LaMET Matching

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LaMET (Large Momentum Effective Theory) Ji '13

- Quasi-PDF $\lambda^\mu = (0,0,0,1)$

$$\widetilde{q}(x,\mu^2,P^z) \quad = \quad \int \frac{dz}{4\pi} e^{-ixzP^z} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(z\lambda) \right| P \right\rangle$$

- Factorization theorem: (~HQET: power corrections + matching)

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{M^2}{P_z^2}\right) + \ldots$$

UV, PQCD

- Proof? (Ma, Qiu; Izubuchi, Ji, Jin, Stewart, Zhao)

Matching Factor

- Computed by quark diagrams?

Recall

$$\sigma(eP \to e'X) = \int_0^1 dx f_i(x,\mu)\sigma(ei \to e'i',\mu)$$

$$\Lambda_{\text{QCD}} \ll \mu \ll Q$$

μ dependence of PDF same as the quark diagram.

Matching Factor Z

- UV difference of quasi-PDF and PDF
- PDF in MS-bar, quasi-PDF in lattice spacing--- lattice action dependent, LPT (Ishikawa, Ma, Qiu, Yoshida; Xiong, Luu, Meissner; Constantinou et al.) slow convergence.
- Wilson line

$$\Gamma[0, \eta\lambda] = \exp\left(i \int_0^\eta d\rho \lambda \cdot A(\rho\lambda) \right)$$

$$i\lambda \cdot D\{\theta(\eta)\Gamma[0, \eta\lambda]\} = i\delta(\eta)$$

~ heavy quark propagator in coordinate space, induces a heavy quark mass set by 1/a. Wilson line mass subtraction scheme (JWC, Ji, Zhang)

Matching Factor Z

- Multiplicative renormalization (Ji, Zhang, Zhao): the quasi PDF operator is formed by (a) two heavy-light currents and (b) one heavy quark propagator with log divergence in (a) and linear divergence in (b)
- NPR (non-perturbative renormalization possible) replacing lattice regulator by another one, so lattice action dependence vanishes in the continuum limit

(a) Ratio scheme (Radyushkin)

(b) RI/MOM (Zhao & Stewart; Constantinou et al)

Ratio Scheme

$$\frac{\langle P|\bar{O}_q^B(z)|P\rangle}{\langle P=0|\bar{O}_q^B(z)|P=0\rangle} = \frac{\langle P|\bar{O}_q^R(z)|P\rangle}{\langle P=0|\bar{O}_q^R(z)|P=0\rangle}$$

- Multiplicative renormlaiztion needed
- Ratio UV finite. Can compute matrix elements in the numerator and denominator by MS-bar scheme.

RI/MOM Scheme

- RI/MOM = Regularization Invariant Momentum Subtraction Scheme
- Subtract all the loop contribution for a quark matrix element at highly off-shell kinematics in a fixed gauge (e.g. Landau gauge in 1706.01295 (LP3))
- Continuum limit could exist provided gauge fixing error vanishes first (ChQCD)

Hybrid Renormalization

- Ratio scheme: long (> Zs~0.3 fm) Wilson line counterterm has non-perturbative IR effect; replaced by Wilson line mass subtraction scheme (X. Ji et al, 2008.03886)

Example: Quark Quasi-PDF in (Hybrid-)Ratio Scheme

$$\frac{\langle P|\bar{O}_q^B(z)|P\rangle}{\langle P=0|\bar{O}_q^B(z)|P=0\rangle} = \frac{\langle P|\bar{O}_q^R(z)|P\rangle}{\langle P=0|\bar{O}_q^R(z)|P=0\rangle}$$

To get the UV part of the RHS, we can compute the quark level diagrams on the LHS (in e.g. Feynman gauge).

