# *Anomalous Casimir Effect and Related Subjects*

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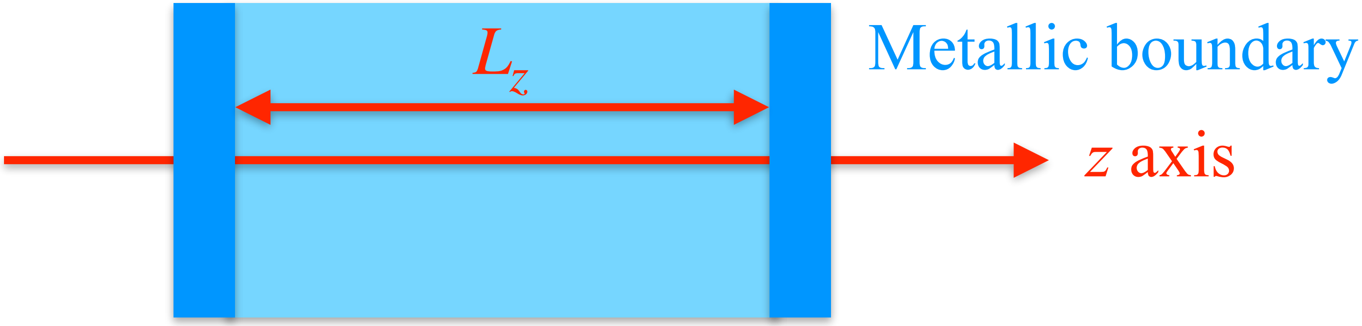
# *Casimir Effect*

## Vacuum full of quantum fluctuations

**Zero-point oscillation energy of photon**




$$E = \hbar c L_x L_y \int \frac{dk_x dk_y}{(2\pi)^2} \left( \frac{1}{2}\omega_0 + \sum_{n=1}^{\infty} \omega_n \right)$$

# *Casimir Effect*

## Dirichlet boundary condition

$$E = \hbar c L_x L_y \int \frac{dk_x dk_y}{(2\pi)^2} \left( \frac{1}{2}\omega_0 + \sum_{n=1}^{\infty} \omega_n \right)$$

$$\omega_n = \sqrt{k_x^2 + k_y^2 + (n\pi/L_z)^2}$$

## Casimir force

$$F_z = -\frac{\partial E}{\partial L_z}$$

**Negative — Attractive Force**
**Positive — Repulsive Force**

# *Casimir Effect*

## Casimir force vs. Casimir energy ?

**The energy is not well-defined (divergent).**

**Even the force is subtle…**

$L_z$

$$F_0 = -\frac{\partial (L_z \varepsilon_0)}{\partial L_z} \sim -\varepsilon_0$$

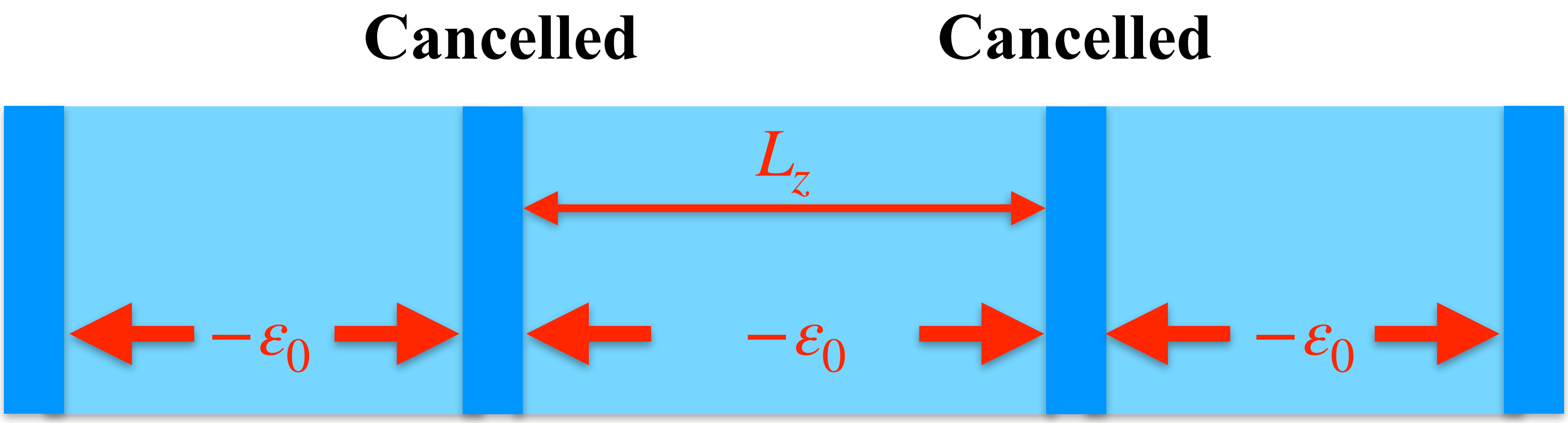
**If the "energy density" is constant (which could be divergent), then the energy is proportional to $L_z$ .**

