{\sigma}{\mu_D(T)}(1 - e^{-\mu_D(T)r}) - \frac{\alpha}{r}e^{-\mu_D(T)r},\ \ \mu_D(T) = 1/r_D(T)$$

F. Karsch, M.T. Mehr,and H. Satz,
Z. Phys. C37 (1988) 617



Quarkonium as thermometer (A. Mocsy, arXiv:0811.0337)

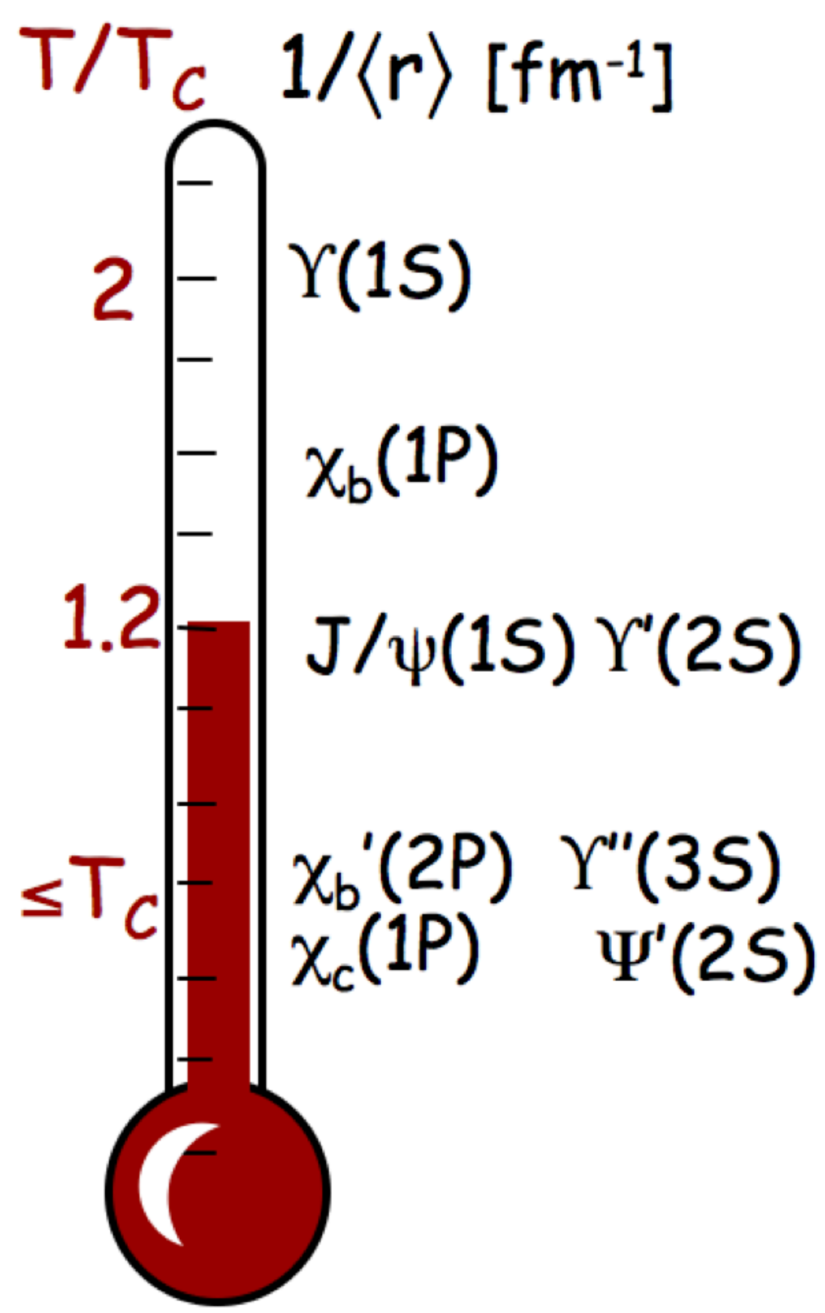


Fig. 5. The QGP thermometer.

precise quarkonium properties cannot be determined this way, but the upper limit can be estimated. The decrease in binding energies with increasing temperature, observed in all the potential models on the market, can yield significant broadening, not accounted for in the currently shown spectral functions from these models. The upper limit estimated using the confining potential predicts that all bound states melt by $1.3T_c$, except the Upsilon, which survives until $2T_c$. The large threshold enhancement above free propagation seen in the spectral functions even at high temperatures, again observed in all the potential models on the market, compensates for melting of states (yielding flat correlators), and indicates that correlation between quark and antiquark persists. Lattice results are thus consistent with quarkonium melting.

And What's Next?

Implications of the QGP thermometer of figure 5 for heavy ion collisions should be considered by phenomenological

Quarkonium as thermometer
(A. Mocsy, arXiv:0811.0337)