$$\tilde{Q}_q^B(z,p^z,\tilde{\mu},\epsilon) = \tilde{Q}^{(0)}(z,p^z) + \tilde{Q}^{(1)}(z,p^z,\tilde{\mu},\epsilon) + \mathcal{O}(\alpha_s^2)$$

$$\tilde{Q}^{(0)}(z,p^z) = e^{-ip^z z}$$

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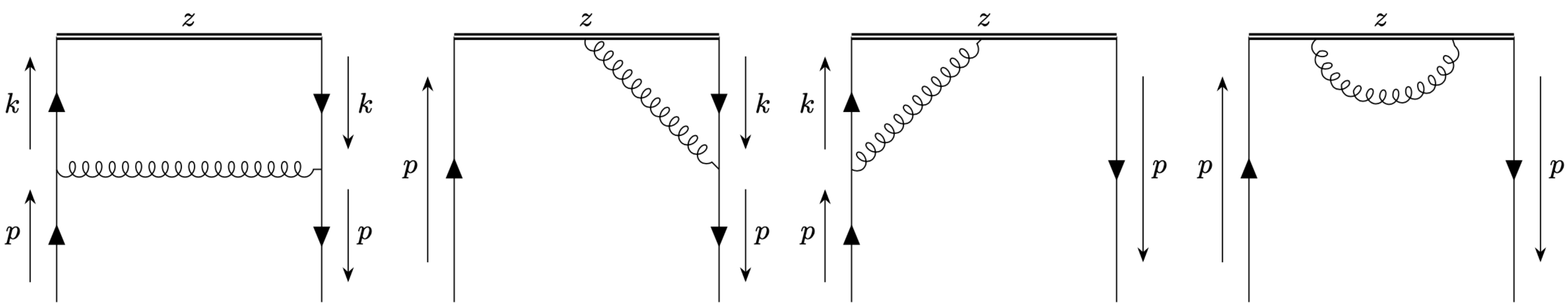
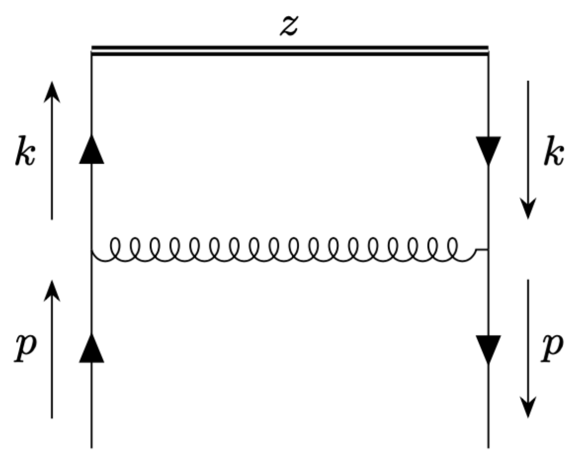


FIG. 1. Non-singlet quark quasi-PDF Feynman diagrams at one loop: the vertex(left), sail(middle two), and tadpole(right) diagrams.

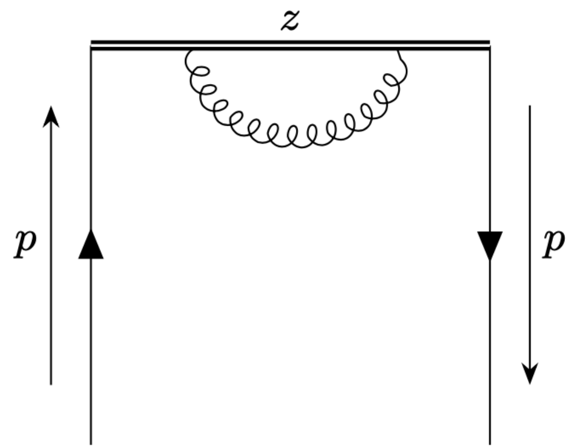


$$\iota = e^{\gamma_E}/4\pi$$

$$\tilde{Q}^{(1)}_{vertex}(z,p^z,\epsilon,\tilde{\mu}) = \frac{\iota^\epsilon \tilde{\mu}^{2\epsilon}}{2p^t}\bar{u}(p)\int \frac{d^dk}{(2\pi)^d}(-igT^a\gamma^\mu)\frac{i\not{k}}{k^2}\gamma^t\frac{i\not{k}}{k^2}(-igT^a\gamma^\nu)\frac{-ig_{\mu\nu}}{(p-k)^2}u(p)e^{-ik^z z}$$

$$\tilde{Q}^{(1)}_{w.f.}(z,p^z,\epsilon,\tilde{\mu}) = -\frac{\alpha_s C_F}{4\pi}\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right)e^{-ip^z z}$$

$\frac{1}{\epsilon_{IR}}$ should match the PDF IR singularity (factorization requirement). PDF contribution obtained by taking p^z to infinity before p^x,p^y integration.

