Pursuit of Subatomic Structures---
a Family of Parton Distributions

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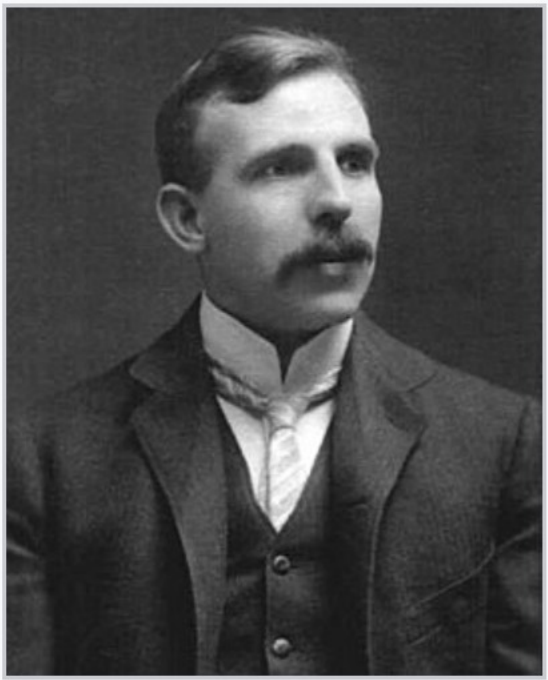
Rutherford’s Atomic Model



Ernest Rutherford

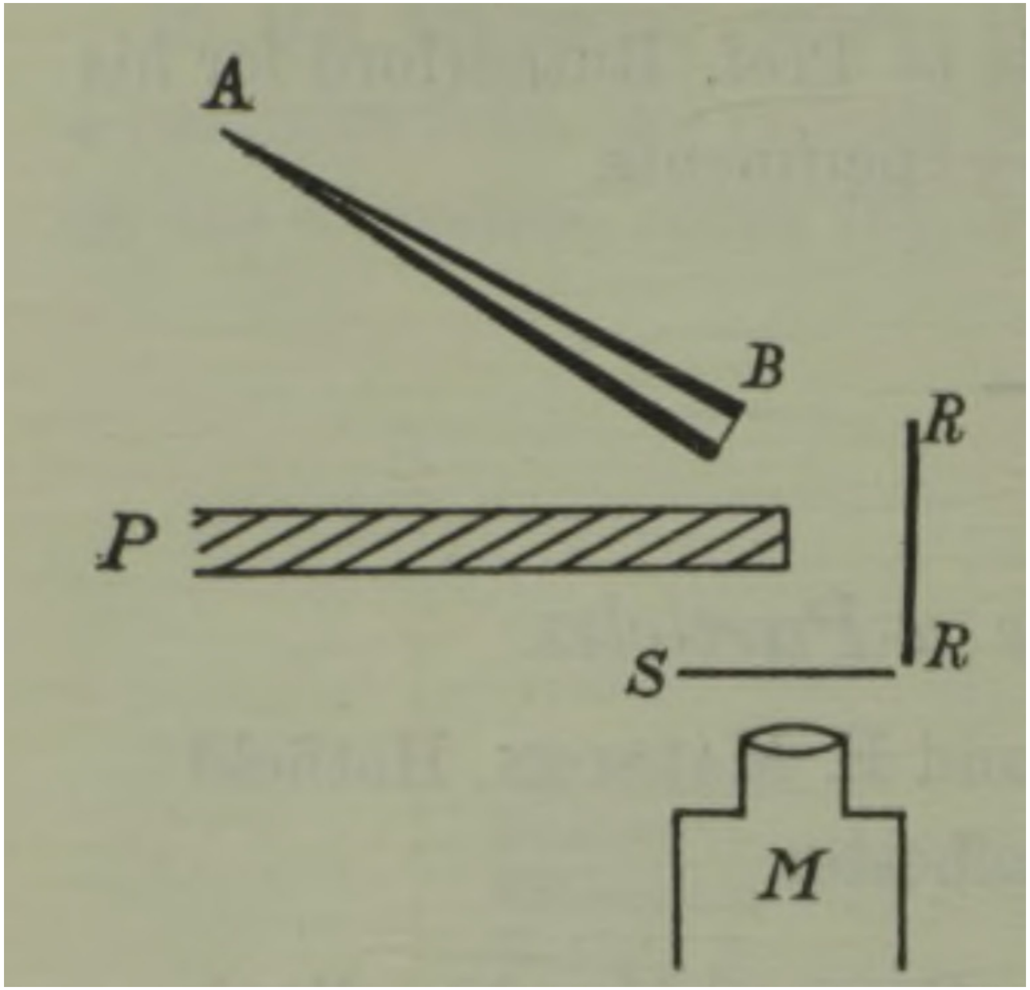
Hans Geiger

Ernest Marsden

Rutherford's Atomic Model

Figure 2: Rutherford's group at Manchester University in 1912. Rutherford is seated second row, center. Also present: C. G. Darwin, J. M. Nuttall, J. Chadwick; H. Geiger, H. G. J. Moseley, and E. Marsden.