Ágnes Mócsy: Potential Models for Quarkonia

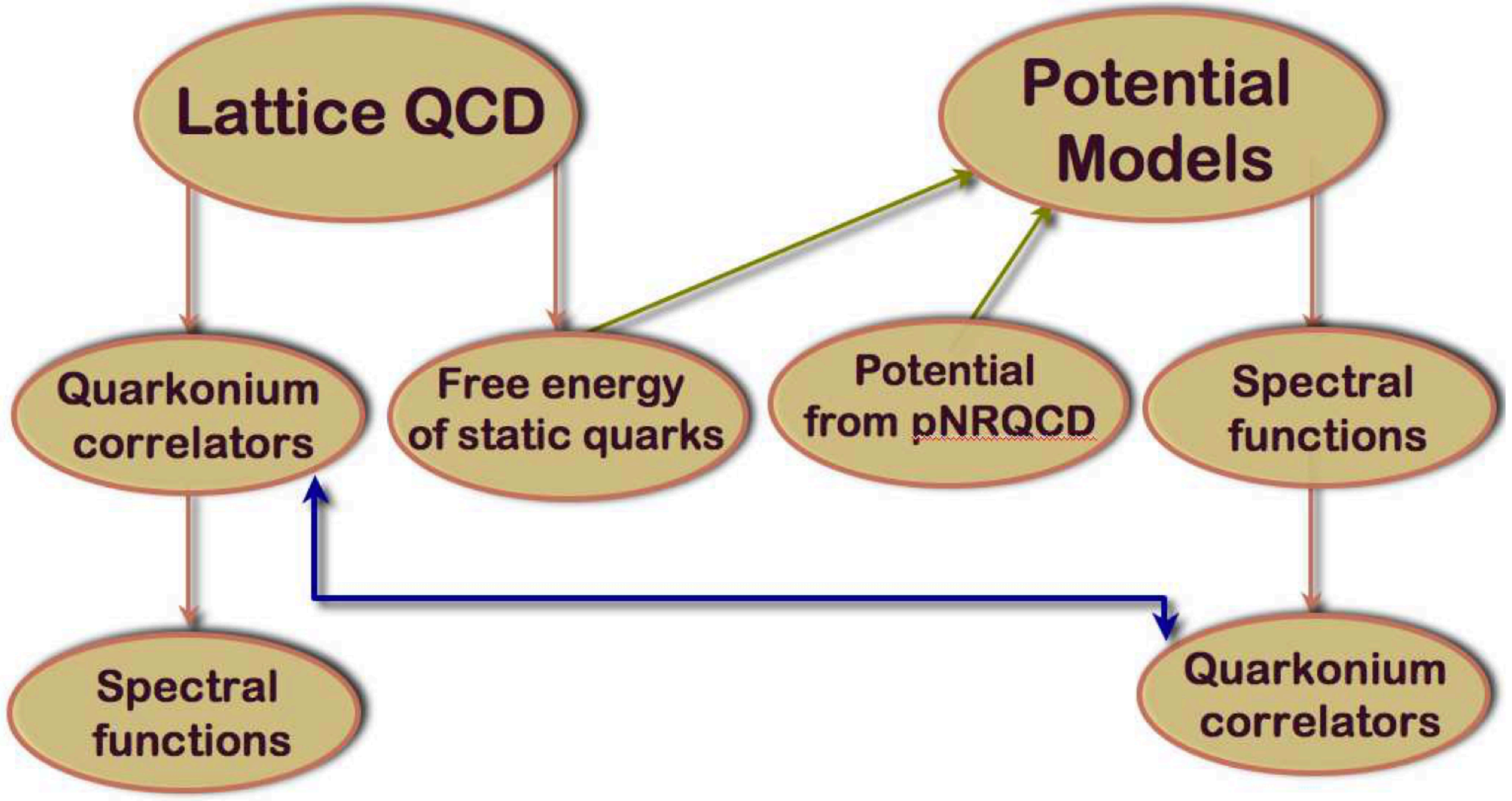


Fig. 3. Structural chart of lattice QCD and potential model calculations.

FASTSUM collaboration

- Originally, Frascati–Argonne–Swansea–Trinity–Sejong–Utah–Maynooth
- M.–P. Lombardo (INFN, Frascati -> Firenze)
- D.K. Sinclair (Argonne National Laboratory): former member
- G. Aarts, A. Allton, M. Anwar, E. Bennet, T. Burns, S. Chen, M. Favoni, M. Gibbons, A. Smecca (Swansea University)
- R. Bignell, S. Ryan (Trinity College Dublin)
- S. Kim (Sejong University)
- M.B. Oktay (University of Utah): former member
- R. D'Arcy, D. Lawlor, J.-I. Skullerud (Maynooth University)
- B. Jaeger (Southern Denmark University)

FASTSUM Characteristics

- Improved Gauge Action/Improved Wilson Quark Action
- Anisotropic Lattices
- QCD Thermodynamics：fixed lattice spacing, varying N_t
- based on Chroma (Gen1), OpenQCD (Gen2, Gen2L, Gen2P, Gen3)

Generation	1	2	2L	2P	3
Gauge Action	Symanzik improved				
Fermion Action	Hamber-Wu stout link	Clover	Clover	Clover	Clover
N_f	2	2+1	2+1	2+1	2+1
a_t [fm/Gev]	0.0268(1)/7.35(3)	0.03506(23) /5.63(4)	0.032459(71)/ 6.079(13)	0.032459(71) /6.079(13)	0.01753/11.26
a_s [fm]	0.162(4)	0.1205(8)	0.11208(31)	0.11208(31)	0.12
xi=a_s/a_t	6.03	3.444(6)	3.453(6)	3.453(6)	6.85
N_s	12	24	32	32	24/32
T_c [Mev]	219	181	167	–	–
M_pi [Mev]	490	384(4)	239(1)	–	

Gen1							
N_t	16	18	20	24	28	32	80
MeV	458	408	368	306	263	230	90
T/T_c	2.06	1.86	1.68	1.40	1.20	1.05	0.42
N_cfg	1000	1000	1000	500	1000	1000	250

Gen2								
N_t	16	20	24	28	32	36	40	128 (N_s = 12)
MeV	352	281	235	201	176	156	141	~0
T/T_c	1.90	1.52	1.27	1.09	0.95	0.84	0.76	~0
N_cfg	499	502	503	998	1001	1002	1000	1042

Gen2L								
N_t	16	20	24	28	32	36	40	128 (N_s = 32)
MeV	380	304	253	217	190	169	152	47
T/T_c	2.27	1.82	1.51	1.29	1.13	1.01	0.91	0.284
N_cfg	1102	1030	1016	1031	1090	1119	1102	1042