**This implies that the nonzero force always exists!?**

**How to treat $L_z$ independent energy density?**

# *Casimir Effect*

## Casimir force vs. Casimir energy ?




**(Infinitely large) force or pressure remains, but the plates do not feel this force. We can drop it!**

**Physically meaningful force is the one depending on the plate separation.**

# *Casimir Effect*

## Conventional calculation

**Computational procedures are similar to finite-*T* QFT.**

**Abel-Plana formula:**

$$\sum_{n=0}^{\infty} f(n) = \int_0^{\infty} dx f(x) + \frac{1}{2} f(0) + i \int_0^{\infty} dx \frac{f(ix) - f(-ix)}{e^{2\pi x} - 1}$$

**Finite-*T* like term**

$$\frac{1}{2}\omega_0 + \sum_{n=1}^{\infty} \omega_n - L_z \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \omega_k = \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \ln(1 - e^{-2L_z\omega})$$

**Fukushima-Ohta (2000)**

# *Casimir Effect*

## Conventional calculation

$$\frac{\partial}{\partial L_z}\int\frac{d^3k}{(2\pi)^3}\ln(1-e^{-2L_z\omega_k}) = 2\int\frac{d^3k}{(2\pi)^3}\omega_k\sum_{n=1}^{\infty}e^{-2nL_z\omega_k}$$

$$= \frac{\Gamma(4)\zeta(4)}{\pi^2(2L_z)^4} = \frac{\pi^2}{240L_z^4} \quad \longrightarrow \quad F_z/A = -\frac{\pi^2}{240L_z^4}$$

$$\sim 0.016\,\mathrm{dyn}\,\frac{(\mu\mathrm{m})^4}{L_z^4}/\mathrm{cm}^2$$

**Attractive force per unit surface area *A* between two plates with separation $L_z$ .**

# Casimir Effect

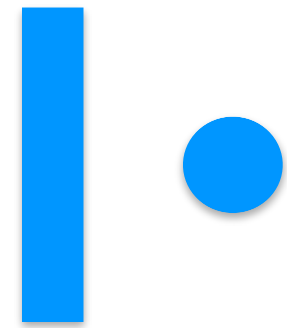
## Experimental signature

VOLUME 78, NUMBER 1 PHYSICAL REVIEW LETTERS 6 JANUARY 1997

**Demonstration of the Casimir Force in the 0.6 to 6 $\mu$m Range**

S.K. Lamoreaux*
*Physics Department, University of Washington, Box 35160, Seattle, Washington 98195-1560*
(Received 28 August 1996)

The vacuum stress between closely spaced conducting surfaces, due to the modification of the zero-point fluctuations of the electromagnetic field, has been conclusively demonstrated. The measurement employed an electromechanical system based on a torsion pendulum. Agreement with theory at the level of 5% is obtained. [S0031-9007(96)02025-X]