Alpha-A Large Angle Scattering



Geiger and Marsden (1909)

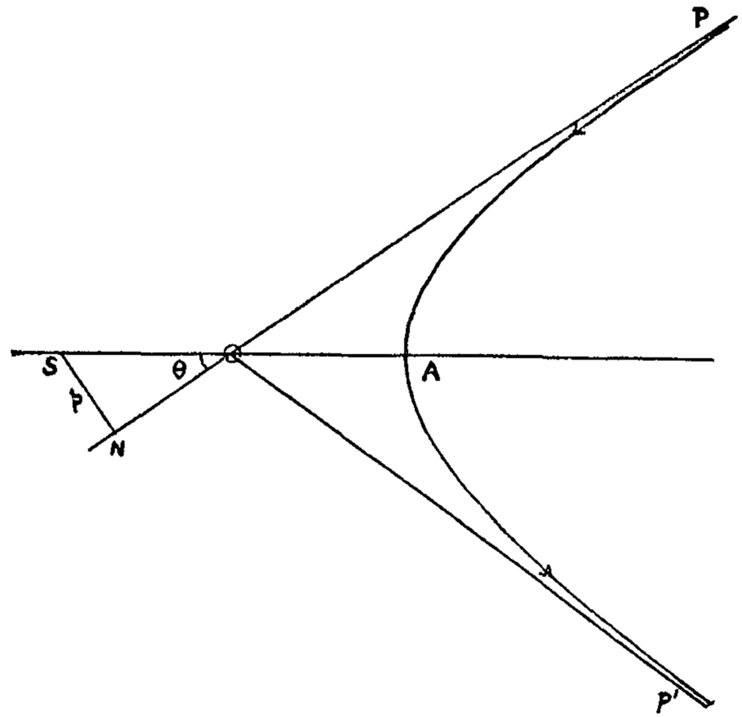
Rutherford:

"It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

[Andrade, 1964].

Rutherford's Atomic Model

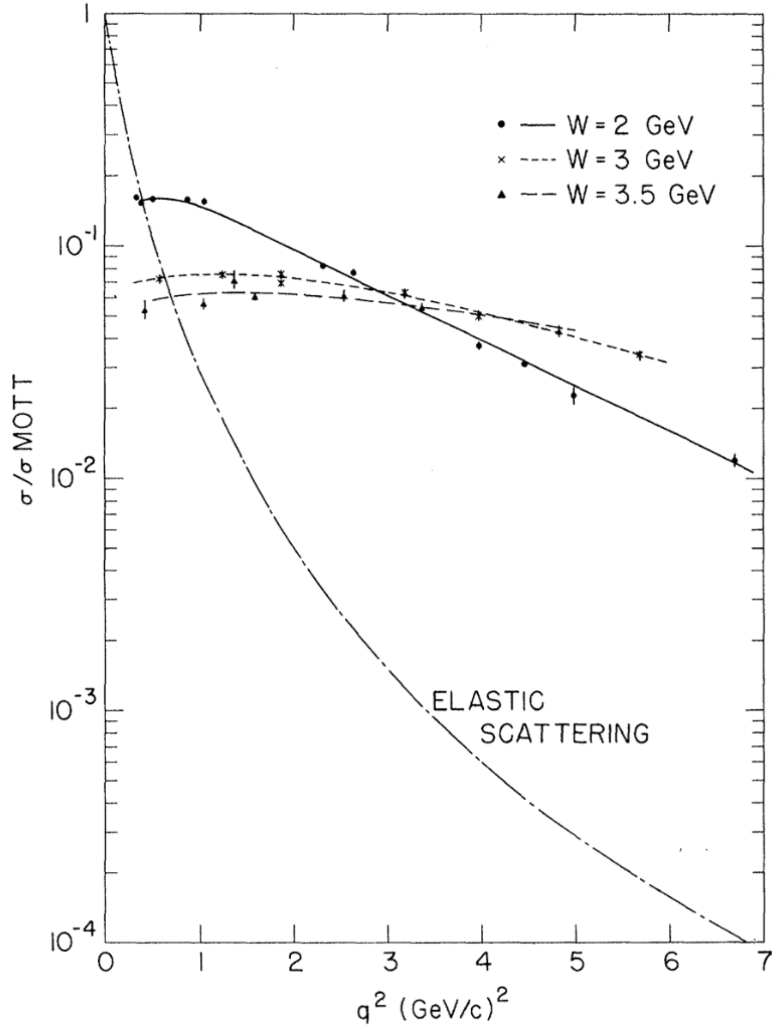
Consider an atom which contains a charge $\pm Ne$ at its centre surrounded by a sphere of electrification containing a charge $\mp Ne$ supposed uniformly distributed throughout a sphere of radius R. e is the fundamental unit of charge,



Rutherford (1911)

Explains Z^2 dependence of deflection rate;
Indirect prediction: nuclear size < 34 fm.

$e + p \to e' + X$ SLAC-MIT Exp.



Breidenbach *et al.* (1969)

The Nobel Prize in Physics 1990

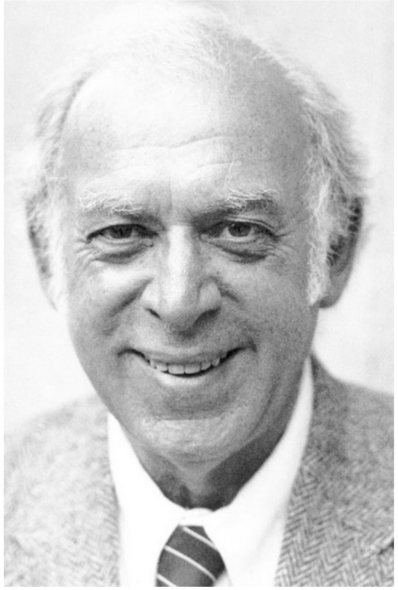
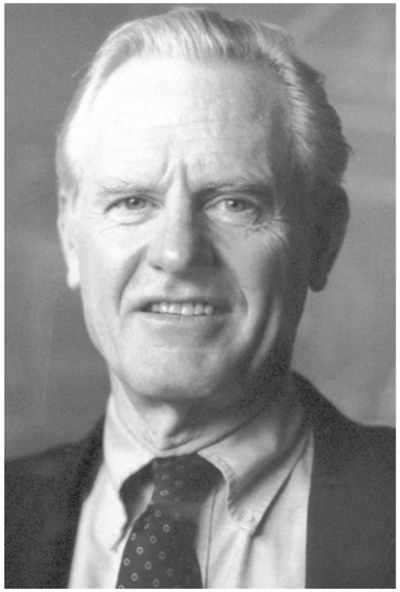


Photo from the Nobel Foundation archive.