NRQCD Quarknium correlators and spectral functions

$$G(\tau) = \int_0^\infty d\omega\ K(\omega, \tau)\rho(\omega)$$

$$K(\omega, \tau) = \frac{\cosh(\omega\tau - \omega/2T)}{\sinh(\omega/2T)} \to e^{-\omega\tau} \quad \text{for } M \to \infty$$

$$G(\tau)_{\mathrm{NRQCD}} \simeq \int_0^\infty d\omega\ e^{-\omega\tau}\rho(\omega)$$

- Spectral function is obtained by inverse Laplace transform
- Euclidean correlators and the periodic boundary condition from the finite temperature condition

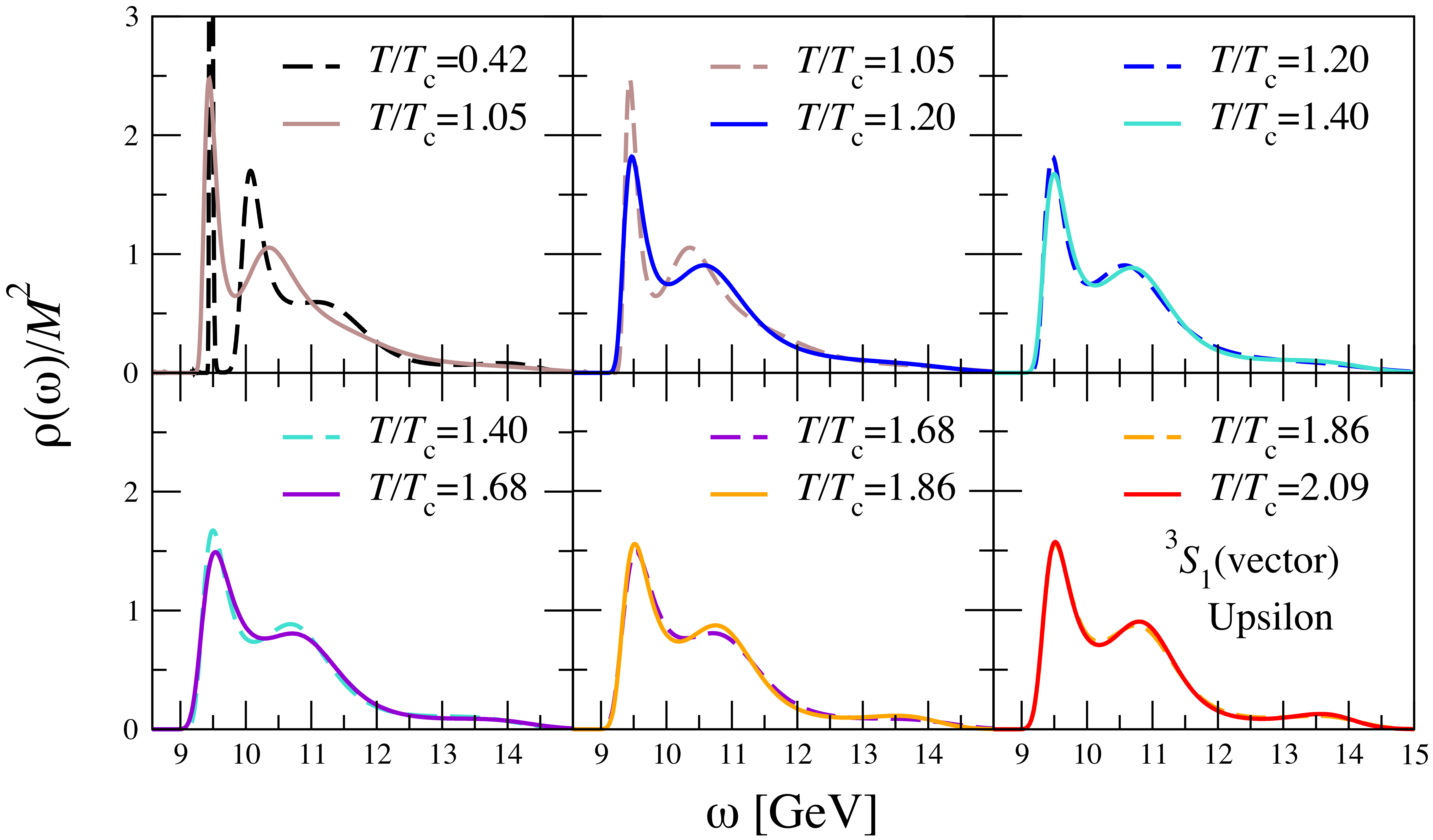
Inverse Problem:
A. Asakawa, T. Hatsuda, Y. Nakahara,
Prog. Part. Nucl. Phys. 46 (2001) 459

- For given $G(\tau)$, obtaining $\rho(\omega)$ is one of the famous ill-posed problems
- The spectral function is a solution of integral equation
- There are infinitely many solutions which satisfies the integral equation
- Maximum Entropy Method (MEM) is one possibility