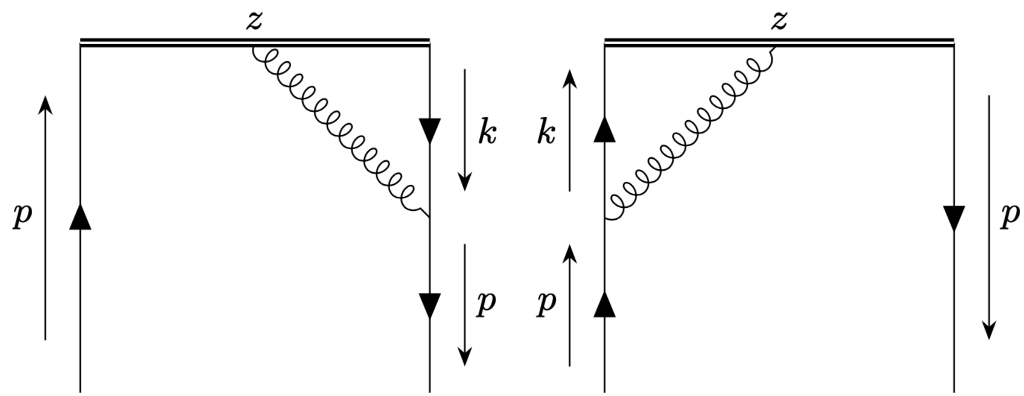


$$\tilde{Q}^{(1)}_{tadpole}(z, p^z, \epsilon, \tilde{\mu})$$

$$= \frac{\iota^\epsilon \tilde{\mu}^{2\epsilon}}{2p^t} \bar{u}(p) \int \frac{d^d k}{(2\pi)^d} (-g^2) C_F \gamma^t \delta^\mu_z \delta^\nu_z \underbrace{\left(\frac{e^{-ip^z z} - e^{-ik^z z}}{(p^z - k^z)^2} - \frac{z e^{-ip^z z}}{i(p^z - k^z)} \right)} \frac{-i g_{\mu\nu}}{(p-k)^2} u(p)$$

$$e^{-ip^z z} \int_0^z dz_1 \int_0^{z_1} dz_2 A_z(z_1) A_z(z_2)$$

$$\to e^{-ip^z z} \int \frac{d^d k}{(2\pi)^d} \int_0^z dz_1 \int_0^{z_1} dz_2 \frac{e^{-i(k^z - p^z)(z_1 - z_2)}}{(p-k)^2}$$



$$\tilde{Q}^{(1)}_{sail}(z, p^z, \epsilon, \tilde{\mu}) =$$

$$\iota^{\epsilon}\tilde{\mu}^{2\epsilon} \frac{}{2p^t}\bar{u}(p)\int \frac{d^dk}{(2\pi)^d}\Bigg\{(igT^a\gamma^t)\frac{1}{i(p^z-k^z)}\left(e^{-ip^z z}-e^{-ik^z z}\right)\delta^{\mu}_{z}\frac{i\not{k}}{k^2}(-igT^a\gamma^{\nu})\frac{-ig_{\mu\nu}}{(p-k)^2}$$

$$+(-igT^a\gamma^{\nu})\frac{i\not{k}}{k^2}(igT^a\gamma^t)\frac{1}{i(p^z-k^z)}\left(e^{-ip^z z}-e^{-ik^z z}\right)\delta^{\mu}_{z}\frac{-ig_{\mu\nu}}{(p-k)^2}\Bigg\}u(p),$$

$$e^{-ip^z z}\int_0^z dz_1 A_z(z_1)$$

$$\rightarrow e^{-ip^z z}\int_0^z dz_1 e^{i(p^z-k^z)z_1}$$

Ratio Scheme

$$\frac{\langle P|\bar{O}_q^B(z)|P\rangle}{\langle P=0|\bar{O}_q^B(z)|P=0\rangle} = \frac{\langle P|\bar{O}_q^R(z)|P\rangle}{\langle P=0|\bar{O}_q^R(z)|P=0\rangle}$$