Jerome I. Friedman

Prize share: 1/3



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Henry W. Kendall

Prize share: 1/3

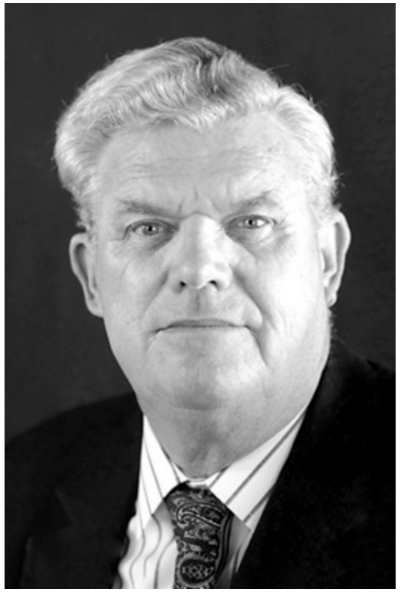


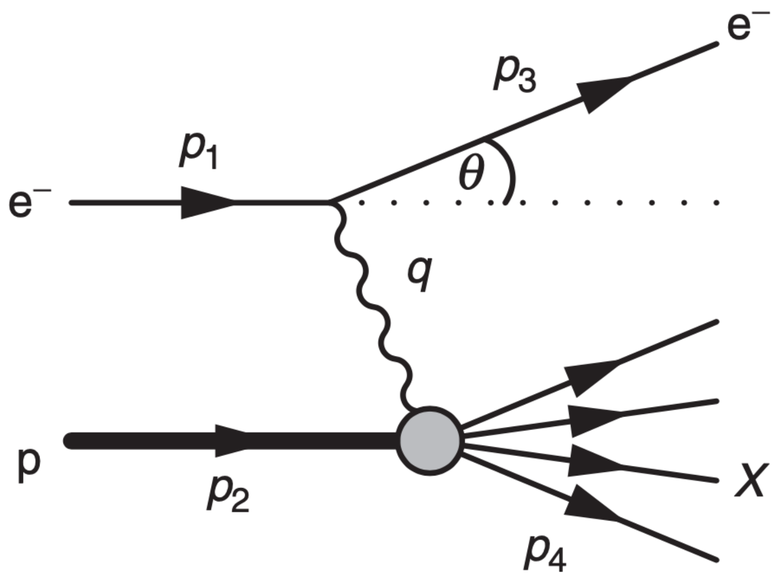
Photo: T. Nakashima

Richard E. Taylor

Prize share: 1/3

Kinematics in $e + p \to e' + X$ scattering

$$q \equiv P_e - P_{e'}$$



$$Q^2 = -q^2 \geq 0$$

$$(P_p + q)^2 = P_X^2$$

$$-Q^2 + 2P_p \cdot q = M_X^2 - M_p^2 \geq 0$$

$$x \equiv \frac{Q^2}{2P_p \cdot q} \equiv \frac{Q^2}{2M_p \nu}$$

$$0 \leq x \leq 1$$

Structure functions: Elastic (Q^2); Inelastic (x, Q^2)

Bjorken Scaling

James D. "BJ" Borken in a photo taken in 1982 at SLAC's 20th Anniversary Conference. Left to right: Edward Leonard Ginzton, Sidney D. Drell, Bjorken, Burton Richter and Wolfgang Kurt Hermann "Pief" Panofsky. (Chuck Painter / Stanford News Service)

Bjorken Scaling

- In Deep Inelastic Scattering (DIS) where

$$Q^2 \to \infty, \quad \nu \to \infty$$

while keeping

$$x = \frac{Q^2}{2M_p\nu}$$

fixed, the structure functions depend on x but not Q^2. This independence of resolution scale implies the processes are governed by point like particles.

