Exploring the phase structure of strongly interacting matter through studies of conserved charge fluctuations in lattice QCD simulations

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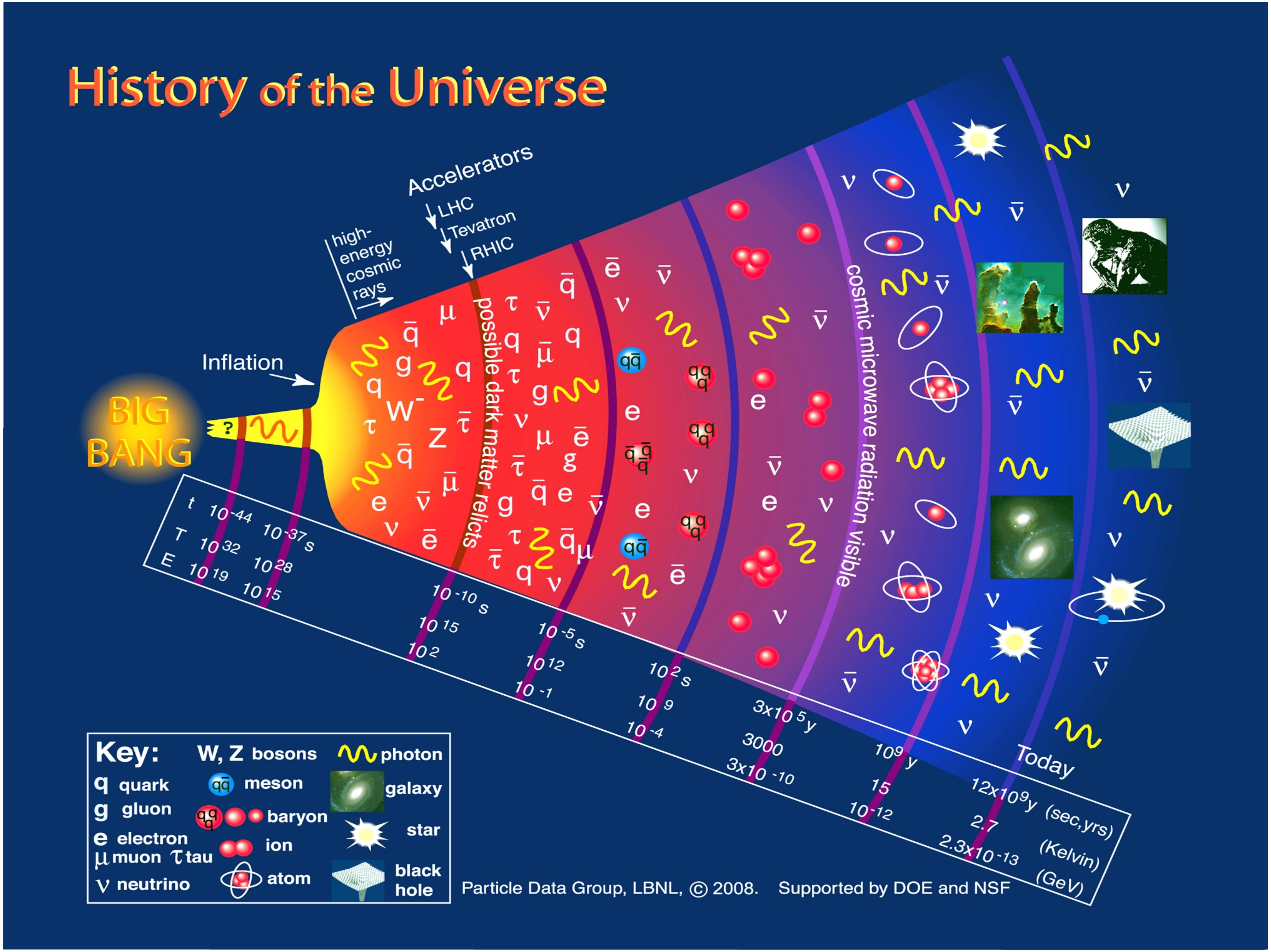
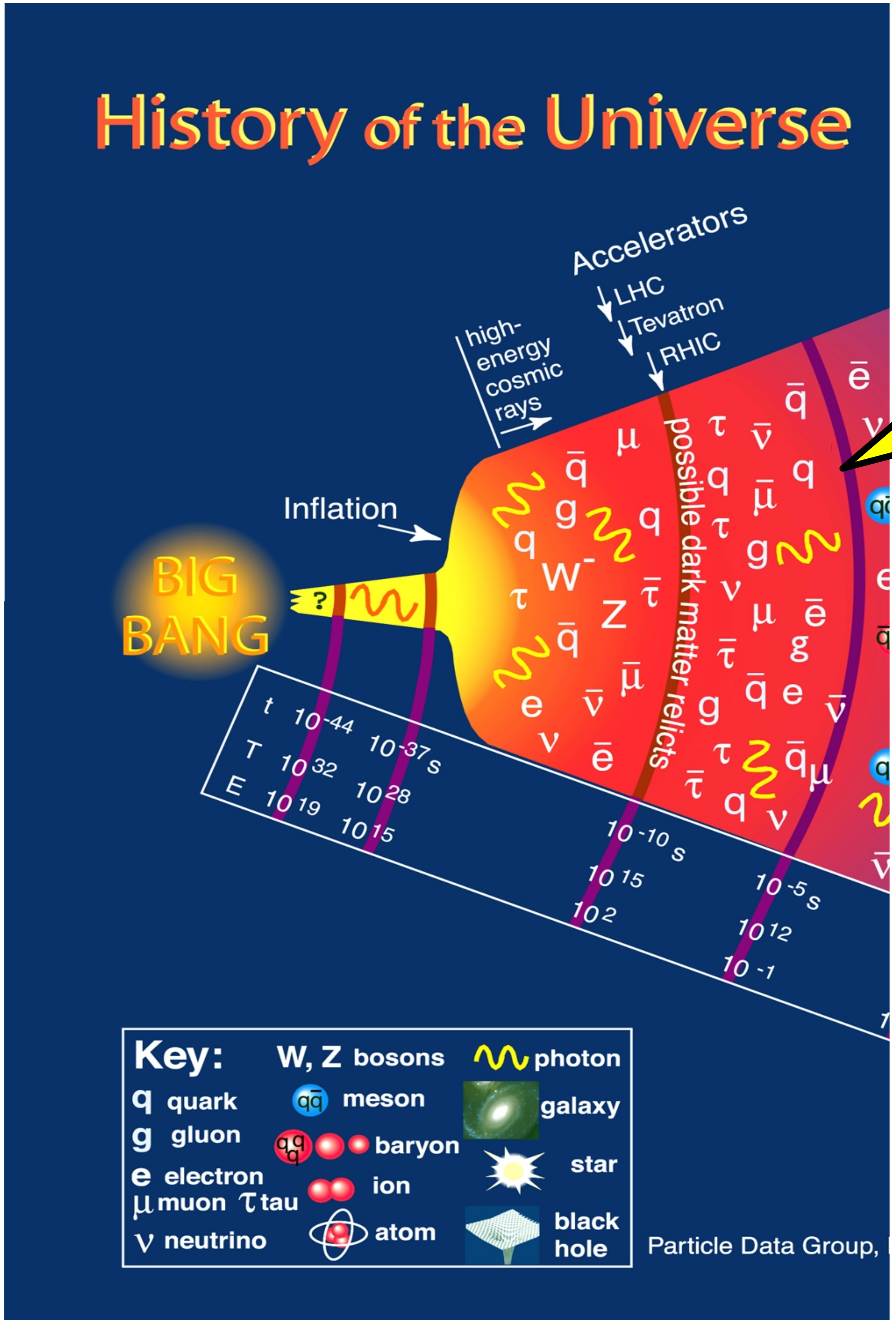
HotQCD collaboration: arXiv: 2107.10011,2202.09184