For our measurement of the Casimir force, the conductors were in the form of a flat plate and a sphere. Our first attempts at measurements using parallel plates were unsuccessful; this is because it is very difficult to maintain parallelism at the requisite accuracy ($10^{-5}$ rad for 1 cm diameter plates). There is no issue of parallelism when

$$F = -2\pi R \cdot f$$

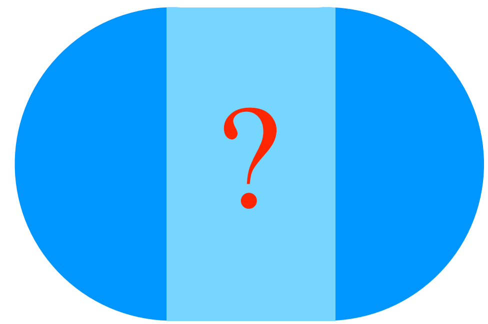
$$f = \frac{1}{3} \cdot \frac{\pi^2}{240 L_z^3}$$

# *Casimir Effect*

## Attractive ? Repulsive ?

**Casimir force depends on:**

**[Geometry]**




**[Dispersion]**

$$\omega_n = \sqrt{k_x^2 + k_y^2 + (n\pi/L_z)^2}$$

**Free energy dispersion**

**Medium effect?**
**Interaction? Phase transition?**

**Flachi, Nitta, Takada, Yoshii (2017-2019)**

# No-go Theorem?

PRL **97,** 160401 (2006) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2006

## Opposites Attract: A Theorem about the Casimir Force

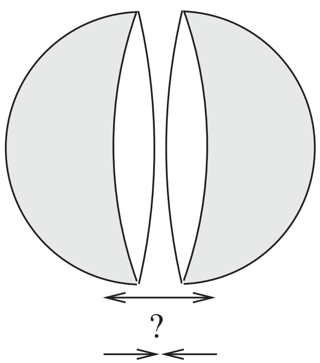
Oded Kenneth[1] and Israel Klich[2,*]

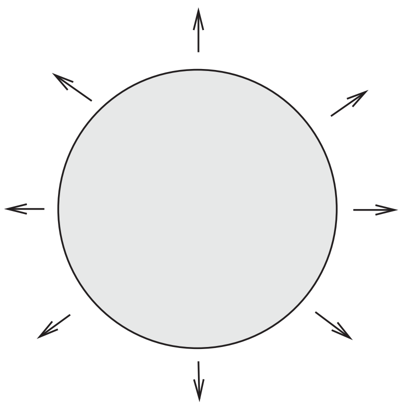
[1]*Department of Physics, Technion, Haifa 32000 Israel*

[2]*Department of Physics, California Institute of Technology, MC 114-36 Pasadena, California 91125, USA*

(Received 10 January 2006; published 18 October 2006)

We consider the Casimir interaction between (nonmagnetic) dielectric bodies or conductors. Our main result is a proof that the Casimir force between two bodies related by reflection is always attractive, independent of the exact form of the bodies or dielectric properties. Apart from being a fundamental property of fields, the theorem and its corollaries also rule out a class of suggestions to obtain repulsive forces, such as the two hemisphere repulsion suggestion and its relatives.



