Parton Distributions on a Euclidean Lattice---the LaMET Method

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Parton Model

- Leading twist t (t = 2) contribution in QCD
- Factorization

$$\sigma(eP \to e'X) = \int_0^1 dx f_i(x,\mu)\sigma(ei \to e'i',\mu)$$

$$\Lambda_{QCD} \qquad\qquad Q$$

- Expansion in powers of Λ_{QCD}/Q

  Power correction

$$\left(\frac{\Lambda_{QCD}}{Q}\right)^{t-2}$$

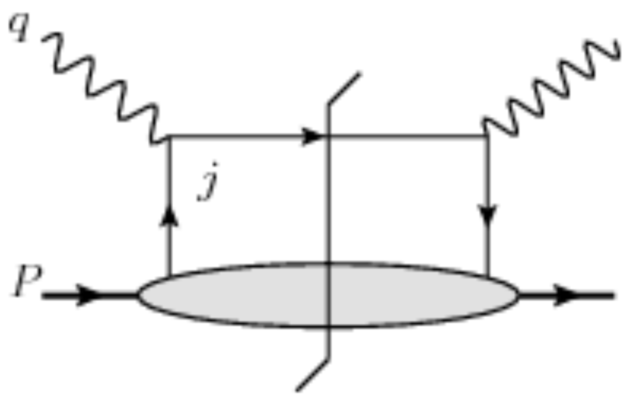
Main result

- Twist-2 matrix element

$$\langle P|\hat{O}^{\mu_1\cdots\mu_n}(x)|P\rangle = 2\langle x^{n-1}\rangle P^{(\mu_1}\ldots P^{\mu_n)}$$

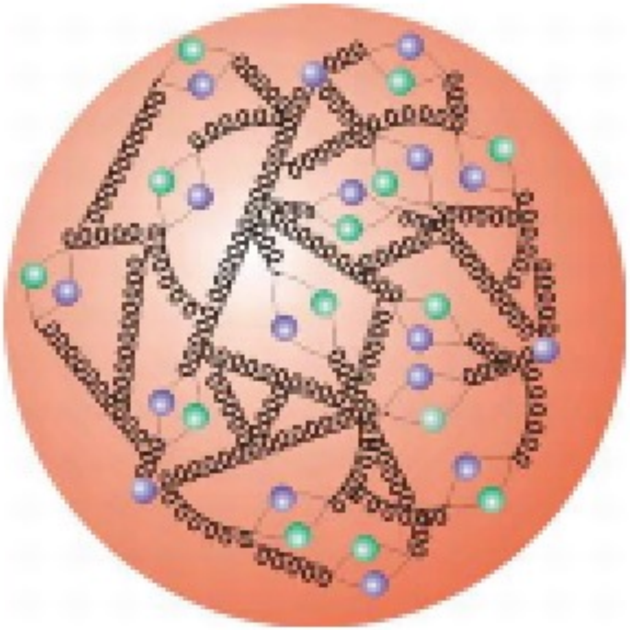
Wilson line

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P\,|\overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda)|\,P\rangle$$



The struck parton moves on a light cone yields a sum of twist-2 operators after OPE.

Golden Age for Proton Structure Studies



Parton structures: 1d mom+spin PDF to 3d GPD & TMD to Wigner (and beyond?) [BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …] to applications (Higgs, new physics…)

PDFs from QCD---Why is it so hard?

- The number of quark anti-quark pairs diverges (manifestation of non-perturbative nature of the problem): .

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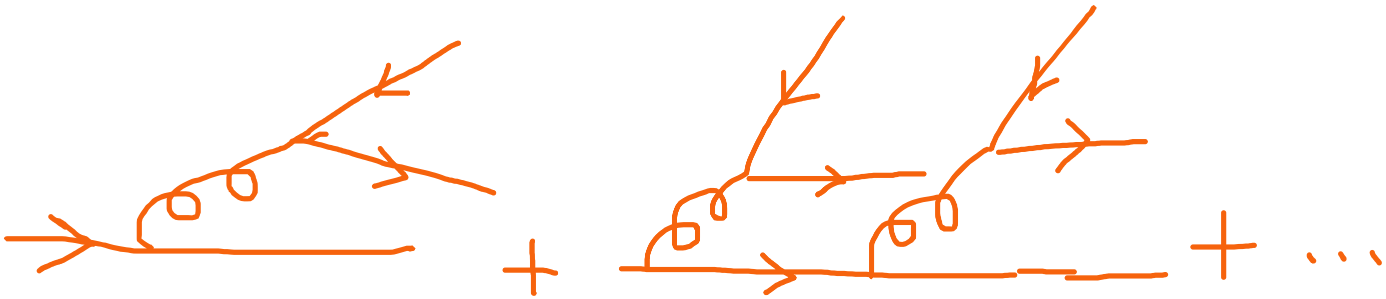

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PDF $\propto \delta(x)$. While short distance expansion is natural in Euclidean space, lightcone expansion is not.