Bjorken Scaling

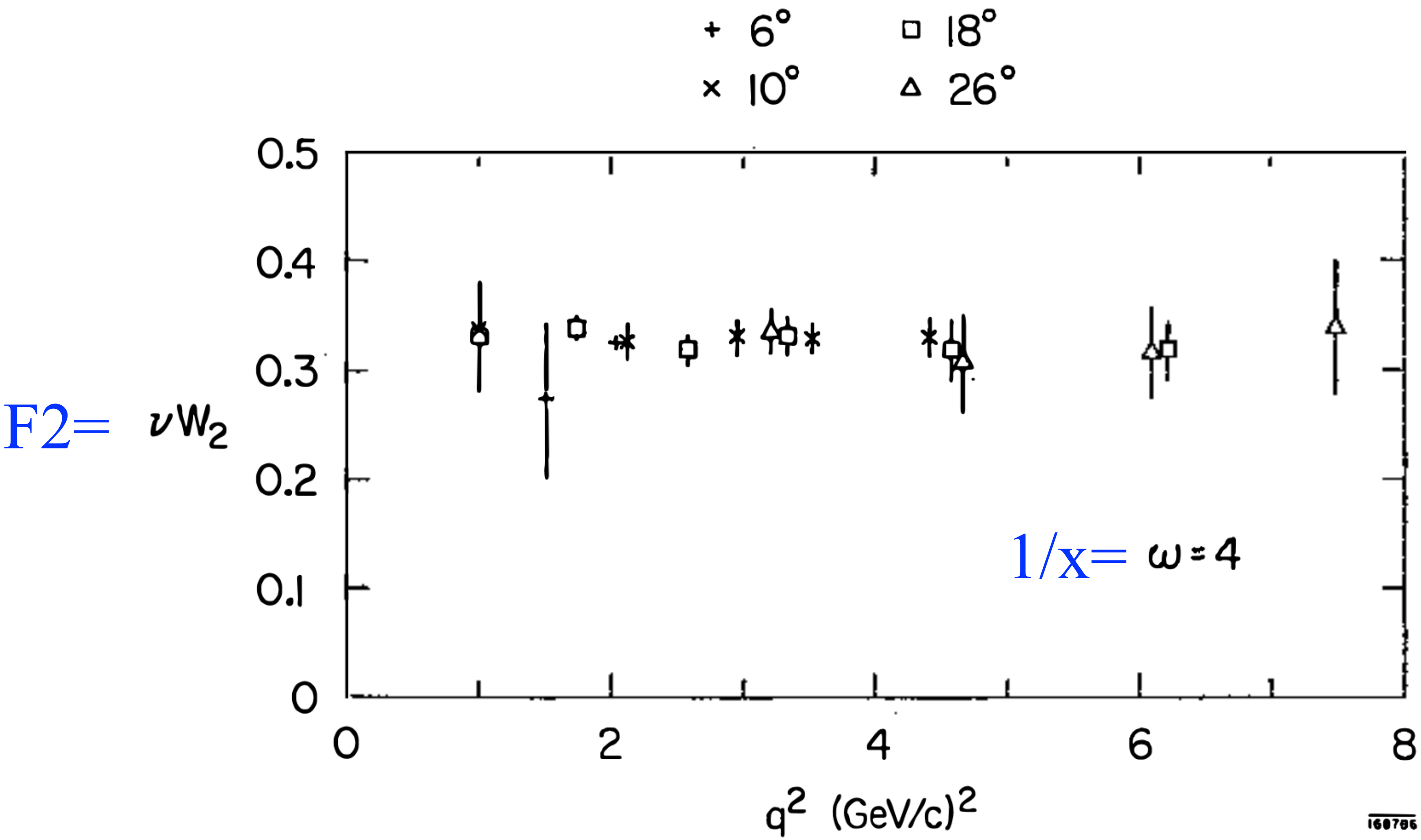
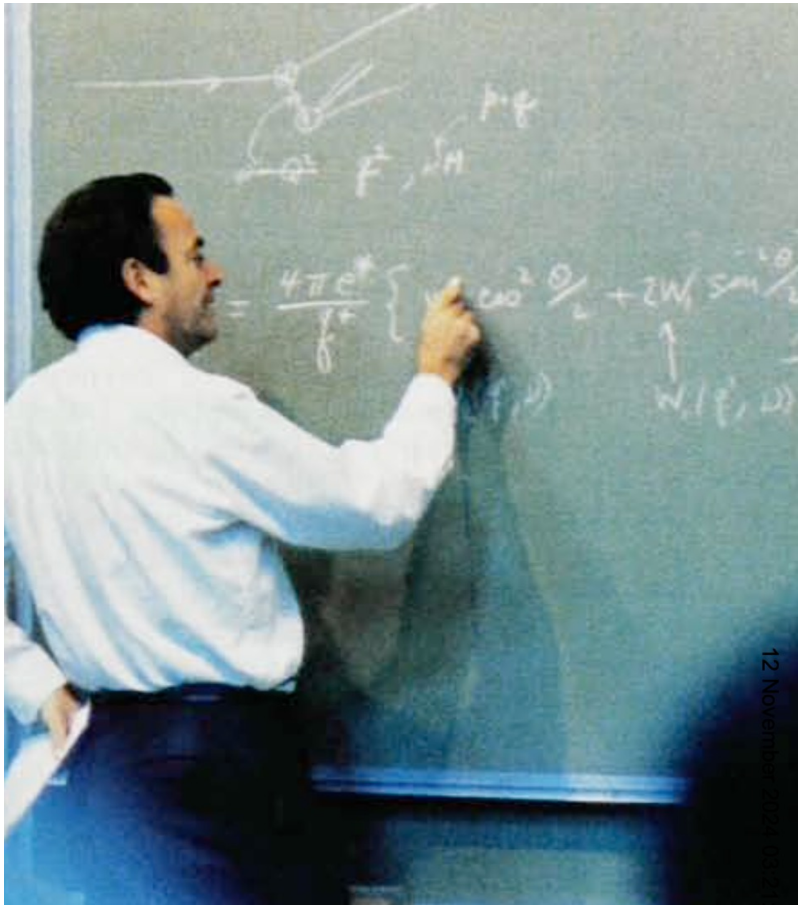


FIGURE 17. νW_2 for the proton as a function of q^2 for $W > 2$ GeV, at $\omega = 4$.

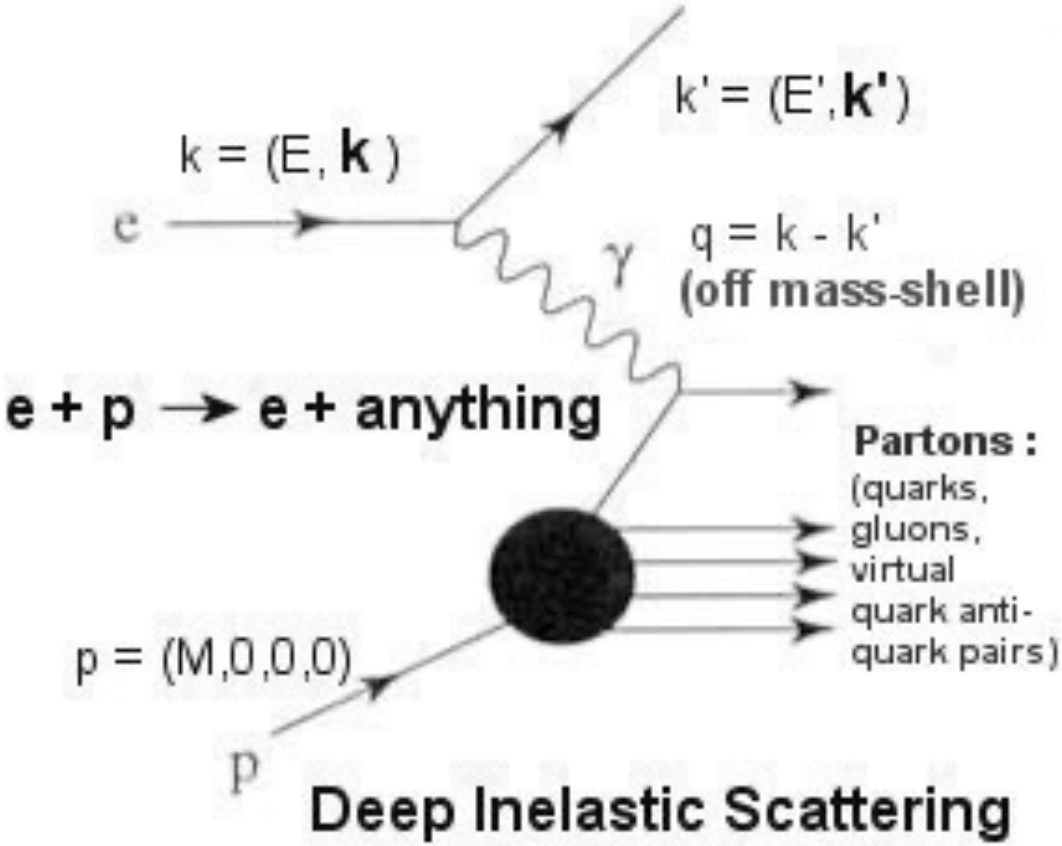
Friedman and Kendall (1972)