**Reflection Symmetry**

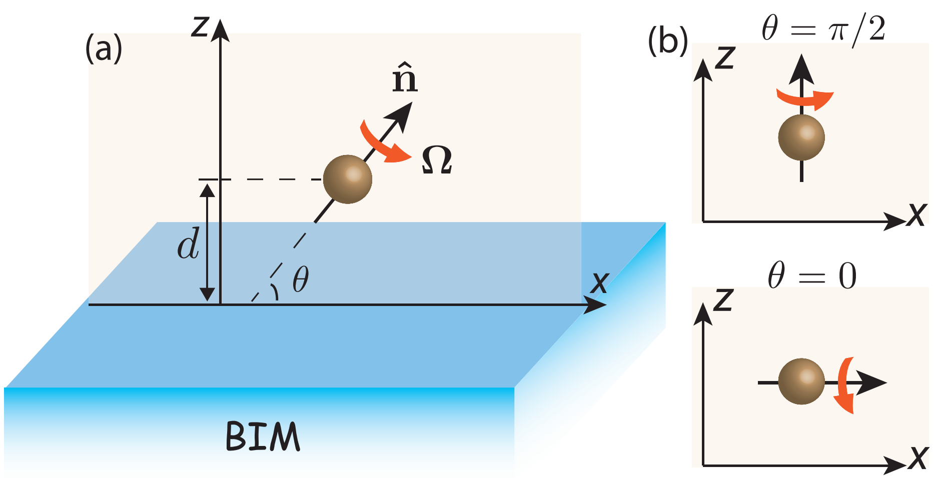
⬇

**Attractive Force**

# *Axial Casimir Force*

## Qing-Dong Jiang and Frank Wilczek (2018)

**I attended a workshop at NORDITA in September 2018 and heard a talk by Frank Wilczek and also met Qing-Dong Jiang who was a student there at that time…**