Prelude



History of the Universe
Accelerators
LHC
Tevatron
RHIC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10^-44 10^-37 s
T 10^32 10^28
E 10^19 10^15
10^-10 s
10^15
10^2
10^-5 s
10^12
10^-1
10^2 s
10^9
10^-4
3x10^5 y
3000
3x10^-10
10^9 y
15
10^-12
Today
12x10^9 y (sec,yrs)
2.7 (Kelvin)
2.3x10^-13 (GeV)
Key:
W, Z bosons
photon
q quark
meson
galaxy
g gluon
baryon
star
e electron
ion
µ muon τ tau
black hole
ν neutrino
atom
Particle Data Group, LBNL, © 2008.
Supported by DOE and NSF

Prelude



The "phase" transition from the quark-gluon plasma to hadronic matter

1/100000 seconds after the big bang quarks and gluons recombine to hadrons

The temperature at this time was about 100000 times that of the interior of the sun

In HIC experiments, QGP is formed and cool back to hadronic matter at low temperature.

Nature of the phase transition ??

“A remark on chiral phase transition in QCD”

QCD Lagrangian is symmetric under , $SU(2)_L \times SU(2)_R \times U(1)_A \times U(1)_V$ **, for** $m_u = m_d = 0$**.**

- For physical values of quark masses, $m_u = m_d \neq 0$, the transition is smooth analytic crossover. Aoki et al, Nature 443, 675-678. (2006)
- At, $m_u = m_d = 0$ the transition becomes second order $O(4)$.
- For physical values of quark masses the pseudo-critical temperature of QCD transition :

$$T_{pc} = (156.5 \pm 1.5) \text{ MeV} \qquad (158.0 \pm 0.6) \text{ MeV}$$

A. Bazavov et al [HotQCD], arXiv:1812.08235

S. Borsanyi et al., arXiv:2002.02821

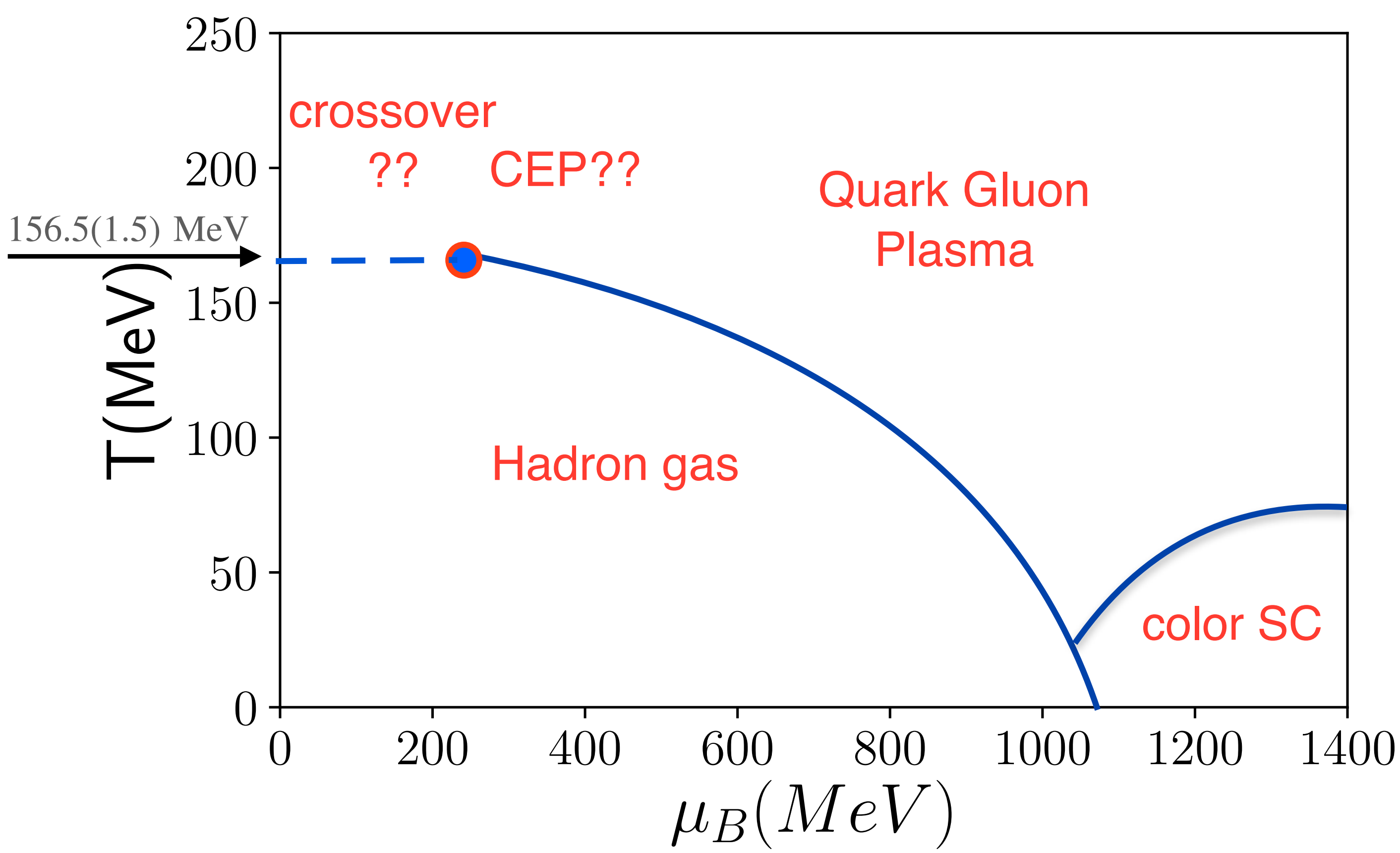
- Success on comparison of LQCD with experiments: Hadrons freeze out temperature from ALICE : $T_f = 156.5$ MeV i.e. close to the QCD crossover.

A. Andronic et al., Nature 561 (2018) 321

Outline

Bigger Picture : Understand the thermodynamics at the QCD crossover, Study the QCD phase diagram, Indication of the location of the critical point.....