- Short distance ln z dependence cancelled by the counterterm. The long distance one, however, survived. The counterterm has non-perturbative IR effects. Need to change to the Wilson line mass subtraction scheme at long distance. This scheme is the hybrid-ratio scheme.

$$\tilde{q}^X(x, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dy}{|y|} C^X\left(\frac{x}{y}, y, \tilde{\mu}, \mu, P^z\right) q^{\overline{\mathrm{MS}}}(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{QCD}^2}{P_z^2}\right), \quad ($$

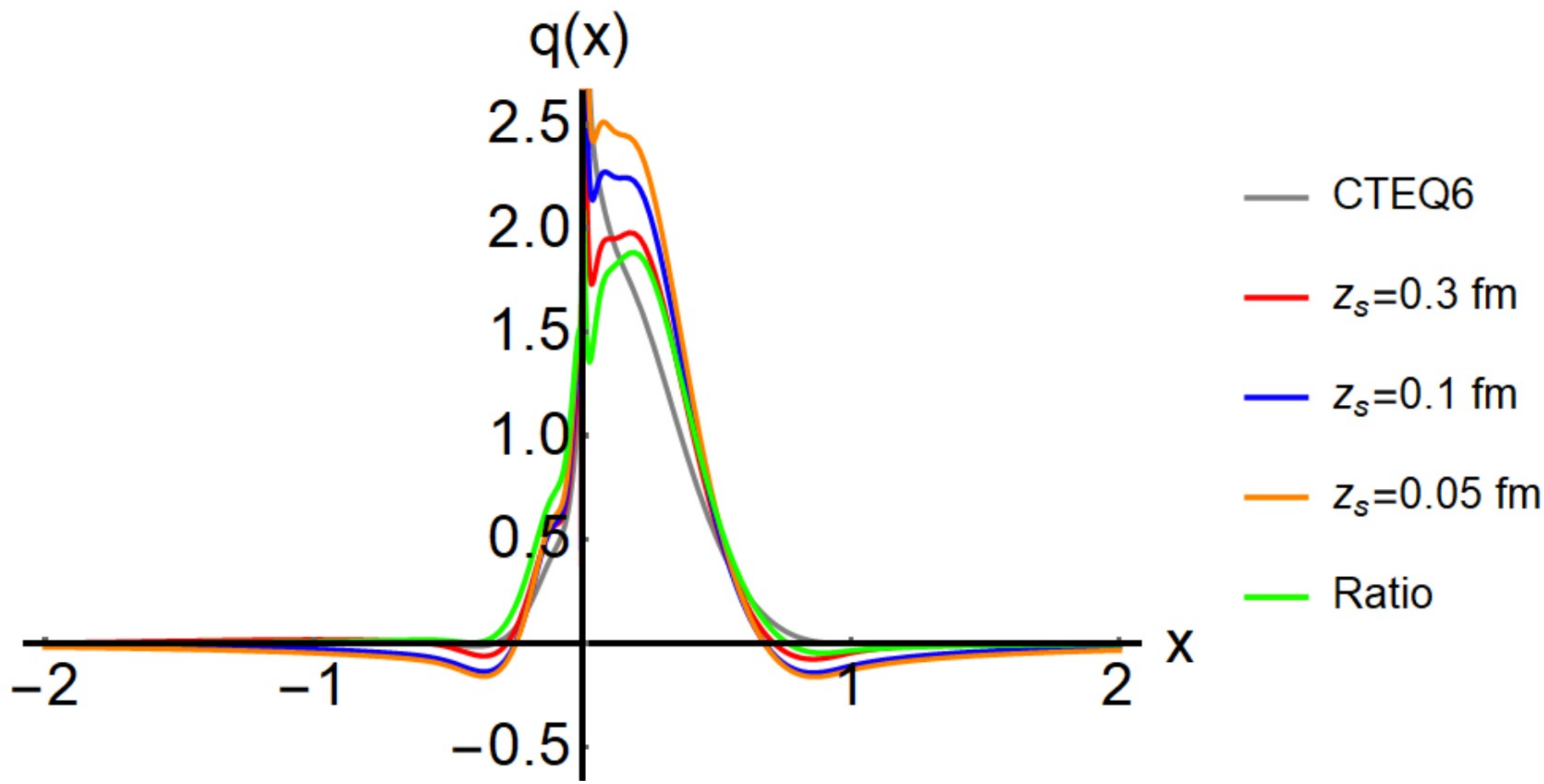
$$C_{\gamma^t}^{\text{hybrid-ratio}}\left(\xi, yz_sP^z, \frac{\mu}{yP^z}\right)$$