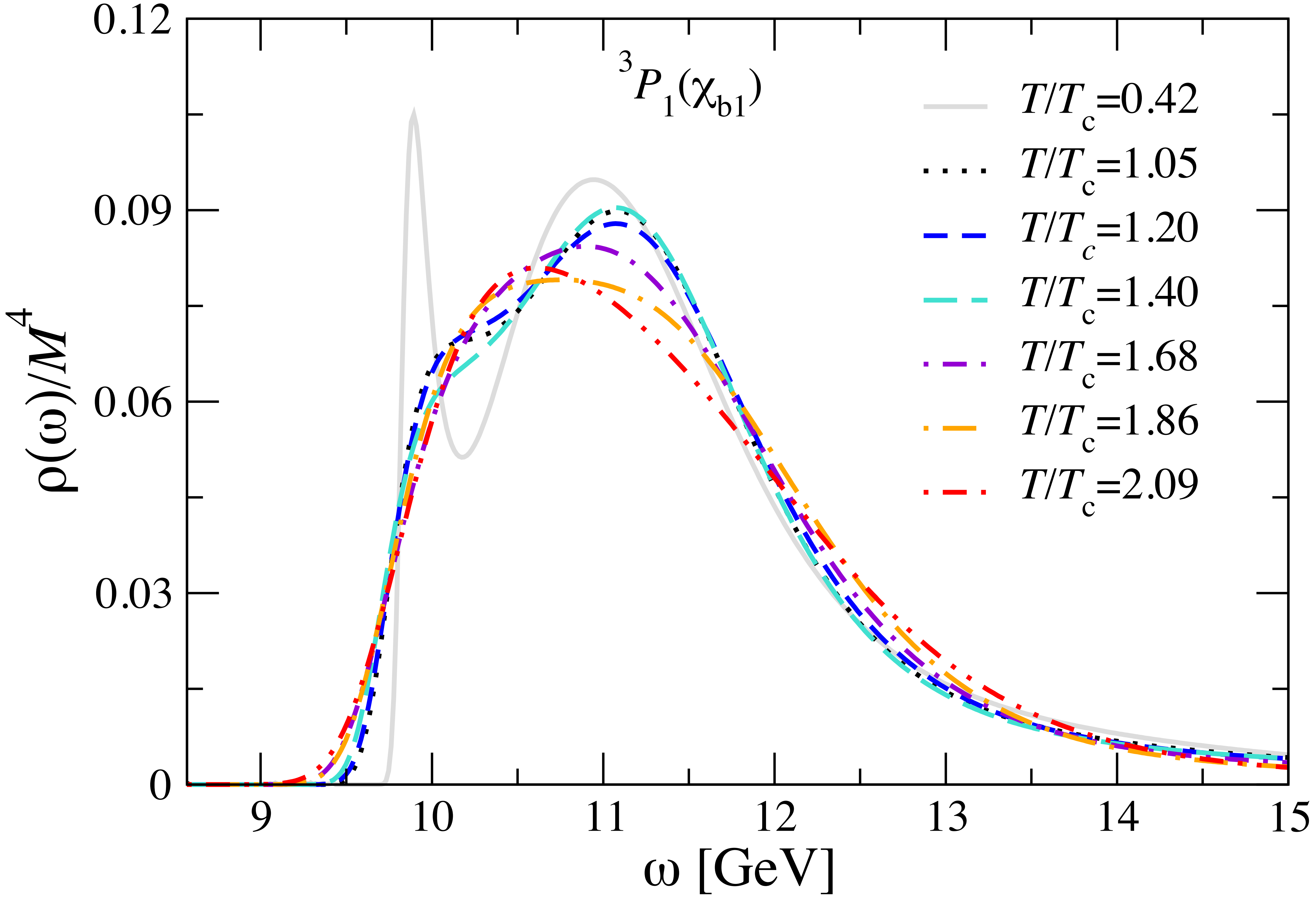
FASTSUM NRQCD project

- Use NRQCD to calculate heavy quark propagator
- Calculate the in-medium quarkonium correlators for various channel
- Use Maximum Entropy Method (MEM) to calculate the spectral functions of given channels
- Investigate the spectral functions as a function of temperature

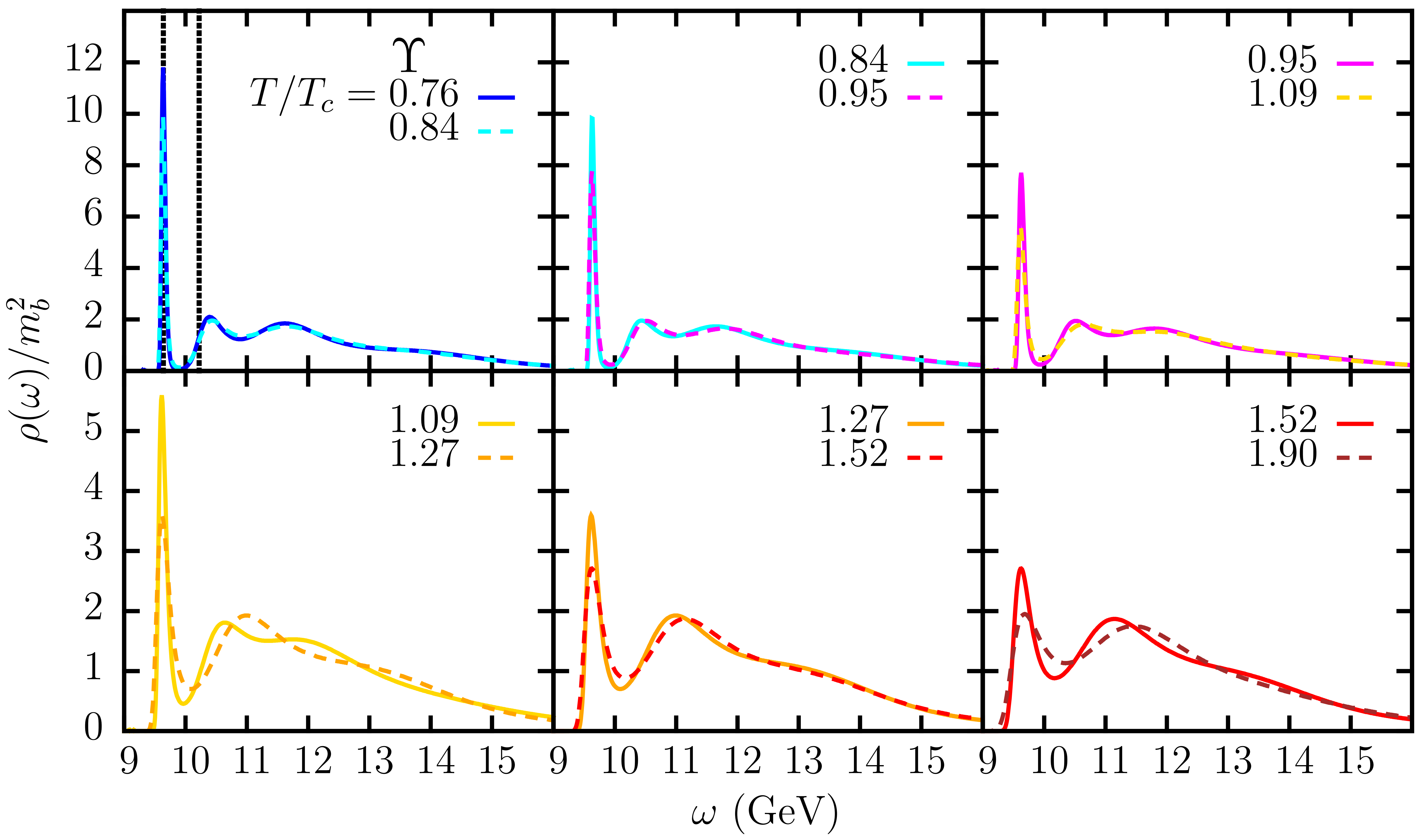
Upsilon spectral function (Gen1): arXiv:1109.4496