QCD Phase diagram

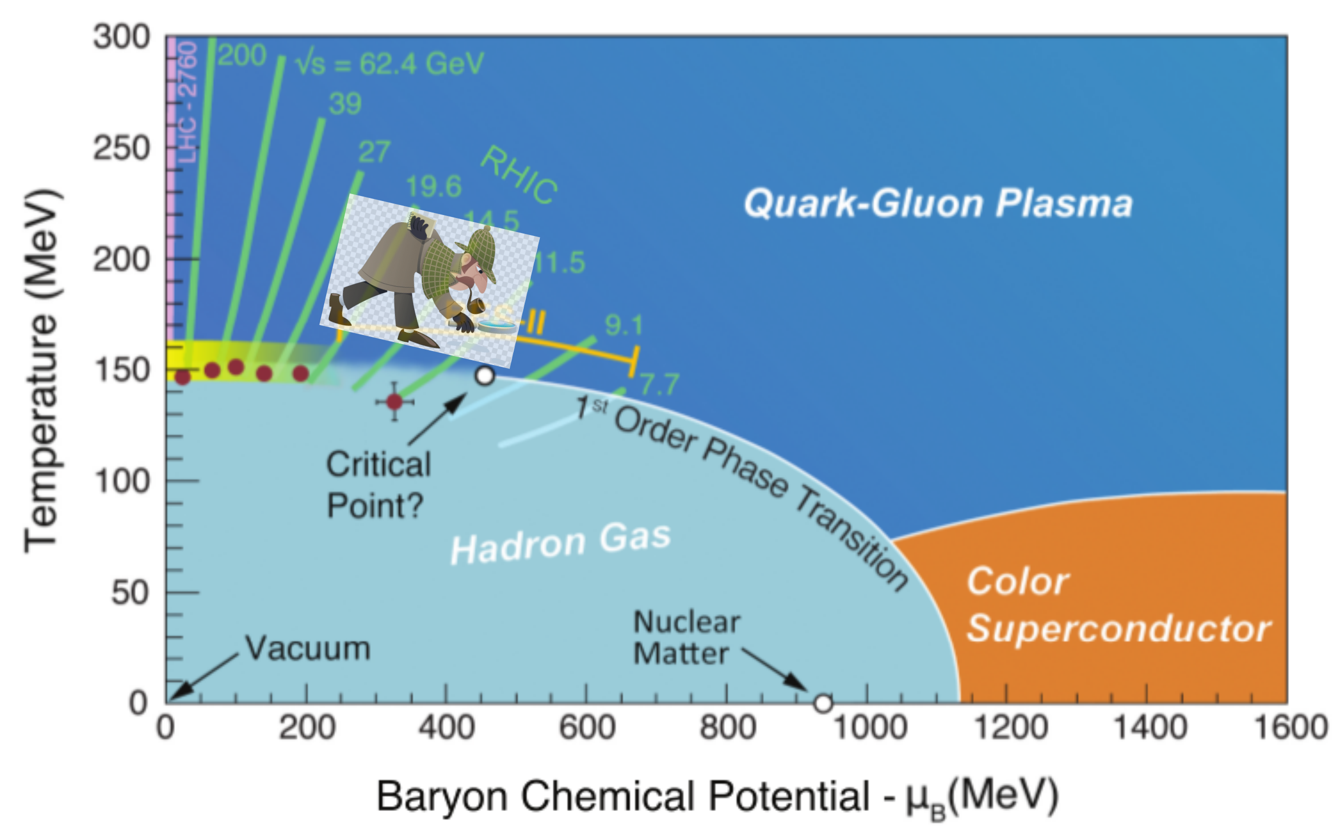


Q1. Degrees of freedom close to the QCD transition temperature (T_{pc})??

Detailed Comparison of Lattice QCD calculations with HRG models at finite chemical potentials.

Q2. Possible location of the critical point on the QCD phase diagram?

Examining the cumulants at non zero chemical potential.



Chemical potential on the lattice

$$\mathcal{Z}_{QCD} = \int \mathcal{D}U \ [\det \mathrm{M}] e^{-S_G(U)}, \text{ Where, } \mathrm{M} = \mathrm{D} + \mathrm{m}$$

- The prescription for introducing chemical potential on the lattice,

$$U_0 \to \exp(a\mu) U_0$$

$$U_0^\dagger \to \exp(-a\mu) U_0^\dagger$$

P. Hasenfratz, F. Karsch , *Phys.Lett.B* 125 (1983) 308-310

$$\mathcal{Z}_{QCD} = \int \mathcal{D}U \ \det[M(m_u, \mu_u)]^{1/4} \det[M(m_d, \mu_d)]^{1/4} \ \det[M(m_s, \mu_s)]^{1/4} \ e^{-S_G(U)}$$

Complex determinant !!

Standard Monte Carlo methods fails !!

Chemical potential on the lattice

$$\mathcal{Z}_{QCD} = \int \mathcal{D}U \ \det[M(m_u, \mu_u)]^{1/4} \det[M(m_d, \mu_d)]^{1/4} \ \det[M(m_s, \mu_s)]^{1/4} \ e^{-S_G(U)}$$

Complex determinant !!

Standard Monte Carlo methods fails !!

Direct Method

Taylor expansion : HotQCD collaboration, Gavai and Gupta, Bielefeld-Swansea collaboration

Analytic continuation from Imaginary chemical potential : M. D'Elia and M. P. Lombardo , Wuppertal Budapest collaboration

Resummation Method

Padé approximant : Gavai and Gupta, Bielefeld Parma collaboration, HotQCD collaboration

There is more,

arXiv:2202.05574 , 2106.03165

Thermodynamics using Lattice QCD

The partition function of QCD:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}U \det[M(m_u, \mu_u)]^{1/4} \det[M(m_d, \mu_d)]^{1/4} \det[M(m_s, \mu_s)]^{1/4} e^{-S_G(U)}$$

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \ ,$$

$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \ ,$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

Calculations at $\mu = 0$

- The Taylor series of the QCD pressure at finite temperature and density: $\frac{P(T, \vec{\mu})}{T^4} = \frac{1}{VT^3} ln\mathcal{Z}_{QCD} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k \ , \ \hat{\mu} = \mu/T$

- Cumulants at vanishing chemical potential,

$$\chi_{ijk}^{BQS}(T,0) = \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_X^{i,j,k}} \bigg|_{\mu_X=0} \ , \ X = B, Q, S$$