PDFs from QCD

- Moments of PDF given by local twist-2 operators (twist = dim - spin); limited to first few moments but carried out successfully

$$\langle x^n \rangle$$

Beyond the first few moments

- Smeared sources: Davoudi & Savage
- Gradient flow: Monahan & Orginos
- Current-current correlators: K.-F. Liu & S.-J. Dong; Braun & Müller; Detmold & Lin; QCDSF; Qiu & Ma
- Xiangdong Ji (Phys. Rev. Lett. 110 (2013) 262002): quasi-PDF: computing the x-dependence directly. (variation: pseudo-PDF, Radyushkin; w/ Karpie, Orginos, Zafeiropoulos)

Ji's observation

For a twist-2 operator,

$$\hat{\mathcal{O}}^{\mu\nu} = \bar{\psi}(\frac{i\gamma^{\mu}D^{\nu} + i\gamma^{\nu}D^{\mu}}{2} - \frac{i}{4}g^{\mu\nu}\not{D})\psi$$

$$\langle P|\hat{\mathcal{O}}^{zz}|P\rangle = 2\langle x\rangle_q(P^zP^z - \frac{1}{4}g^{zz}M^2)$$

$$\langle P|\bar{\psi}i\gamma^z D^z\psi|P\rangle = 2\langle x\rangle_q P^z P^z \left[1 + \mathcal{O}(\frac{M^2}{P_z^2}) + \mathcal{O}(\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2})\right]$$

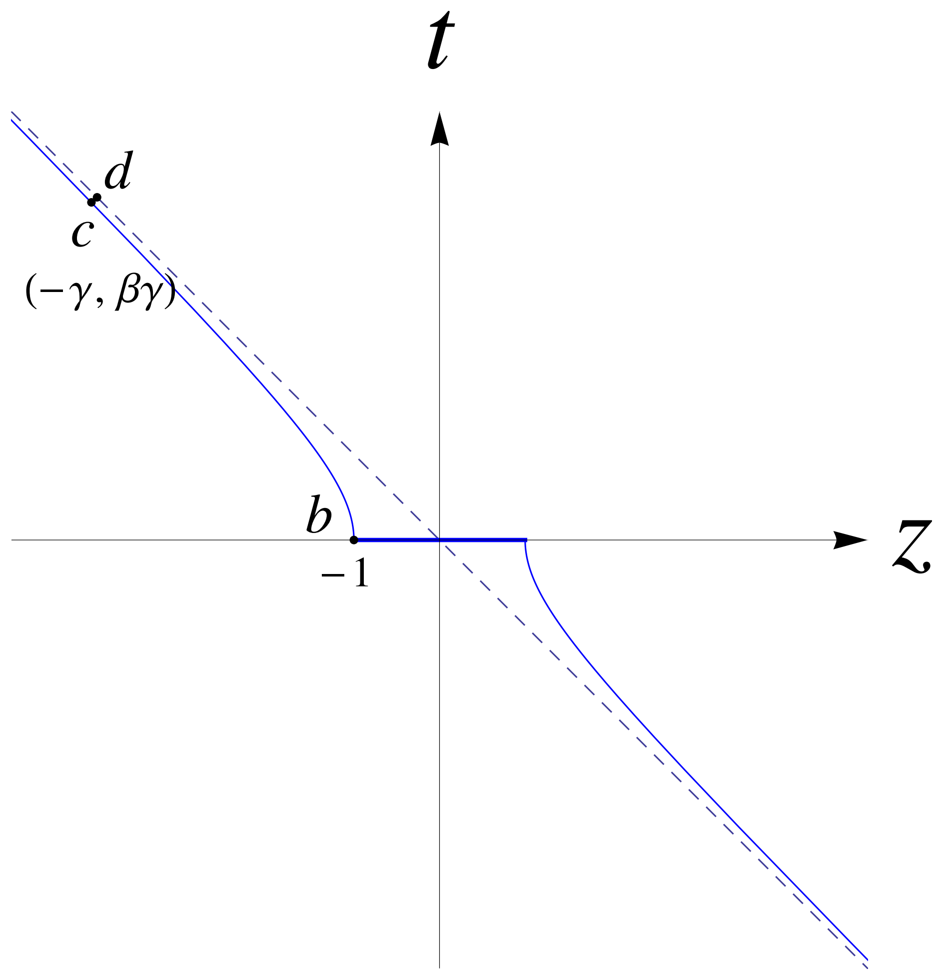
An equal time quark bilinear operator along the z direction gives the PDF of a proton moving in the z direction?