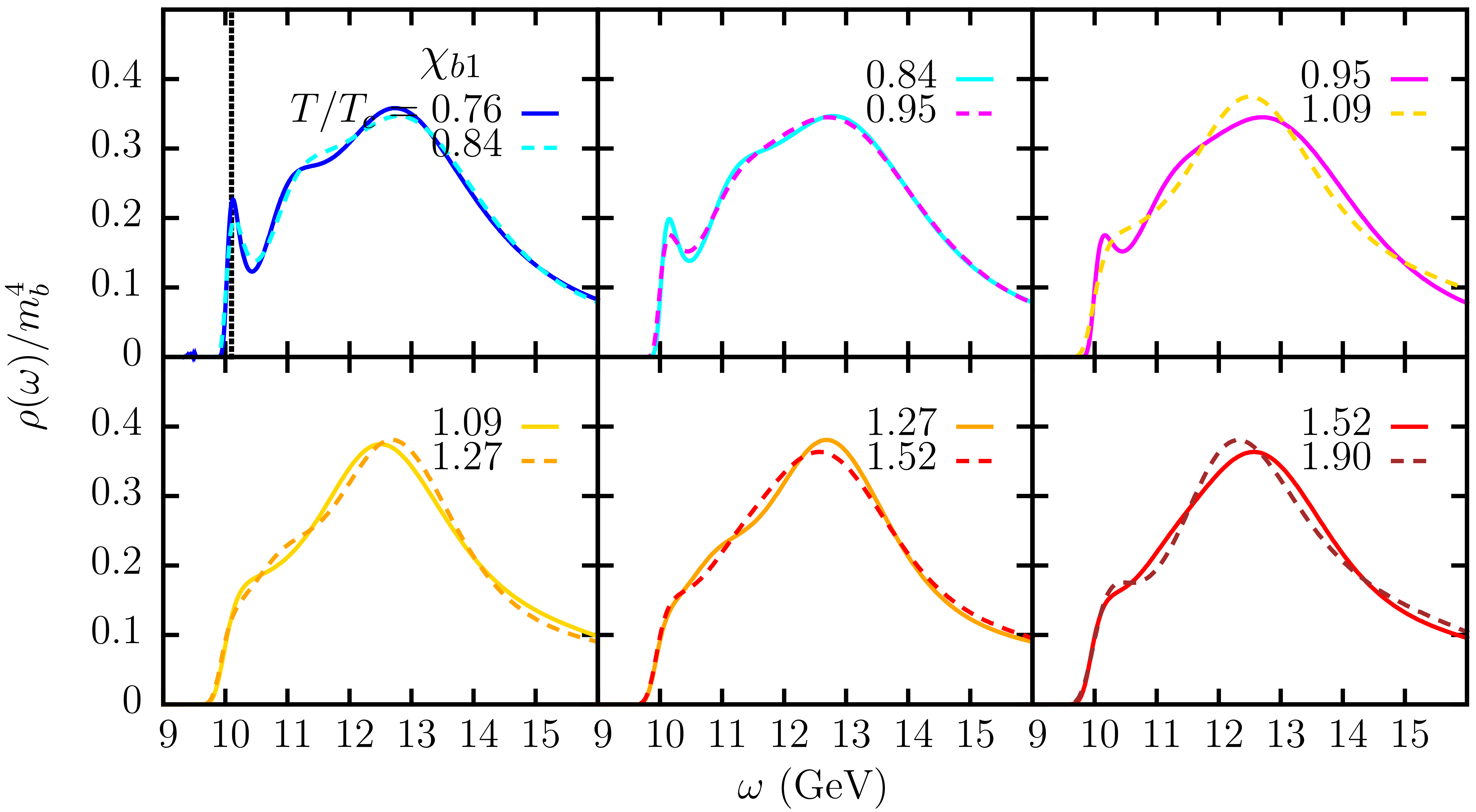
χ_{b1} spectral function (Gen1)
arXiv:1310.5467