**Breaking…**

**Time-Reversal Symmetry**

**Parity Symmetry**

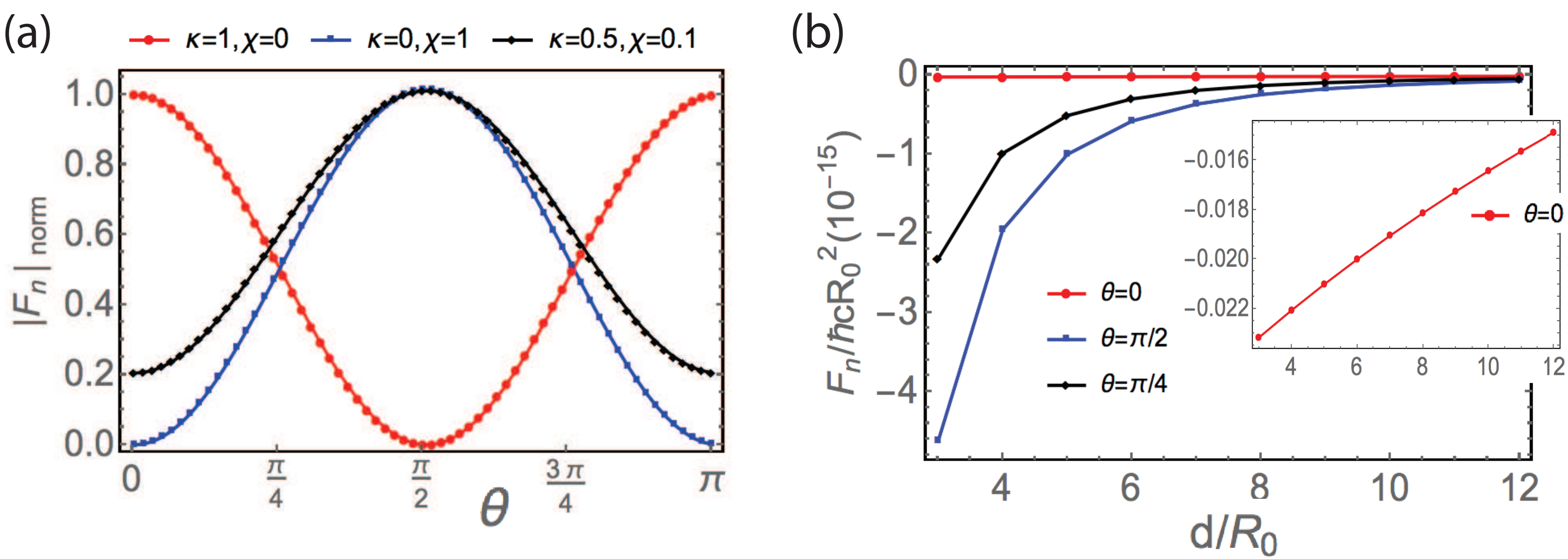
**Both…**

# *Axial Casimir Force*

**Qing-Dong Jiang and Frank Wilczek (2018)**

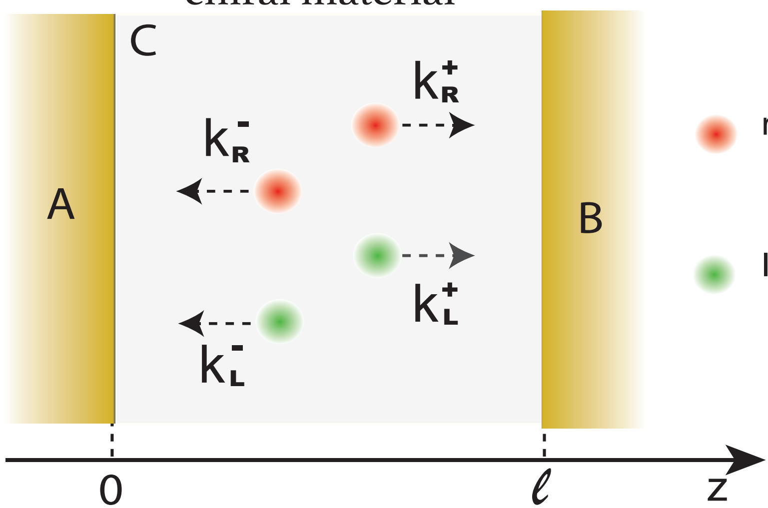
**One can use the Casimir force to diagnose matter!**

$$\mathbf{D} = \epsilon\mathbf{E} + (\chi - i\kappa)\sqrt{\epsilon_0\mu_0}\mathbf{H}$$ **[magnetoelectric effect]**