Feynman's Parton Model



Feynman's first seminar on parton at SLAC in Oct. 1968.

Feynman's Parton Model



$$(p+q)^2 = p'^2$$

$$-Q^2 + 2p \cdot q = 0$$

$$p = xP_p$$

$$x = \frac{Q^2}{2M_p \nu}$$

$$\sigma(eP \to e'X) = \int_0^1 dx f_i(x) \sigma(ei \to e'i')$$

But why…

- Incoherent sum
- No final state interaction. But no free partons detected experimentally
- Bjorken scaling exact?
- Factorization

Twist-Expansion: OPE in DIS

(Note: Largely follows Schwartz's QFT book)

$$\left(\frac{d\sigma}{d\Omega\, dE'}\right)_{\text{lab}} = \frac{\alpha_e^2}{4\pi m_p q^4}\frac{E'}{E}L^{\mu\nu}W_{\mu\nu}$$

$$W^{\mu\nu} = 2\mathrm{Im}T^{\mu\nu}$$

$$T_{\mu\nu}(\omega, Q) = i\int d^4x\, e^{iq\cdot x}\langle P|T\{J_\mu(x)J_\nu(0)\}|P\rangle$$

Bjorken limit $Q^2 \to \infty, \;\; \nu \to \infty, \;\; \frac{Q^2}{\nu}\, fixed$ implies

$$\frac{|q|}{\nu} \to 1 \;\; or \;\; x^2 \to 0$$

a lightcone expansion or twist expansion rather than a short distance expansion $(x^\mu \to 0)$

OPE and EFT

- OPE (Operator Product Expansion): low energy EFT (effective field theory) in coordinate space, e.g. EFT of W boson exchange

$$\mathcal{L}_W \sim g^2 \int d^4x\, d^4y\, \bar{\psi}(x)\gamma^\mu\psi(x)\, D^{\mu\nu}(x,y)\, \bar{\psi}(y)\gamma^\nu\psi(y)$$
$$\sim G_F \int d^4x \left[\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma^\mu\psi - \bar{\psi}\gamma^\mu\psi\frac{\Box}{m_W^2}\bar{\psi}\gamma^\mu\psi + \bar{\psi}\gamma^\mu\psi\frac{\Box^2}{m_W^4}\bar{\psi}\gamma^\mu\psi + \cdots\right]$$

- OPE:

$$\lim_{x\to y} \mathcal{O}_1(x)\mathcal{O}_2(y) = \sum_n C_n(x-y)\,\mathcal{O}_n(x)$$

- Now back to DIS:

$$i\int d^4x\, e^{iqx}\langle p|T\{J^\mu(x)J^\nu(0)\}|p\rangle$$ (a quark external state)

$$= -\bar{u}(p)\frac{\gamma^\mu(\not{p}+\not{q})\gamma^\nu}{(p+q)^2+i\varepsilon}u(p) - \bar{u}(p)\frac{\gamma^\nu(\not{p}-\not{q})\gamma^\mu}{(p-q)^2+i\varepsilon}u(p)$$

$$= \frac{1}{Q^2}\bar{u}(p)\gamma^\mu(\not{p}+\not{q})\gamma^\nu u(p)\sum_{n=0}^{\infty}\left(\frac{2p\cdot q+\cancel{p^2}}{Q^2}\right)^n + \cdots$$

$$= \frac{2}{Q^2}\bar{u}(p)(p^\mu\gamma^\nu+\gamma^\mu p^\nu)\sum_{n=0,2,\cdots}^{\infty}\left(\frac{2q\cdot p}{Q^2}\right)^n u(p)$$

$$- g^{\mu\nu}\frac{2}{Q^2}\bar{u}(p)\not{q}\sum_{n=1,3,\cdots}^{\infty}\left(\frac{2q\cdot p}{Q^2}\right)^n u(p)$$

$$\frac{1}{(p+q)^2} = \frac{1}{-Q^2+2q\cdot p+p^2} = -\frac{1}{Q^2}\sum_{n=0}^{\infty}\left(\frac{2p\cdot q+\cancel{p^2}}{Q^2}\right)^n$$

$$\gamma^\mu(\not{p}+\not{q})\gamma^\nu + \gamma^\nu(\not{p}+\not{q})\gamma^\mu = 2\gamma^\mu(p^\nu+\cancel{q^\nu}) + 2\gamma^\nu(p^\mu+\cancel{q^\mu}) - 2g^{\mu\nu}(\cancel{\not{p}}+\not{q})$$