Ji's idea

- Quark PDF in a proton: $(\lambda^2 = 0)$

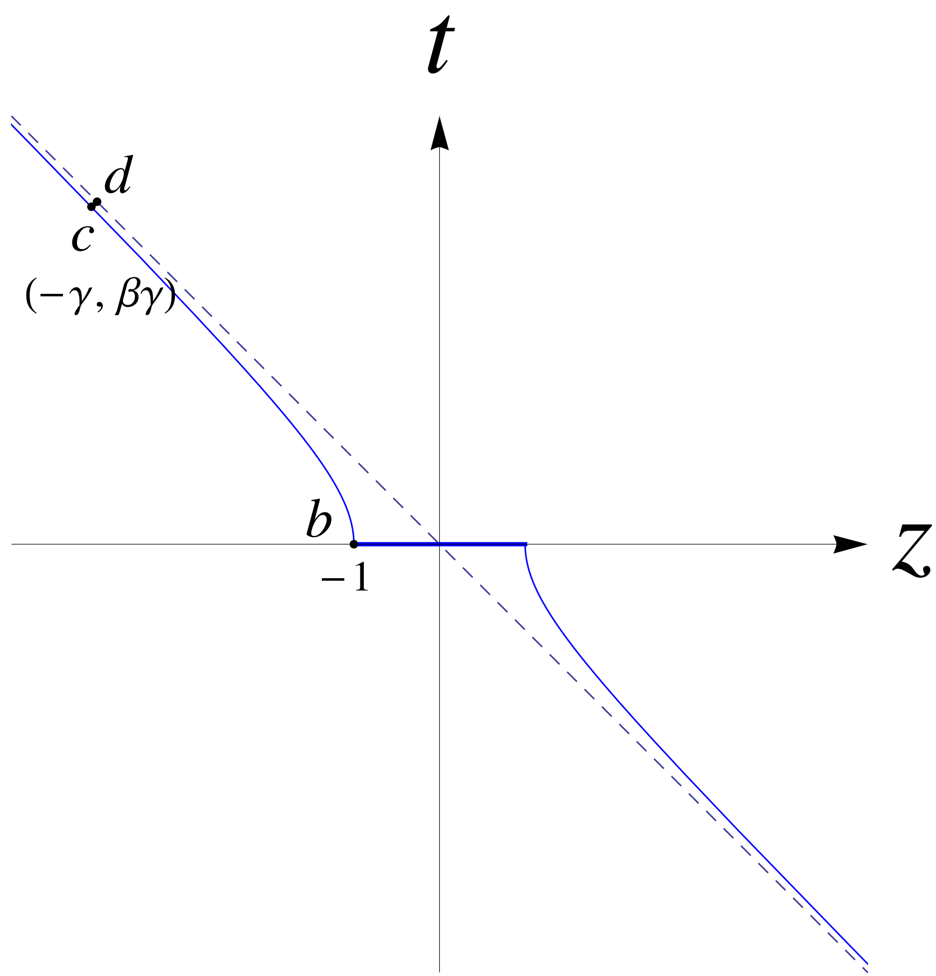
$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \,|\overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda)|\, P\rangle$$

- Boost invariant in the z-direction, rest frame OK
- Quark bilinear op. always on the light cone
- What if the quark bilinear is slightly away from the light cone (space-like) in the proton rest frame?



t
d
c
(−γ, βγ)
b
−1
Z

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.



- Analogous to HQET: need power corrections & matching---LaMET (Large Momentum Effective Theory)

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$$\widetilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{-ixzP^z} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(z\lambda) \right| P \right\rangle$$

$$\lambda^\mu = (0,0,0,1)$$

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2},\frac{M^2}{P_z^2}\right) + \ldots$$

UV, PQCD

Comment: Boosting makes the short distance expansion look bigger---physics depends on z Pz!