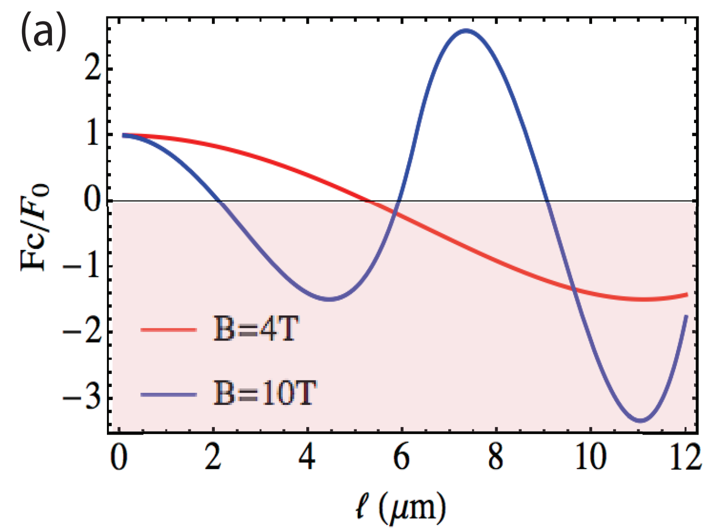


# *Chiral Casimir Forces*

**Qing-Dong Jiang and Frank Wilczek (2018)**




**Polarized photons**




**Tunable by the strength of the magnetic field.**

# *Chiral Casimir Forces*

**I found these works extremely interesting, but…**

**Calculations are intriguing but complicated:**

**They develop the scattering formalism.**
**See also Jaffe (2005) (no need of "vacuum")**

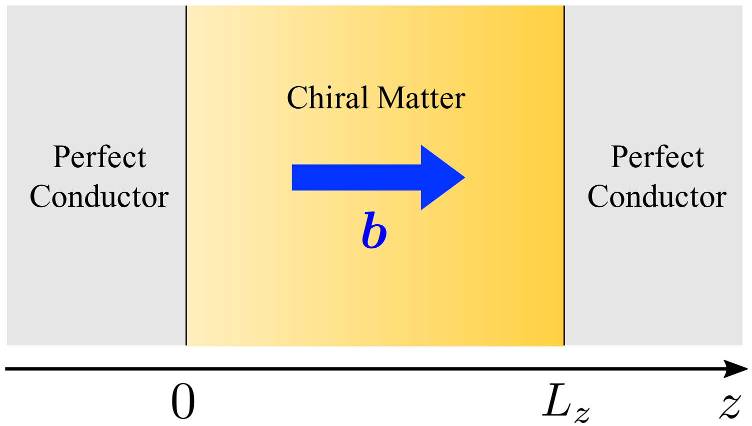
**Experimental realization is quite challenging:**

**[ACF] "Note that the very recent experiments have already achieved a superfast rotation of nanoparticles, making the ACF within the experimental reach".**

**[CCF] "Faraday materials" and "Octically active materials"**

# *Anomalous Casimir Force*

**Another ACF proposed by Fukushima-Imaki-Qiu (2019)**




$$\mathcal{L}_{\text{axion}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$b_0(x) \equiv \partial_t \theta(x)\,, \qquad \boldsymbol{b}(x) \equiv -\boldsymbol{\nabla}\theta(x)$$

$$\mathbf{D} = \epsilon\mathbf{E} + (\chi - i\kappa)\sqrt{\epsilon_0\mu_0}\mathbf{H}$$

**Similar to magnetoelectric effect**

**No need to complexify the theta angle.**
**Spatially changing theta is common in topological matter.**

# *Anomalous Casimir Force*

## Another ACF proposed by Fukushima-Imaki-Qiu (2019)

$$\mathcal{L}_{\rm axion} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$b_0(x) \equiv \partial_t \theta(x)\,, \qquad \boldsymbol{b}(x) \equiv -\boldsymbol{\nabla}\theta(x)$$

**Constant theta would not change the EoM.**

**Inhomogeneous theta generates new terms.**

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = -\boldsymbol{b}\cdot\boldsymbol{B}\,,$$

$$\boldsymbol{\nabla}\times\boldsymbol{B} - \frac{\partial \boldsymbol{E}}{\partial t} = b_0\boldsymbol{B} + \boldsymbol{b}\times\boldsymbol{E},$$

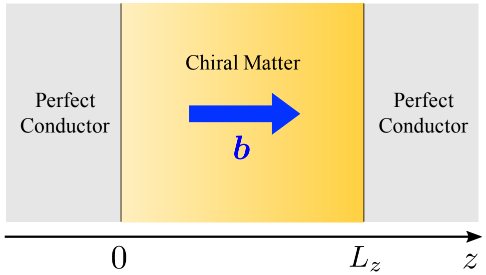
$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0\,,$$

$$\boldsymbol{\nabla}\times\boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0\,.$$

# *Anomalous Casimir Force*

## Another ACF proposed by Fukushima-Imaki-Qiu (2019)

**Background *E*-*B* are unnecessary!**




$$\boldsymbol{b} = b\hat{\boldsymbol{z}}$$

**Dispersion relations are modified as:**

**[Scalar / Longitudinal]**

$$\omega_1 = \boldsymbol{k} \ , \omega_2 = \boldsymbol{k}$$

**Cancelled by ghosts**

**[Transverse]**

$$\omega_+ = \sqrt{k_\perp^2 + (\sqrt{k_z^2 + b^2/4} + b/2)^2} \qquad \omega_- = \sqrt{k_\perp^2 + (\sqrt{k_z^2 + b^2/4} - b/2)^2}$$