$$=\delta(1-\xi) + \frac{\alpha_s C_F}{2\pi}\begin{cases} \left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1\right)_{+(1)}^{[1,\infty]} & ,\xi > 1 \\ \left(\frac{1+\xi^2}{1-\xi}\left(-\ln\frac{\tilde{\mu}^2}{y^2P_z^2}+\ln 4\xi(1-\xi)-1\right)+1\right)_{+(1)}^{[0,1]} & ,0<\xi<1 \\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi}-1\right)_{+(1)}^{[-\infty,0]} & ,\xi<0 \end{cases}$$

$$+\frac{3\alpha_s C_F}{2\pi^2}\left(\frac{\mathrm{Si}((1-\xi)|y|z_sP^z)}{(1-\xi)}\right)_{+(1)}^{[-\infty,\infty]} + \mathcal{O}(\alpha_s^2), \quad ($$

CY Chou, JWC, 2204.8343

$$(f(x))_{+(c)}^{[a,b]} \equiv \int_a^b dx(f(x)-f(c)), \qquad \mathrm{Si}(x) \equiv \int_0^x \frac{\sin t}{t}dt$$



CY Chou, JWC, 2204.8343

Renormalon Ambiguity in LaMET

In an OPE, one

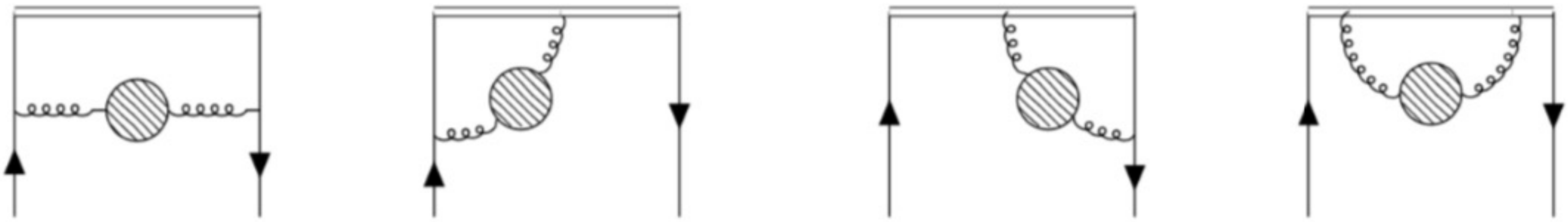
(1) uses Borel transform to improve the convergence of the Wilson coefficients

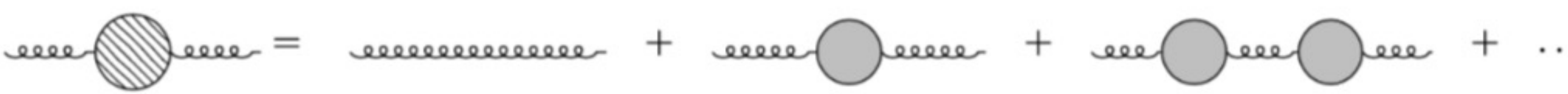
(2) sums the series

(3) then performs inverse Borel transform. Poles in the integrant (renormalons) lead to ambiguity in the contour integrals which can be absorbed by the power corrections.