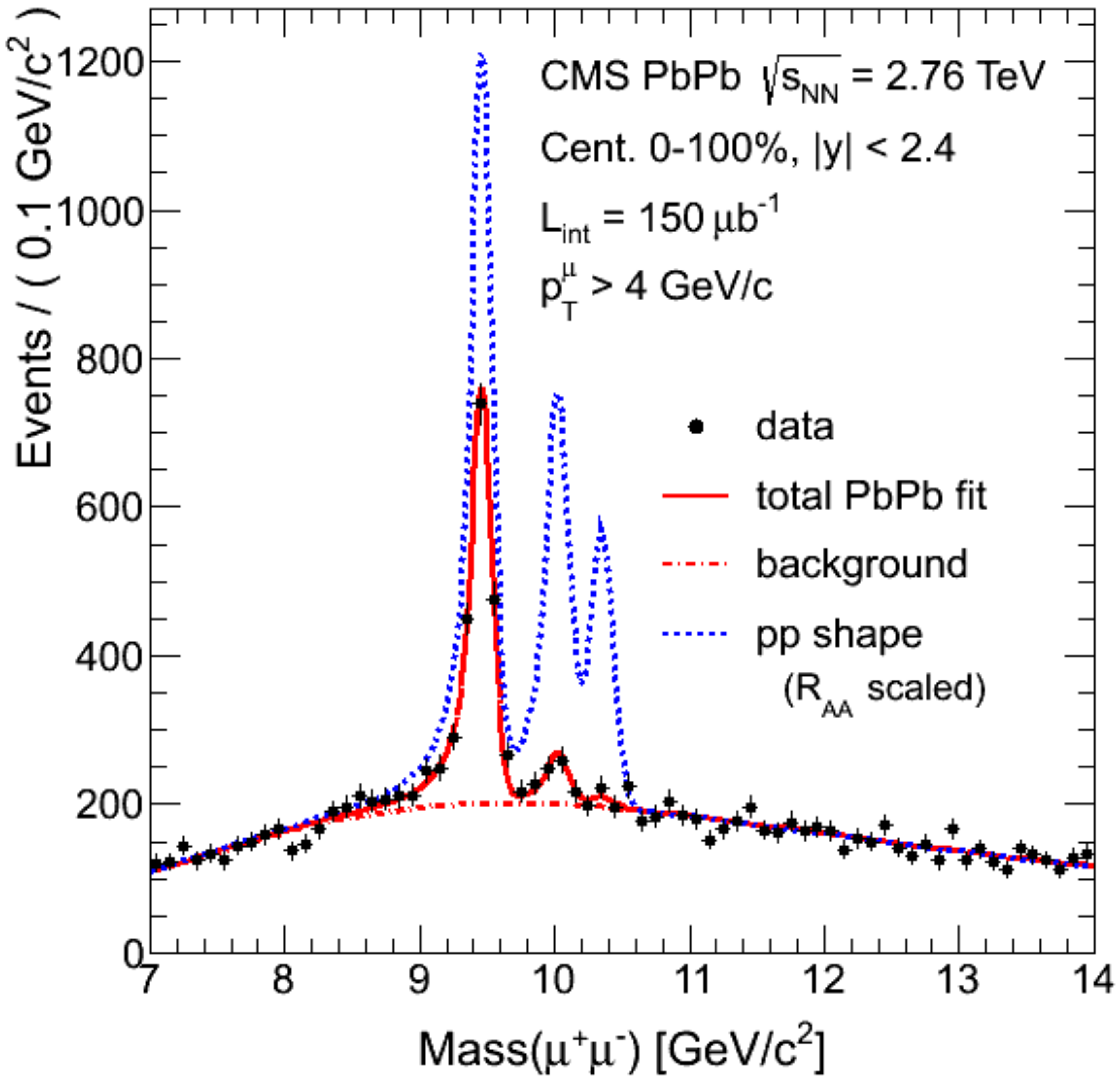
Upsilon spectral function (Gen2): arXiv:1410.6210