Vanishing Chemical potential

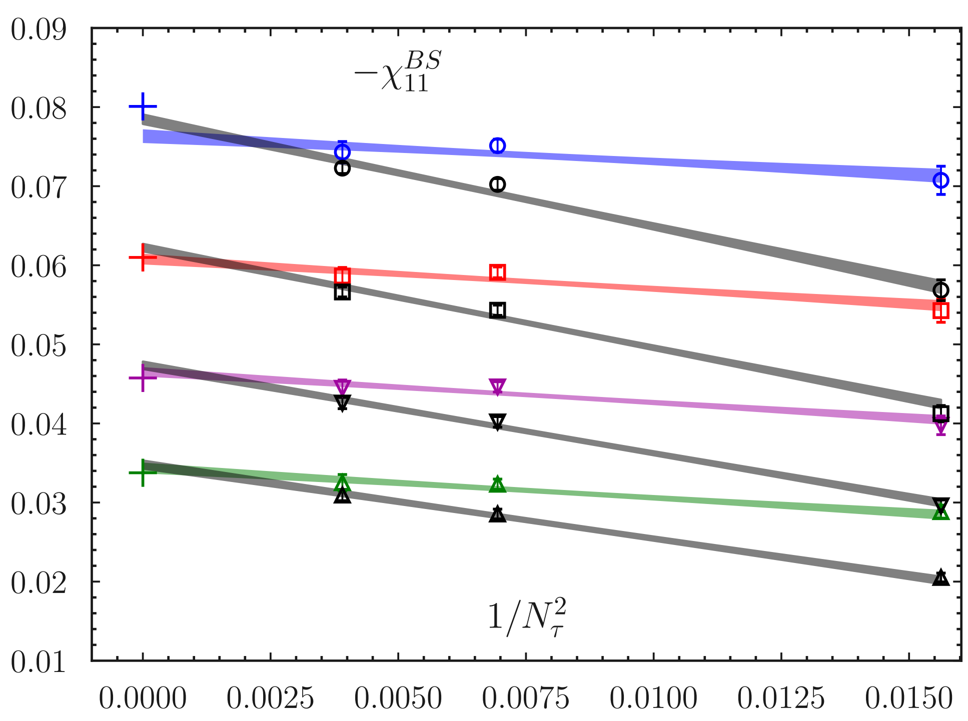
Introduction

- QCD (Lattice) describes the dynamics of the strongly interacting matter both in the high and low temperature regime.
- An approximate model of QCD can be used to identify various hadronic species in the low temperature phase to extract the freeze out parameters from experiment at small and vanishing μ_B.
- Hadron resonance gas (HRG) models are in good agreement with the lowest order cumulants (χ_n^X) calculated in Lattice QCD at $T < T_{pc}$ (T_f), however agreement starts to deteriorate as T approaches T_{pc} (T_f) .

Cumulants →

- **LQCD for small values of chemical potential**
- **Easily obtained from HRG models**
- **Accesible to HIC experiments**

Scale setting and continuum extrapolations



- We used r_1 and f_K to set the scale for the lattice spacing at finite values of the gauge coupling $\beta = 10/g^2$.
- $r_1 f_K$ is solely determined through a lattice calculation.
- The physical value of r_1 is needed, it requires input from experiment.

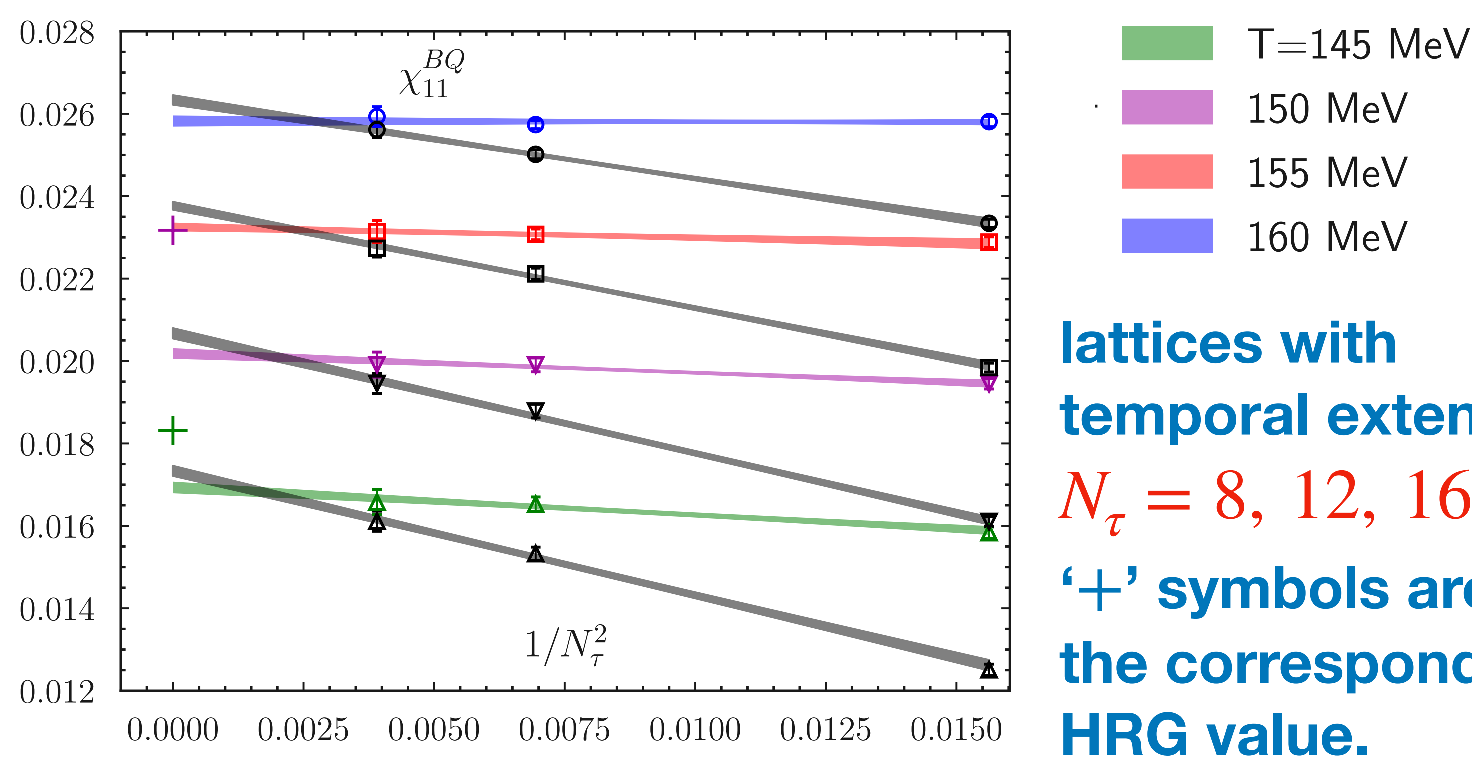
Two different temperature scales produce consistent continuum extrapolation!!

$$\frac{T_{f_K}}{T_{r1}} = \left(\frac{1}{af_K}\frac{a}{r_1}\right) r_1 f_K \ , \ r_1 f_K = 0.1734(9)$$

$$r_1 = 0.3106(8)(14) \ \text{fm}$$

$$f_K = 155.7/\sqrt{2} \ \text{MeV}$$

[Milc collaboration]



lattices with temporal extent $N_\tau = 8, 12, 16$.

'+' symbols are the corresponding HRG value.

Continuum extrapolation based on T_{f_K} (color) and T_{r_1} (black) scale

Hadron Resonance Gas (HRG)

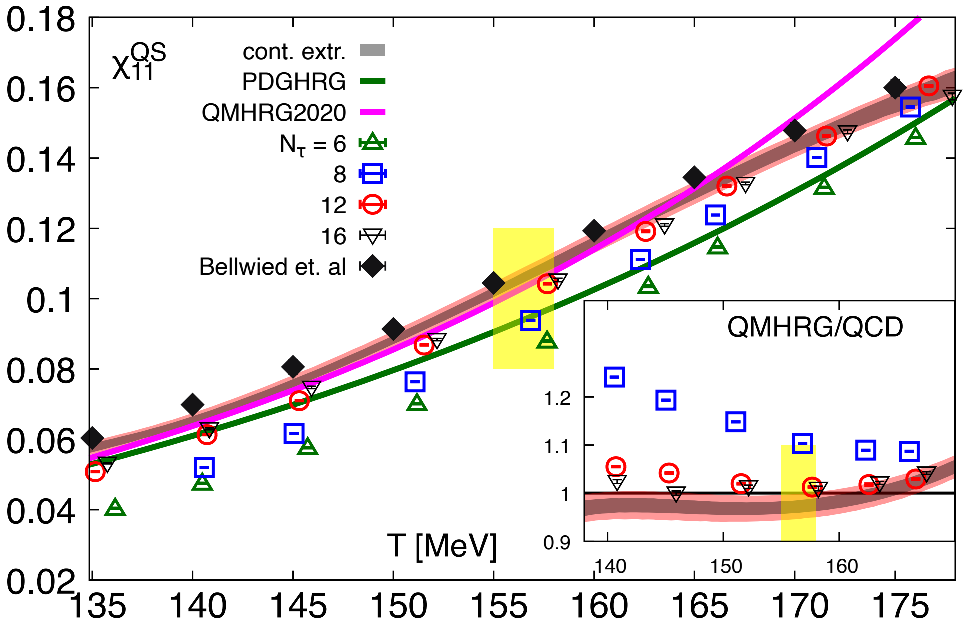
- The pressure of a non-interacting hadron resonance gas model:

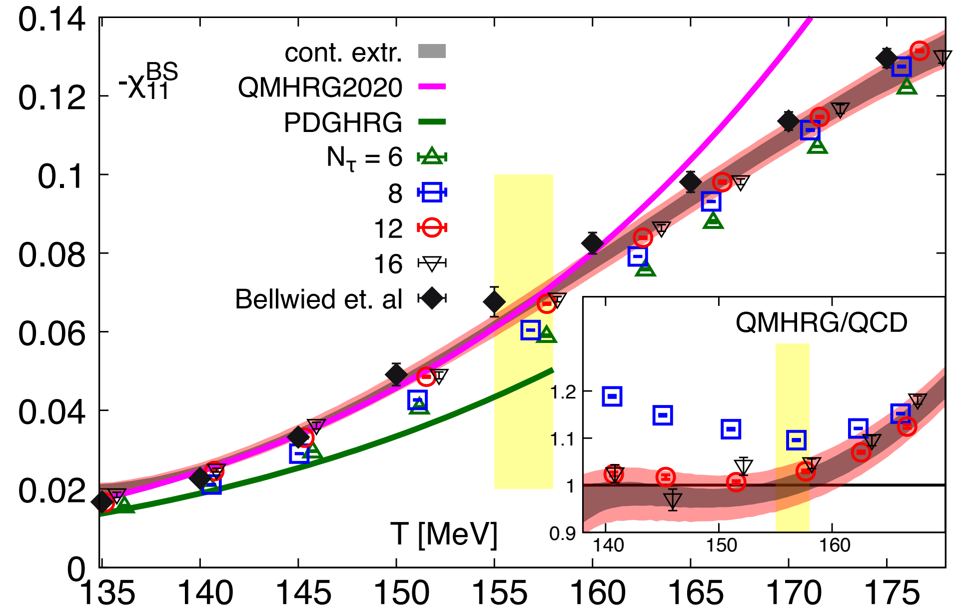
$$P/T^4 = \sum_H \frac{g}{2\pi^2}(m_H/T)^2 \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k^2} K_2\left(\frac{km_H}{T}\right) \exp[k\vec{C}_H . \vec{\mu}/T],\ K_2(m_H/T) \sim \exp(-m_H/T)$$

$$\chi_{lmn}^{BQS}/T^3 = \sum_H B_H^l Q_H^m S_H^n P_H/T^4\ ,\ \vec{C}_H = (B, Q, S)\ ,\ \vec{\mu} = (\mu_B, \mu_Q, \mu_S)$$

For ex. H could be all baryons and mesons listed in the PDG.

LQCD vs HRG





2^{nd} **order cumulants satisfy ($m_u = m_d$),**

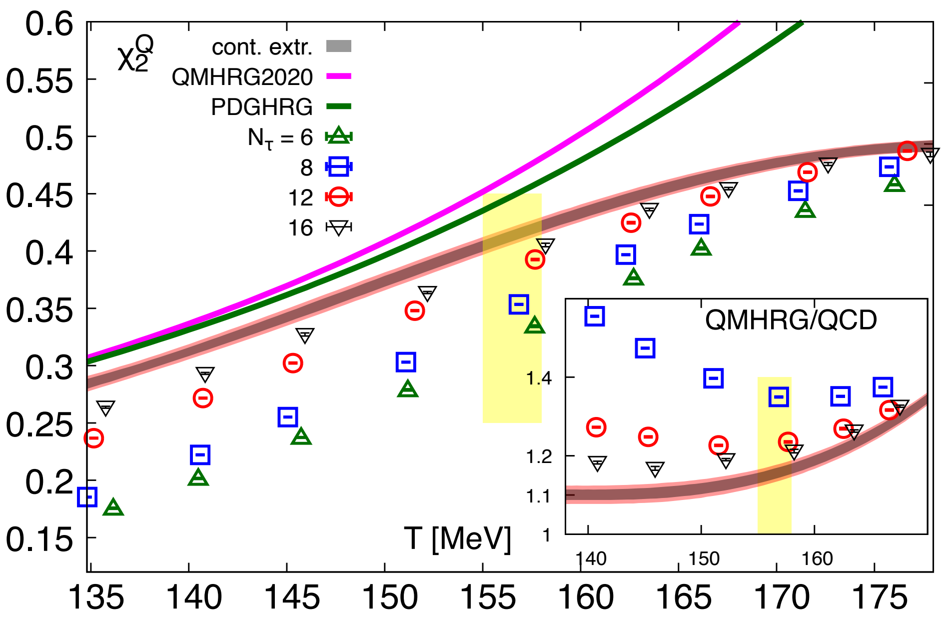
$$\chi_2^S = 2\chi_{11}^{QS} - \chi_{11}^{BS}$$

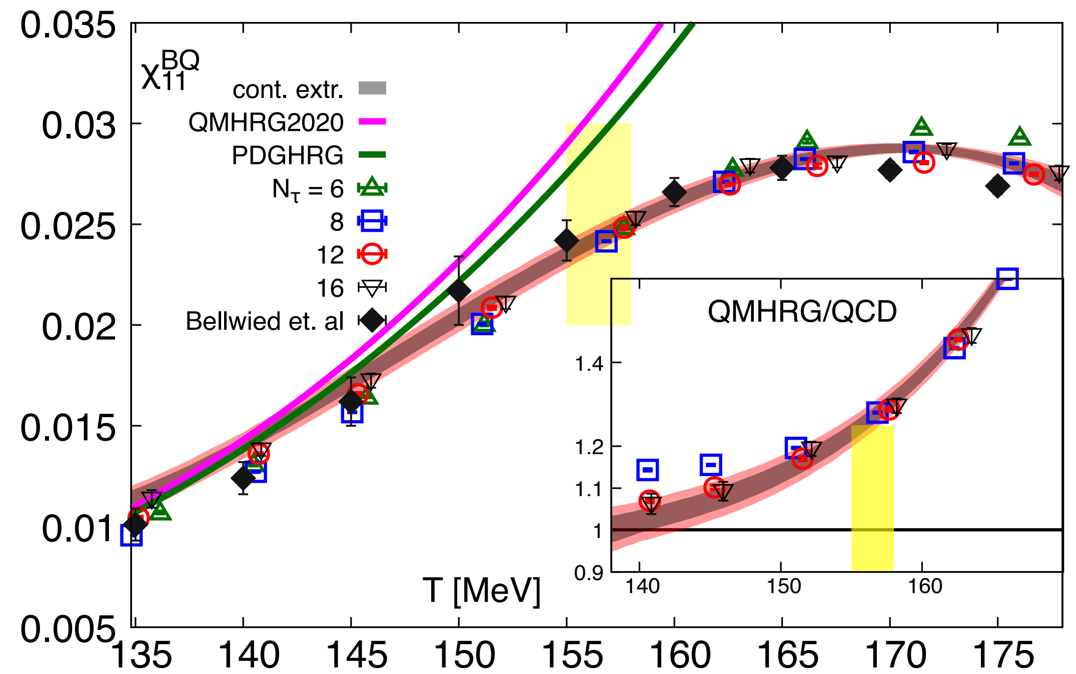
$$\chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS}$$

- **PDGHRG** : Established resonances (3 and 4-star) from PDG
- **QMHRG2020** : Additional resonances from PDG (1 and 2 star) and from Quark Model calculations.
- **Uniqueness**: Identification of 1 and 2-star resonances with QM prediction states.