$$
\begin{aligned}
i \int d^4x\, e^{iqx} \langle p|T\{J^\mu(x) J^\nu(0)\}|p\rangle \\
&= \frac{2}{Q^2}\bar{u}(p)(p^\mu\gamma^\nu + \gamma^\mu p^\nu) \sum_{n=0,2,\cdots}^{\infty} \left(\frac{2q\cdot p}{Q^2}\right)^n u(p) \\
&- g^{\mu\nu}\, \frac{2}{Q^2}\bar{u}(p)\not{q} \sum_{n=1,3,\cdots}^{\infty} \left(\frac{2q\cdot p}{Q^2}\right)^n u(p)
\end{aligned}
$$

$$p_\mu \to i\partial_\mu \to iD_\mu$$

Hadron matrix element largest when the indexes (enclosed by (…)) are symmetric and traceless

$$\hat{\mathcal{O}}^{\mu_1\cdots\mu_n}(x) = \bar{\psi}_q(x)\gamma^{(\mu_1} iD^{\mu_2}\cdots iD^{\mu_n)}\psi_q(x)$$

$$\langle P|\hat{\mathcal{O}}^{\mu_1\cdots\mu_n}(x)|P\rangle = 2\,\mathcal{A}_q^n P^{\mu_1}\cdots P^{\mu_n} - \text{traces},$$

Example

- Quark twist-2 operator of spin-2 (quark energy momentum tensor)

$$\hat{\mathcal{O}}^{\mu\nu} = \bar{\psi}(\frac{i\gamma^\mu D^\nu + i\gamma^\nu D^\mu}{2} - \frac{i}{4} g^{\mu\nu} \not{D})\psi$$

$$g_{\mu\nu}\hat{\mathcal{O}}^{\mu\nu} = 0$$

$$\langle P|\hat{\mathcal{O}}^{\mu\nu}|P\rangle = 2\mathcal{A}_q^2(P^\mu P^\nu - \frac{1}{4}g^{\mu\nu}M^2)$$

- Later will show $\mathcal{A}_q^2 = \langle x \rangle_q$
- Trace part subleading in

$$q_\mu q_\nu \langle P|\hat{\mathcal{O}}^{\mu\nu}|P\rangle$$

The symmetric and traceless index combination

$$\hat{\mathcal{O}}^{\mu_1\cdots\mu_n}(x) = \bar{\psi}_q(x)\gamma^{(\mu_1} iD^{\mu_2}\cdots iD^{\mu_n)}\psi_q(x)$$

has spin s = n, mass dimension d = n + 2 and twist t = d – s = 2. Higher twist operators are suppressed by powers of

$$\left(\frac{\Lambda_{QCD}}{Q}\right)^{t-2}$$

The smallest twist is 2. A gluonic twist-2 operator is (free indexes are symmetric and traceless as well):

$$F^{\alpha(\mu_1} iD^{\mu_2}\cdots iD^{\mu_{n-1}} F_{\alpha}{}^{\mu_n)}$$

Focusing on twist-2

- Twist-2 matrix element

$$\langle P|\hat{O}^{\mu_1\cdots\mu_n}(x)|P\rangle = 2\,\mathcal{A}_q^n P^{\mu_1}\cdots P^{\mu_n} - \text{traces},$$

- w = 1/x, forward Compton amplitude (q^μ terms added, current conservation explicit)

$$T^{\mu\nu} = \sum_q Q_q^2 \left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \sum_{n=2,4,\cdots}^{\infty} \omega^n \mathcal{A}_q^n \right.$$
$$\left. + \frac{4}{Q^2\omega^2}\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right) \sum_{n=2,4,\cdots}^{\infty} \omega^n \mathcal{A}_q^n \right\}$$

- Matching to parton model yields

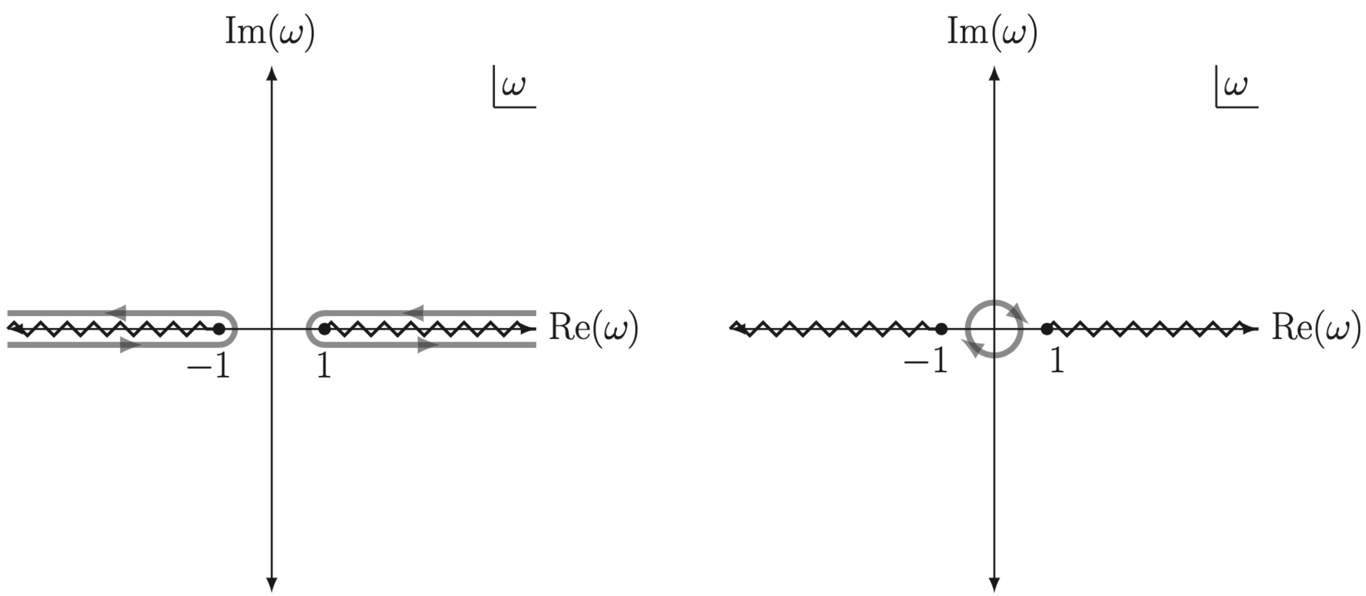
$$f_q(x) = \frac{1}{\pi} \sum_{n=2,4,\cdots} x^{-n} \mathrm{Im}\mathcal{A}_q^n$$