Power Corrections

JWC, Cohen, Ji, Lin, Zhang, hep-ph/1603.06664

M^2/P_z^2 corrections to all orders

$$\lambda_{\mu_1}\cdots\lambda_{\mu_n}P^{(\mu_1}\dots P^{\mu_n)}=\lambda_{(\mu_1}\cdots\lambda_{\mu_n)}P^{\mu_1}\dots P^{\mu_n}$$

(...) means the indices enclosed are symmetric and traceless

$$\lambda_{(\mu_1}\cdots\lambda_{\mu_n)}=\sum_{i=0}^{i_{\max}}B_{n,i}\left(\lambda^2\right)^i\left(\frac{\partial^2}{\partial\lambda_\alpha\partial\lambda^\alpha}\right)^i\lambda_{\mu_1}\cdots\lambda_{\mu_n}$$

$$i_{\max}=\frac{n-\text{Mod}[n,2]}{2}\ \text{ and }\ B_{n,0}=1$$

- $$g^{\mu_1\mu_2}P^{\mu_3}\dots P^{\mu_n}\lambda_{(\mu_1}\cdots\lambda_{\mu_n)}=0,$$

$$\sum_{i=0}^{i_{\max}}B_{n,i}\left(\lambda^2\right)^i\left(\frac{\partial^2}{\partial\lambda_\alpha\partial\lambda^\alpha}\right)^i\lambda^2\left(\lambda\cdot P\right)^{n-2}=0.$$

M^2/P_z^2 corrections to all orders

$$B_{n,i} = -\frac{B_{n,i-1}}{4i(n-i+1)}$$

$$(P^z P^z - \frac{1}{4} g^{zz} M^2)$$

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M^2/P_z^2 corrections to all orders

$$K_n \equiv \frac{\left\langle x^{n-1}\right\rangle_{\tilde{q}}}{\left\langle x^{n-1}\right\rangle_q} = \frac{\lambda_{(\mu_1}\cdots\lambda_{\mu_n)}P^{\mu_1}\ldots P^{\mu_n}}{\lambda_{\mu_1}\cdots\lambda_{\mu_n}P^{\mu_1}\ldots P^{\mu_n}}$$

$$= \sum_{i=0}^{i_{\max}} C_{n-i}^{i} c^{i},$$

where C is the binomial function and $c = -\lambda^2 M^2/4\left(\lambda \cdot P\right)^2$

$= M^2/4P_z^2$ with $\lambda^\mu = (0,0,0,-1)$ and $\lambda \cdot P = P_z$.

M^2/P_z^2 corrections to all orders

$$q(x) = \sqrt{1+4c}\sum_{n=0}^{\infty}\frac{f_-^n}{f_+^{n+1}}\left[(1+(-1)^n)\tilde{q}\left(\frac{f_+^{n+1}x}{2f_-^n}\right)+(1-(-1)^n)\tilde{q}\left(\frac{-f_+^{n+1}x}{2f_-^n}\right)\right]$$

$$= \sqrt{1+4c}\sum_{n=0}^{\infty}\frac{(4c)^n}{f_+^{2n+1}}\left[(1+(-1)^n)\tilde{q}\left(\frac{f_+^{2n+1}x}{2(4c)^n}\right)+(1-(-1)^n)\tilde{q}\left(\frac{-f_+^{2n+1}x}{2(4c)^n}\right)\right]$$

$$f_\pm = \sqrt{1+4c}\pm 1$$

$\Lambda^2_{\mathbf{QCD}}/P_z^2$ correction

$$\tilde{q}(x,\Lambda,P_z) \to \tilde{q}(x,\Lambda,P_z) + \tilde{q}_{\text{twist-4}}(x,\Lambda,P_z)$$

$$\tilde{q}_{\text{twist-4}}(x,\Lambda,P_z) = \frac{1}{8\pi}\int_{-\infty}^{\infty} dz\,\Gamma_0\left(-ixzP_z\right)\langle P\left|\mathcal{O}_{\text{tr}}(z)\right|P\rangle$$

$$\mathcal{O}_{\text{tr}}(z) = \int_0^z dz_1\,\bar{\psi}(0)\Big[\gamma^\nu\Gamma\left(0,z_1\right)D_\nu\Gamma\left(z_1,z\right) + \int_0^{z_1} dz_2\,\lambda\cdot\gamma\Gamma\left(0,z_2\right)D^\nu\Gamma\left(z_2,z_1\right)D_\nu\Gamma\left(z_1,z\right)\Big]\psi(z\lambda).$$

Γ_0 is the incomplete Gamma function $\displaystyle\int_0^1 \frac{dt}{t}e^{ix/t} = \Gamma_0\left(-ix\right)$

Power divergent mixing with $\tilde{q}$ on the lattice?

Next Lecture: Matching Coefficients