D. Bollweg et al ,arXiv:2107.10011 ; J. Goswami et al, arXiv:2109.00268, SQM2021

Electric charge fluctuations and their correlations with baryon-number fluctuations

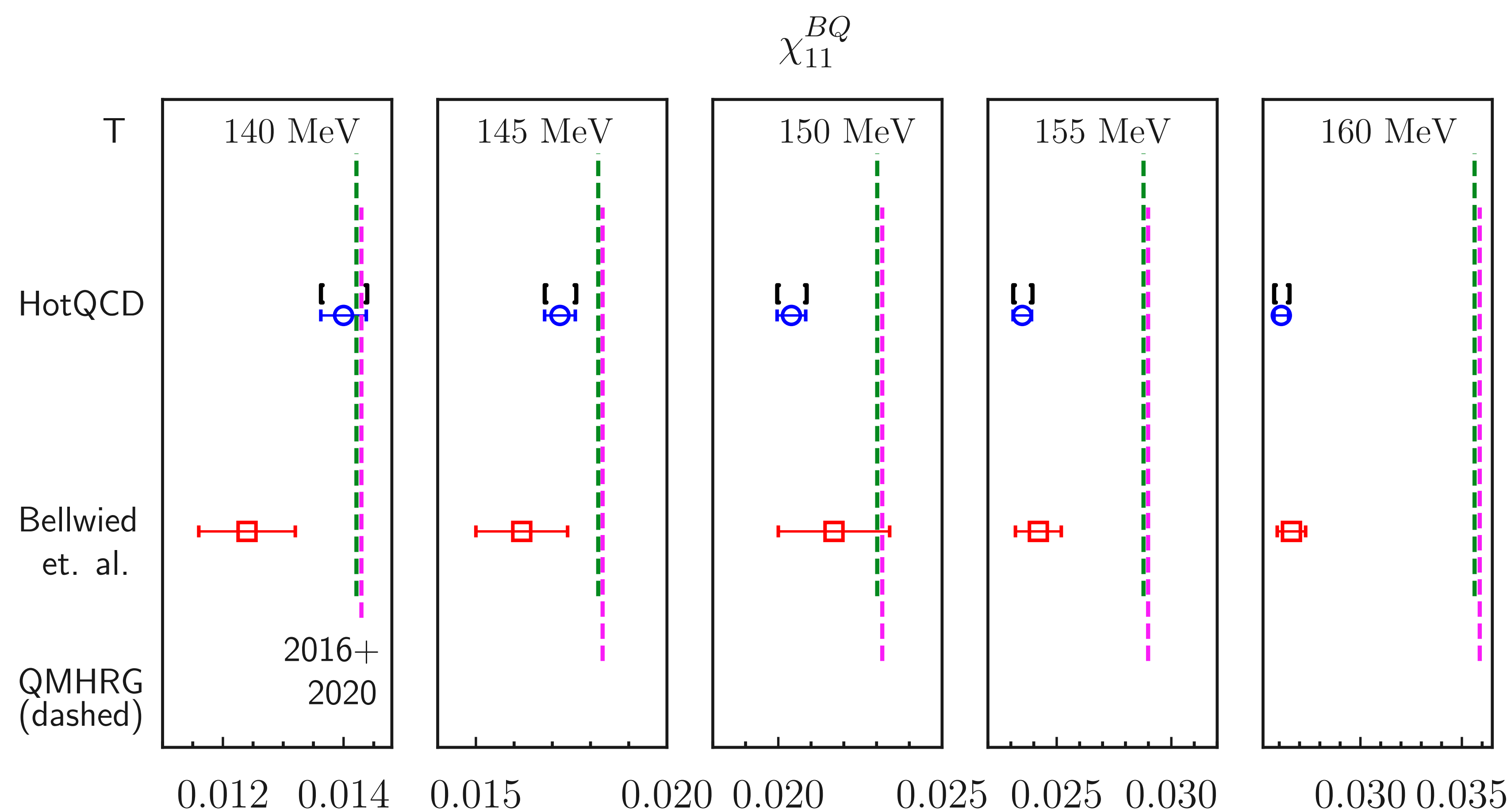




None of the non-interacting HRG models work at T_{pc}.

Deviations from non-interacting HRG are robust for χ_{11}^{BQ} as temperature approaches $T \sim 155$ MeV

Need for interacting HRG at T_{pc} : EVHRG , virial expansions ??



Excluded Volume HRG

Pressure of a HRG model which introduces additional repulsive interactions through excluded volume:

$$P = P_M + P_{\bar{M}} + P_B^{int} + P_{\bar{B}}^{int}$$

$$P_B^{int} = \frac{T}{b} \mathrm{W}\left[\sum_{i \in B} \frac{b}{T} \phi_B(T, \mu)\right], \quad \phi_B = \phi_{non-strange} + \phi_{strange}$$, ϕ_B is baryon pressure

$$= \phi_B - (b/T)\phi_B^2(T) + (3b^2/2T^2)\phi_B^3(T) + \ldots..$$

$\phi_B \sim e^{-m_B/T}$, at high temperature only linear term will survive

$b = 16/3\pi r^3$, where r is hard sphere radius of the hadron.

$$R_B^{EV} = \frac{(\chi_{11}^{BQ})_{EVHRG}}{(\chi_{11}^{BQ})_{HRG}} = \frac{(\chi_{11}^{BS})_{EVHRG}}{(\chi_{11}^{BS})_{HRG}} = \frac{(\chi_2^{B})_{EVHRG}}{(\chi_2^{B})_{HRG}}$$