Dispersion Analysis

$$f_q(x) = \frac{1}{\pi} \sum_{n=2,4,\cdots} x^{-n} \mathrm{Im} \mathcal{A}_q^n$$

$$\langle x^{m-1} \rangle = \int_0^1 dx\, x^{m-1} f_q(x) = \mathrm{Im} \frac{1}{\pi} \int_1^\infty d\omega \sum_n \omega^{n-m-1} \mathcal{A}_q^n$$

$$= \sum_n \frac{1}{2\pi i} \int \frac{d\omega}{\omega^{m-1}} \omega^{n-2} \mathcal{A}_q^n = \mathcal{A}_q^m$$



(See Cheng and Li for a more careful discussion.)

Main result

- Twist-2 matrix element

$$\langle P|\hat{O}^{\mu_1\cdots\mu_n}(x)|P\rangle = 2\langle x^{n-1}\rangle P^{(\mu_1}\ldots P^{\mu_n)}$$

Wilson line

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P\,|\overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda)|\,P\rangle$$

Main result

- Twist-2 matrix element

$$\langle P|\hat{O}^{\mu_1\cdots\mu_n}(x)|P\rangle = 2\langle x^{n-1}\rangle P^{(\mu_1}\cdots P^{\mu_n)}$$

Wilson line

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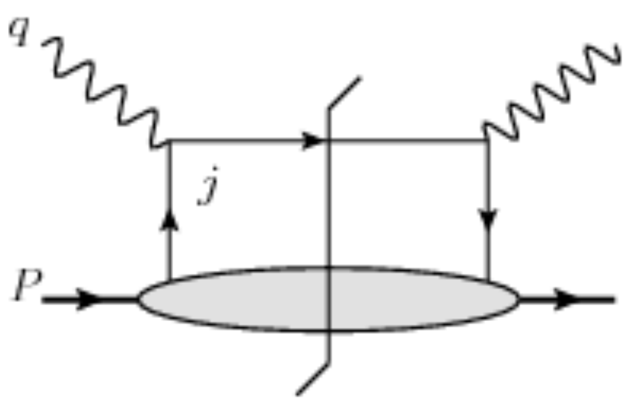
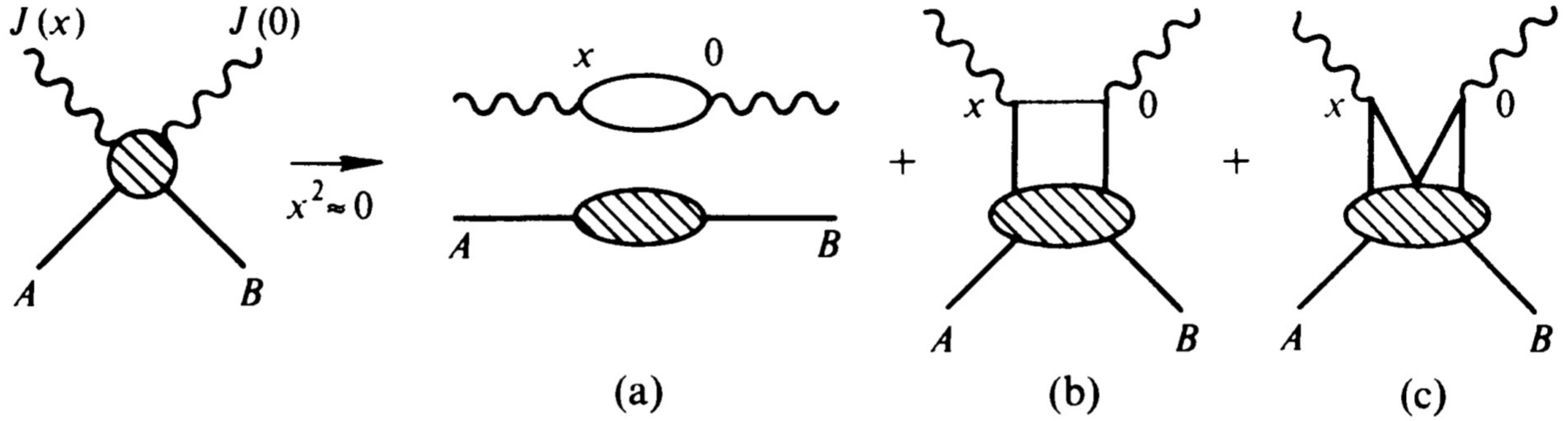


The struck parton moves on a light cone yields a sum of twist-2 operators after OPE.

Back to Parton Model

- Incoherent sum, broken by higher twist effect



Back to Parton Model

- Incoherent sum, broken by higher twist effect
- Factorization, broken by higher twist effect

$$\sigma(eP \to e'X) = \int_0^1 dx f_i(x,\mu)\sigma(ei \to e'i',\mu)$$

$$\Lambda_{QCD} \qquad\qquad Q$$

Back to Parton Model

- Incoherent sum, broken by higher twist effect
- Factorization, broken by higher twist effect
- Bjorken scaling violated (by powers of ln Q), need asymptotically free theory
- No final state interaction. But no free partons detected experimentally. Not affected by hadronization, can only be true in inclusive processes.