(4) Braun, Vladimirov and Zhang (1810.00048): power corrections enhanced at end point $\mathcal{O}\left(\frac{\Lambda^2}{p^2 x^2 (1-x)}\right)$

Studied by bubble chain diagrams

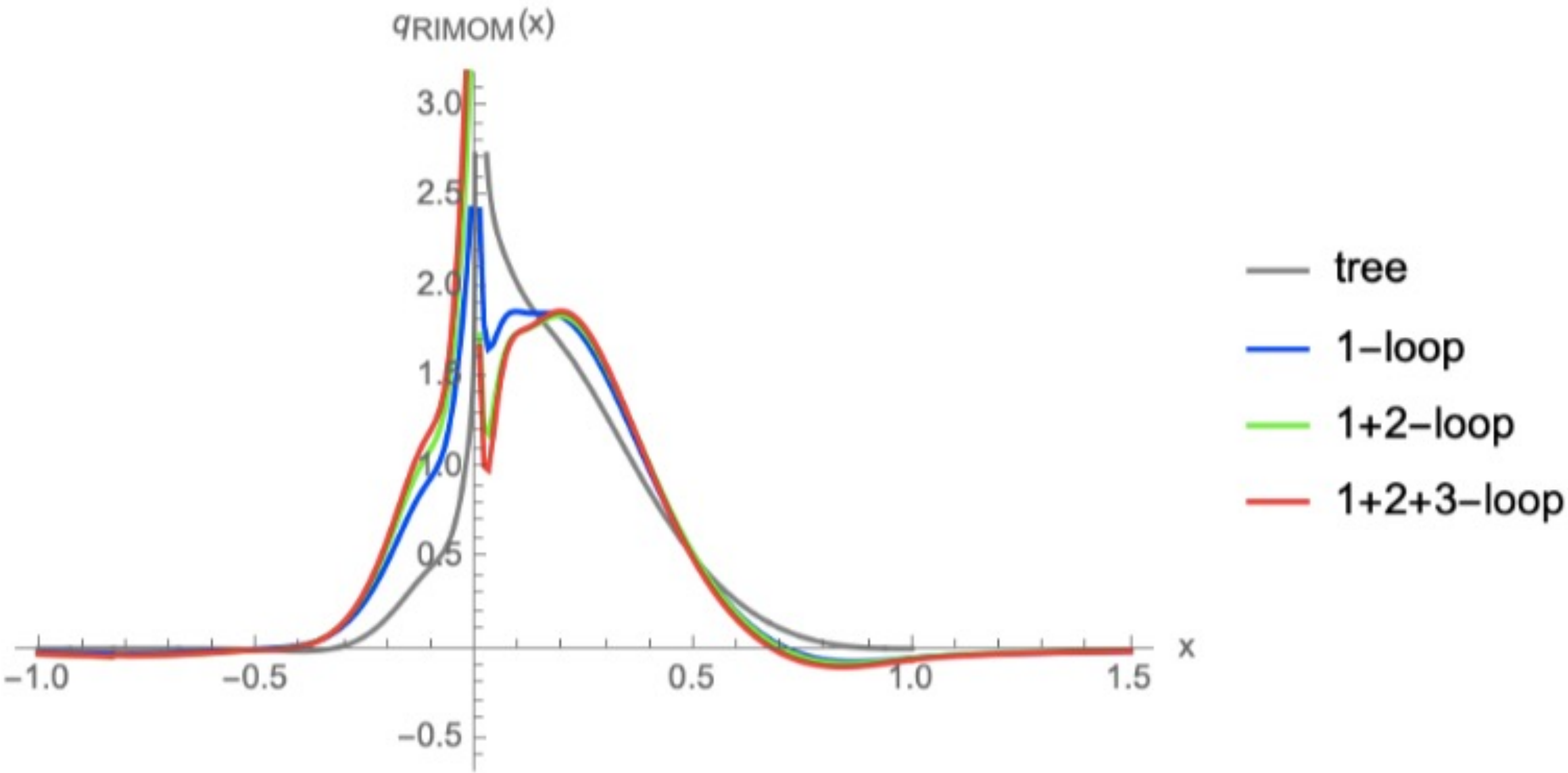




$$\alpha_s n_f \quad \text{as} \quad \mathcal{O}(1)$$

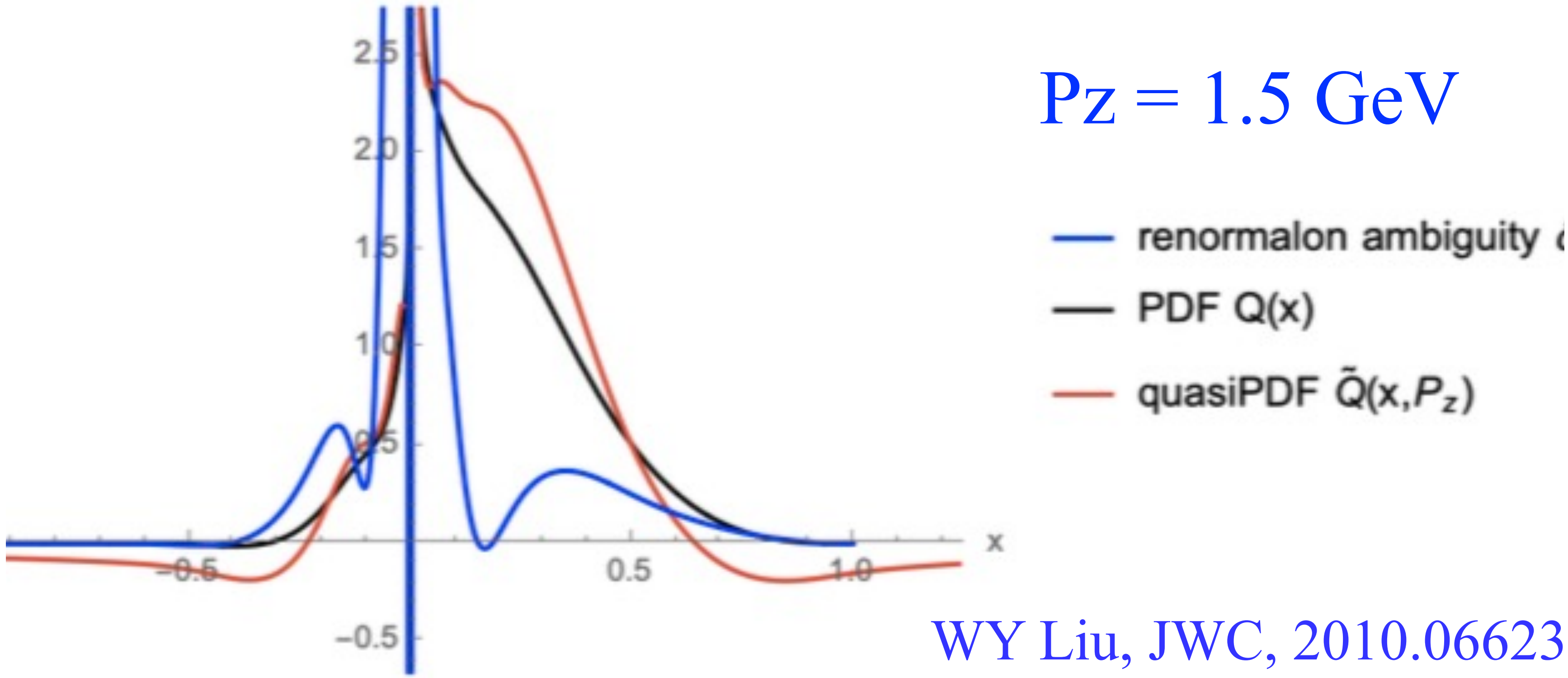
$$\frac{1}{-k^2 - i\epsilon} \to \frac{(\Lambda^2_{\mathrm{QCD}})^w}{(-k^2 - i\epsilon)^{1+w}}$$

Bubble diagram contribution up to 3-loops (RI/MOM to MS-bar)



WY Liu, JWC, 2010.06623

Power Corrections suggested by Renormalon Ambiguity



Pz = 1.5 GeV

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Can this be confirmed by Pz dependence?