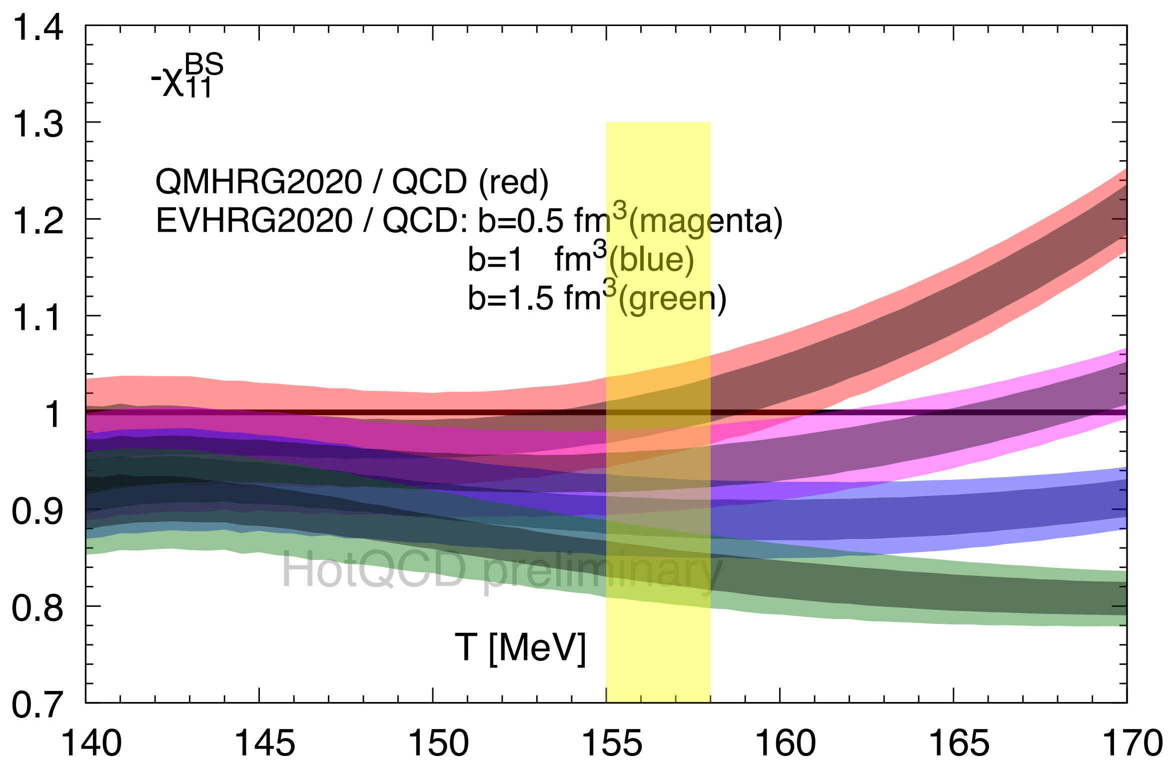
$$= 1 - 4bP_B^{HRG}(T) + \mathcal{O}(b^2)$$

P_B^{HRG} is the total pressure of baryon and anti-baryon.

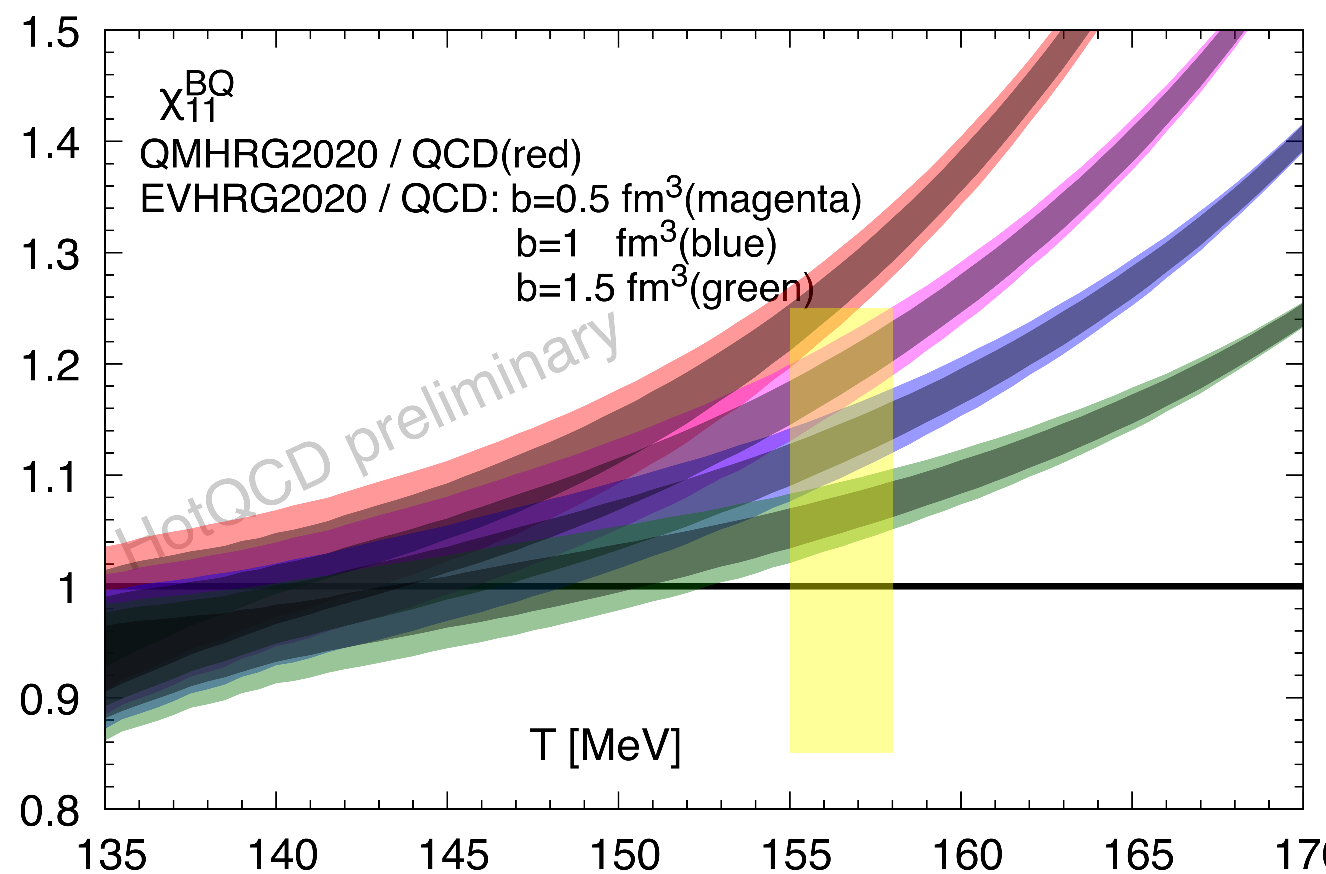
V. Vovchenko et al, Phys. Lett.B 775, 71 (2017), **K. Taradiy et al** PRC 100, 065202,

P. Huovinen et al Phys.Lett.B 777 (2018) 125-130

Excluded volume HRG



It is evident that the differences in the HRG/QCD cannot solely be taken care of in HRG models that use the same size for all baryons.