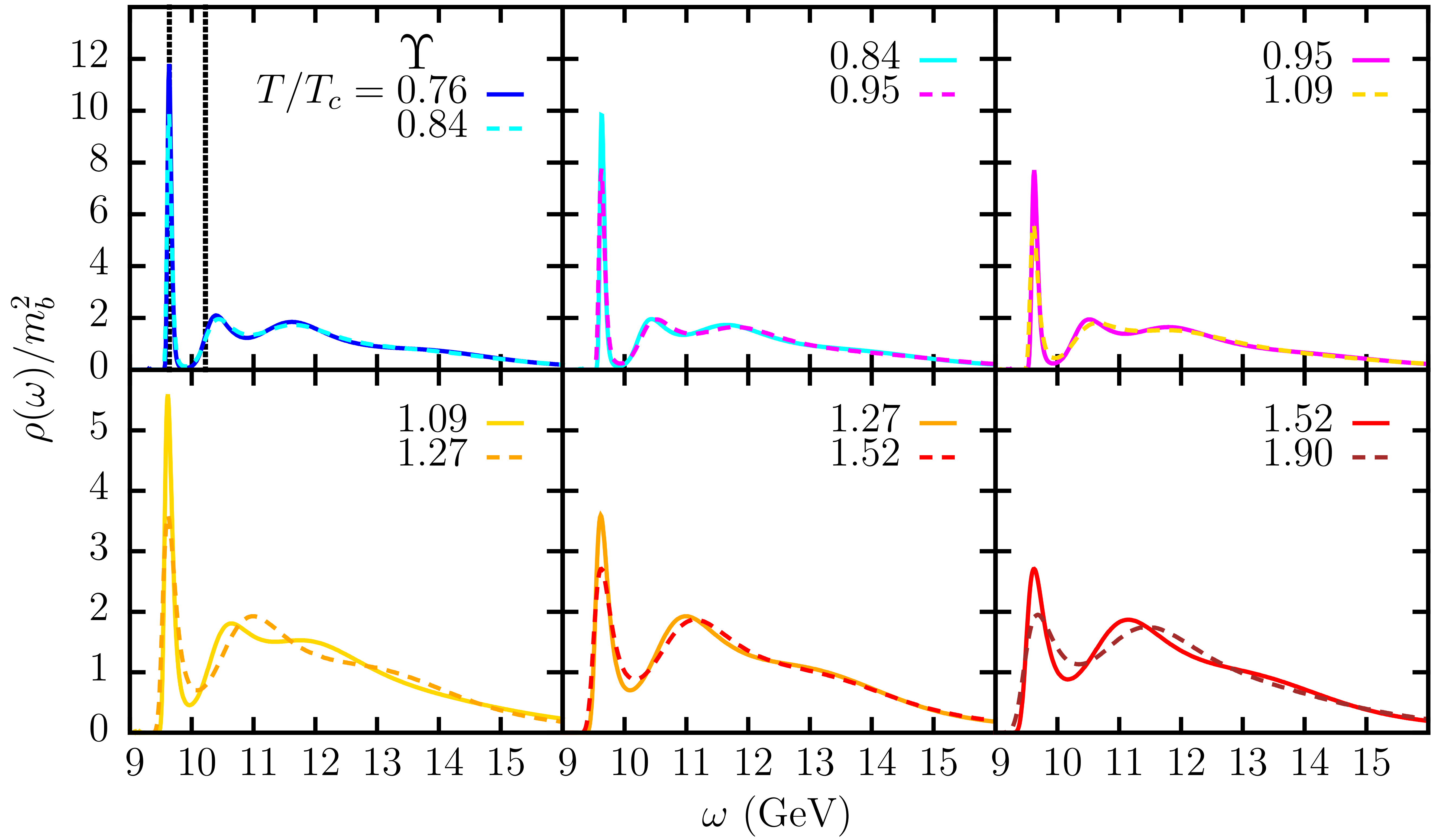


χ_{b1} spectral function (Gen2)
arXiv:1402.6210



Upsilon in $T \neq 0$?





JHEP 07 (2014) 097: arXiv:1402.6210

- We can observe squential melting of quarkonium in nonzero temperature qualitatively
- How crucial do the results depend on the MEM?
- Gen 3 has larger anisotropy → larger number of N_τ → better MEM results

4.Discussion

- Continuum limit?
- Spectral function from MEM: method dependent result?

$$S_{SJ}[\rho] = \alpha \sum_l \left(\rho_l - m_l - \rho_l \log(\frac{\rho_l}{m_l}) \right) \delta\omega_l, \;\; S_{BR}[\rho] = \alpha \sum_; \left(1 - \frac{\rho_l}{m_l} + \log(\frac{\rho_l}{m_l}) \right) \delta\omega_l$$

- Is the pion light enough?
- Is broadening of the peaks method dependent? (cf. P. Petreczky and collaborators)

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Unscreened forces in the quark-gluon plasma?