# *Anomalous Casimir Force*

**Another ACF proposed by Fukushima-Imaki-Qiu (2019)**

$$\omega_+ = \sqrt{k_\perp^2 + (\underbrace{\sqrt{k_z^2 + b^2/4} + b/2}_{\pi\mu_+/L_z})^2} \qquad \omega_- = \sqrt{k_\perp^2 + (\underbrace{\sqrt{k_z^2 + b^2/4} - b/2}_{\pi\mu_-/L_z})^2}$$

$$= \frac{\pi\mu_\pm}{L_z}\sqrt{\tilde{k}_\perp^2 + 1}$$

**Recaled expression for the energy**

$$\mathscr{E} = \frac{\pi^3}{L_z^3}\sum_{\pm,n}\mu_\pm^3(n)\int^{\tilde{\Lambda}_\pm}\frac{d^2\tilde{k}_\perp}{(2\pi)^2}\frac{1}{2}\sqrt{\tilde{k}_\perp^2 + 1}$$

# *Anomalous Casimir Force*

**Another ACF proposed by Fukushima-Imaki-Qiu (2019)**

**After some tedious calculations, we find:**

$$\mathcal{E} = \frac{\pi^3}{L_z^3} \sum_{\pm,n} \mu_{\pm}^3(n) \int^{\tilde{\Lambda}_{\pm}} \frac{d^2\tilde{k}_{\perp}}{(2\pi)^2} \frac{1}{2} \sqrt{\tilde{k}_{\perp}^2 + 1}$$

$$= \mathcal{E}_{\infty} + \frac{b^4 L_z}{16\pi^2} \sum_{m=1}^{\infty} \left[ \frac{K_1(mbL_z)}{mbL_z} - \frac{K_2(mbL_z)}{(mbL_z)^2} \right]$$

**divergent**

$$\mathcal{E}_{\infty} = -\frac{5b^4 L_z}{512\pi^3} \Gamma(0)$$

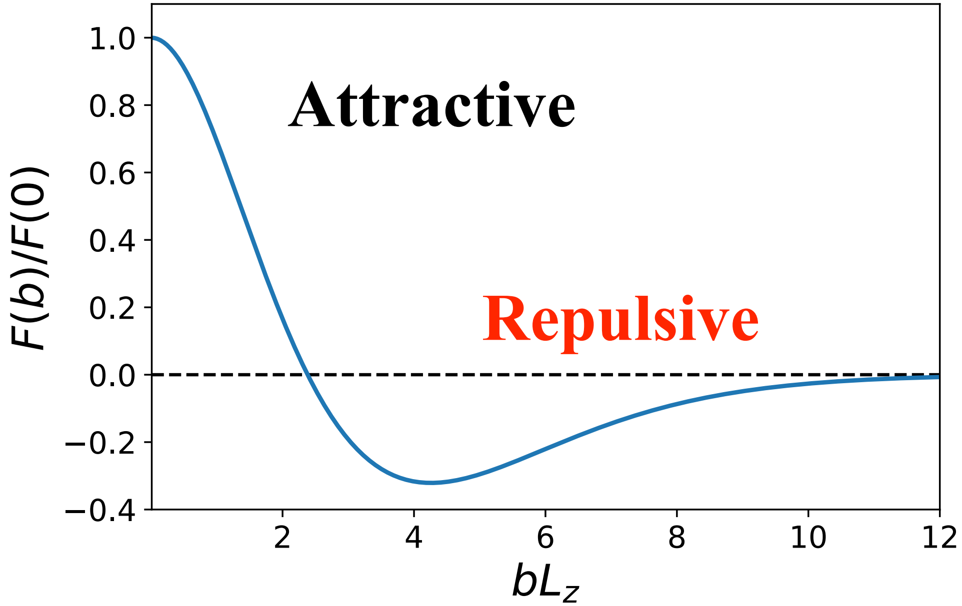
**This "constant" force, which is there, can be dropped!**