GPD's (Generalized Parton Distributions)

Ji, hep-ph/9603249

- Yield one class of 3D parton distributions
- Original motivation: proton spin crisis (quark helicity accounts for ~20% of the proton spin)

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}$$

$$M^{\alpha\mu\nu} = T^{\alpha\nu}x^\mu - T^{\alpha\mu}x^\nu$$

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$$

GPD's (Generalized Parton Distributions)

$$T_q^{\mu\nu} = \frac{1}{2}[\bar{\psi}\gamma^{(\mu} i\overrightarrow{D^{\nu)}}\psi + \bar{\psi}\gamma^{(\mu} i\overleftarrow{D^{\nu)}}\psi]$$

$(\mu\nu)$ denotes symmetrization with respect to μ, ν indices.

$$T_g^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}{}_{\alpha}$$

$$\vec{J}_q = \int d^3x \; \psi^\dagger [\vec{\gamma}\gamma_5 + (\vec{x} \times i\vec{D})]\psi \; ,$$

$$\vec{J}_g = \int d^3x \; (\vec{x} \times (\vec{E} \times \vec{B})) \; ,$$

Proton anugular momentum can be decomposed to three gauge invariant terms. The quark helicity contribution can be measured directly. What about the others?

GPD's (Generalized Parton Distributions)

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(\Delta^2)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M$$
$$+C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \bar{C}_{q,g}(\Delta^2)g^{\mu\nu}M\Big]U(P)$$

$\bar{P}^\mu = (P^\mu + P^{\mu\prime})/2$, $\Delta^\mu = P^{\mu\prime} - P^\mu$, and $U(P)$ is the nucleon spinor.

Momentum sum rule: $A_q(0) + A_g(0) = 1$

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk} \qquad M^{\alpha\mu\nu} = T^{\alpha\nu}x^\mu - T^{\alpha\mu}x^\nu$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right]$$

Angular momentum sum rule:

$$J_q + J_g = \frac{1}{2}$$

DVCS (Deeply Virtual Comption Scattering)

- One photon is highly off-shell s.t. the Compton amplitude

$$i\int d^4y e^{-iq\cdot y}\langle P'|TJ^\nu(y)J^\mu(0)|P\rangle$$

yields a convolution of the GPD's:

$$\int \frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\psi(\lambda n/2)|P\rangle = H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu U(P)$$
$$+E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P)+\ldots$$

$$\int \frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\gamma_5\psi(\lambda n/2)|P\rangle = \tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu\gamma_5 U(P)$$
$$+\tilde{E}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{\gamma_5\Delta^\mu}{2M}U(P)+\ldots$$

Sum Rules

- $$H(x,0,0) = f_1(x), \quad \tilde{H}(x,0,0) = g_1(x)$$

$$\int dx H(x,\Delta^2,\Delta\cdot n) = F_1(\Delta^2) \ ,$$
$$\int dx E(x,\Delta^2,\Delta\cdot n) = F_2(\Delta^2) \ ,$$
$$\int dx \tilde{H}(x,\Delta^2,\Delta\cdot n) = G_A(\Delta^2) \ ,$$
$$\int dx \tilde{E}(x,\Delta^2,\Delta\cdot n) = G_P(\Delta^2) \ .$$

$$\int dx x[H(x,\Delta^2,\Delta\cdot n) + E(x,\Delta^2,\Delta\cdot n)] = A_q(\Delta^2) + B_q(\Delta^2)$$

TMD (Transverse Momentum Distribution)

- Yield one class of 3D parton distributions

$$f_i(x, \mathbf{k}_\perp)$$

- Quark bilinear operator does not completely sit on the light-like direction (defined in the t-z plane) but has transverse displacement in the x-y direction
- Rich phenomena. Easier to be measured than GPD's.

Wigner Distributions

Ji, hep-ph/0304037

- Mother distributions of GPD's and TMD's
- In QM, using wave function to define

$$W(x,p) = \int \psi^*(x-\eta/2)\psi(x+\eta/2)e^{ip\eta}d\eta$$

Integrating over x(p) yields $|\breve{\psi}(p)|^2$ ($|\psi(x)|^2$). W could be negative.

Wigner Distributions

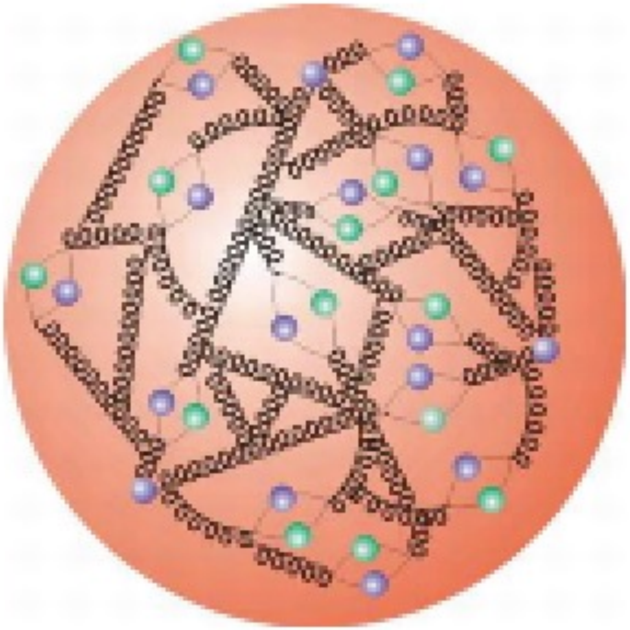
- In QCD,

$$W_\Gamma(\vec{r},\vec{k}) = \int \frac{dk^-}{(2\pi)^2}\int \overline{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2)e^{ik\cdot\eta}d^4\eta$$

$$\Psi(\eta) = \exp\left(-ig\int_0^\infty d\lambda n\cdot A(\lambda n+\eta)\right)\psi(\eta)$$

- Integrating out $\vec{r}$, to get TMD
- Integrating out $\mathbf{k}_\perp$ then Fourier transform $\vec{r}$, to get GPD.

Golden Age for Proton Structure Studies



Parton structures: 1d mom+spin PDF to 3d GPD & TMD to Wigner (and beyond?) [BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …] to applications (Higgs, new physics…)