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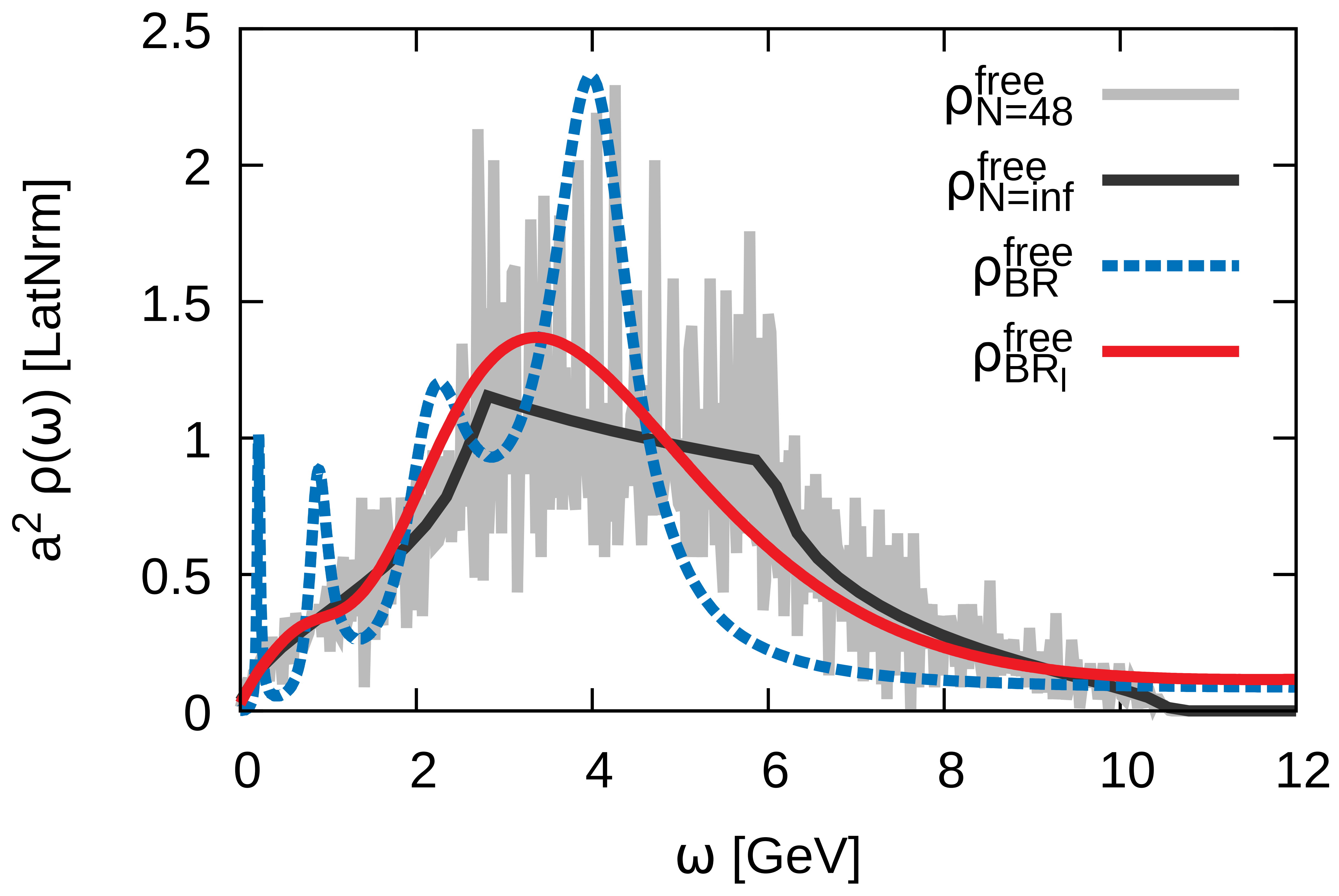
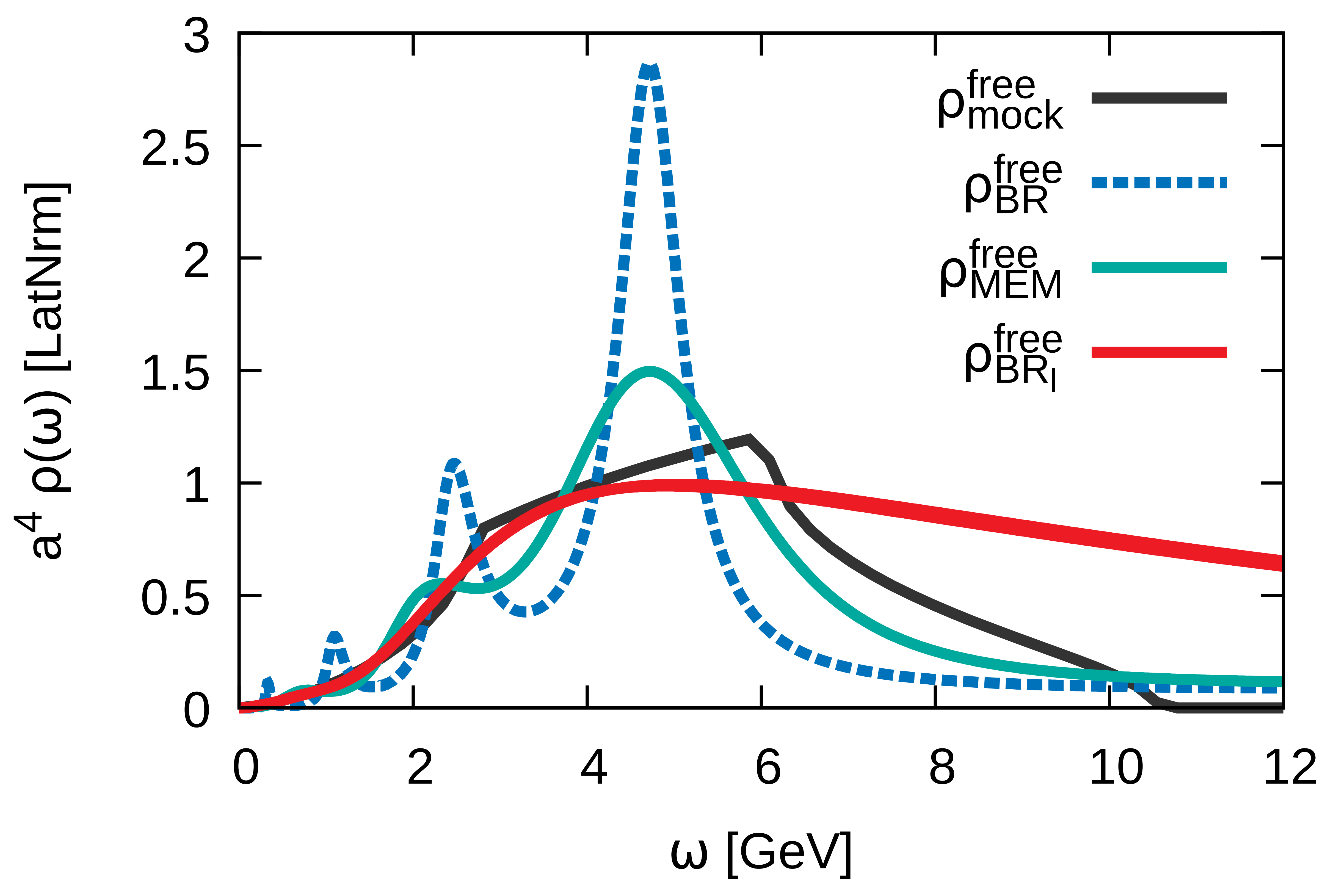
We study the correlator of temporal Wilson lines at nonzero temperature in $2 + 1$ flavor lattice QCD with the aim to define the heavy quark-antiquark potential at nonzero temperature. For temperatures $153\ \mathrm{MeV} \leq T \leq 352\ \mathrm{MeV}$ the spectral representation of this correlator is consistent with a broadened peak in the spectral function, position, or width of which then defines the real or imaginary parts of the heavy quark-antiquark potential at nonzero temperature, respectively. We find that the potential's real part is not screened contrary to the widely held expectations. We comment on how this fact may modify the picture of

Method dependence?



S−wave



P−wave

- Effective field theory understanding of quarkonium in non-zero temperature is highly promising (better accuracy due to the scale separation
- Quarkonium in Quark-Gluon Plasma (QGP) can be understood qualitatively through a modification of the spectral functions from NRQCD quarkonium correlators
- Quarkonium melting via Debye screening or thermal width broadening?
- Quantitative understanding of quarkonium melting is yet to come