# *Anomalous Casimir Force*

**Trivial check:**

$$\lim_{b\to 0} F(b)/A = -\frac{\pi^2}{240 L_z^4}$$

**Repulsive component:**




**Good… but the experimental realization is also challenging… (Separation should be filled in with chiral matter.)**

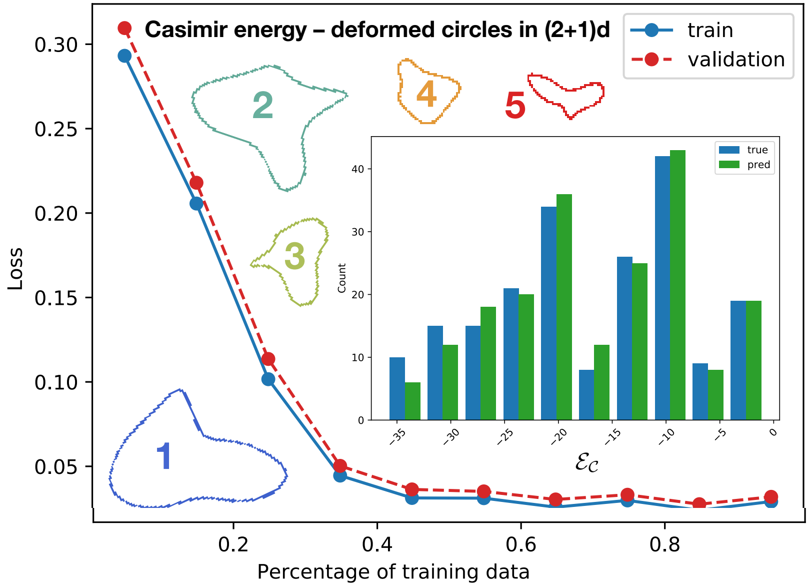
# Comments

## Lattice simulation helps ?

**How to treat divergnet energy / force is very subtle…**

Casimir effect with machine learning **1911.07571**

M. N. Chernodub,[1,2] Harold Erbin,[3] I. V. Grishmanovskii,[2] V. A. Goy,[2] and A. V. Molochkov[2]




$$\mathcal{E}_{\mathcal{S}}(x) = \left\langle T_E^{00}(x) \right\rangle_{\mathcal{S}} - \left\langle T_E^{00}(x) \right\rangle_0$$

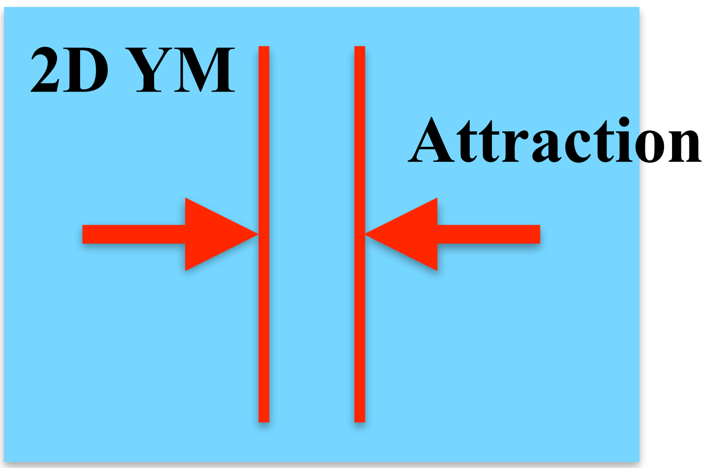
**Maybe, still, a divergent force is not yet subtracted which can be finite in finite-size systems…**

# *Comments*

## Casimir effect helps our understanding ?

Casimir effect in Yang-Mills theory **1805.11887**

M. N. Chernodub,[1,2] V. A. Goy,[2] A. V. Molochkov,[2] and Ha Huu Nguyen[1,3]




**They found exponential suppression (Casimir mass) and it is different from the glueball mass… new scale?**

$$M_{\text{Cas}} = 1.38(3)\sqrt{\sigma}$$

$$M_{0^{++}} = 4.7\sqrt{\sigma}$$

**Ghost? Origin of confinement? Something else?**

# *Conclusions*

- **Casimir effect has some subtleties in the treatment of the divergent energy / force.**

- **No-go theorem (attraction with reflection symmetry) is violated for chiral matter that breaks parity and time-reversal symmetries.**

- **Casimir effect is useful to diagnose the nature of matter and could be a probe to the origin of confinement.**