Improvement from non-interacting HRG,

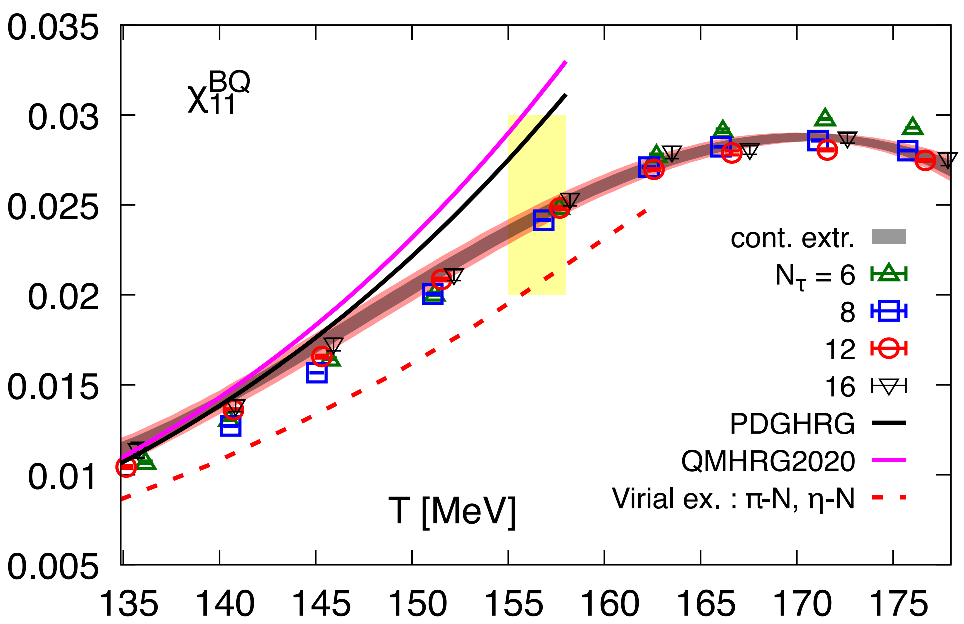
For χ_{11}^{BQ} one needs $b > 1.5\ \text{fm}^3$ and

For χ_{11}^{BS} one needs $b < 0.5\ \text{fm}^3$

$$\chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS}$$

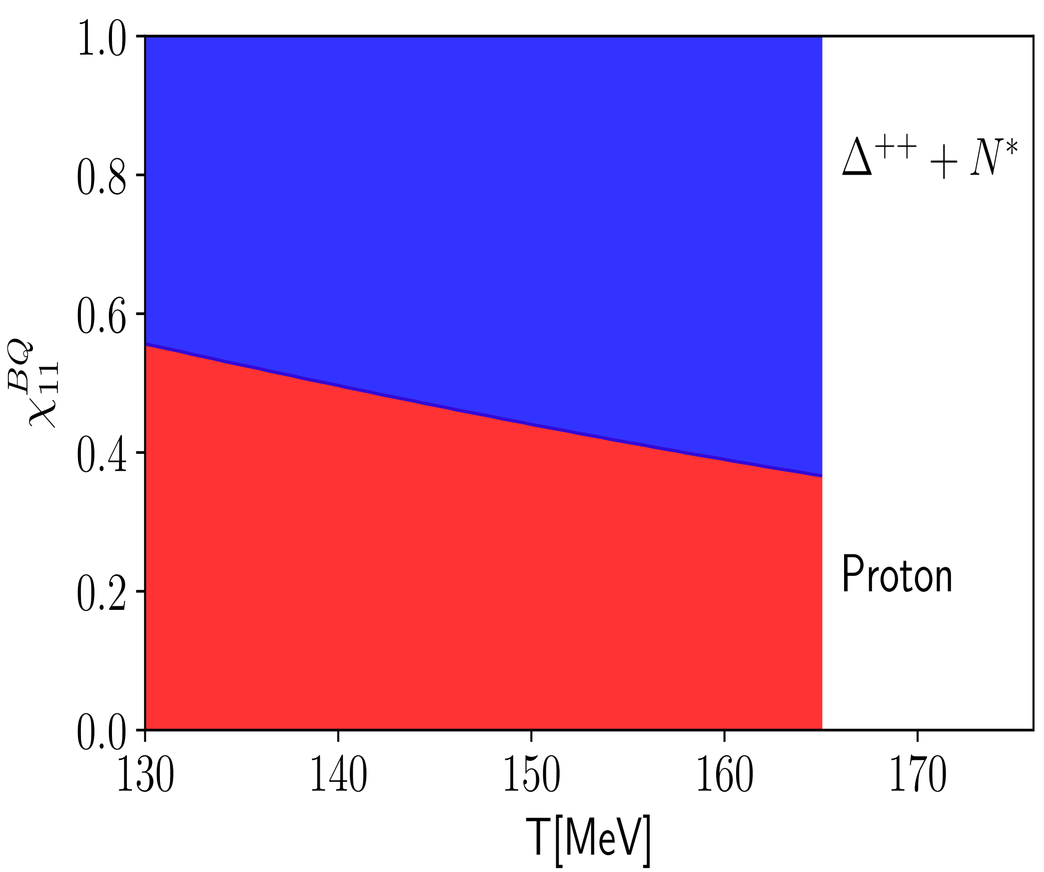
No single b will describe χ_2^B, χ_{11}^{BQ} and χ_{11}^{BS} simultaneously.

Two particle interactions (virial expansion)



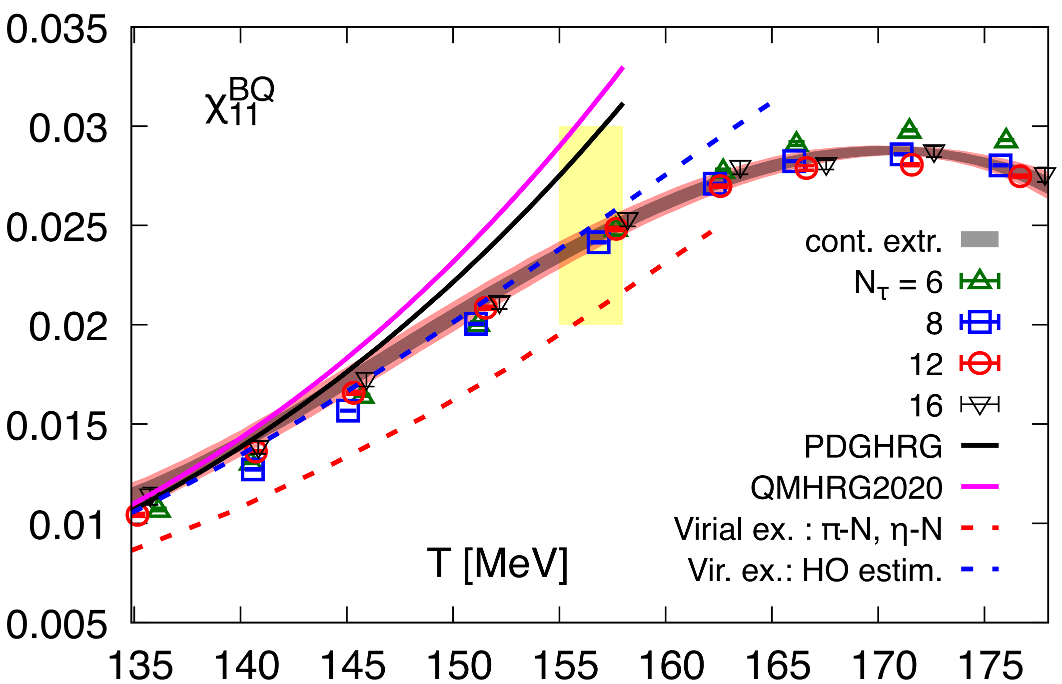
- Fate of the Δ^{++} in a hot hadron gas close to T_{pc} will determine the agreement of QCD and HRG.
- Currently, viral expansion based calculations consider interaction between pion and nucleon.

P. M. Lo, et al, Phys. Lett. B 778, 454 (2018)



Schematic Diagram to illustrate the importance of N^*, Δ^{++} , while ignoring contributions of other Baryons,

Higher order (virial expansion) using LQCD



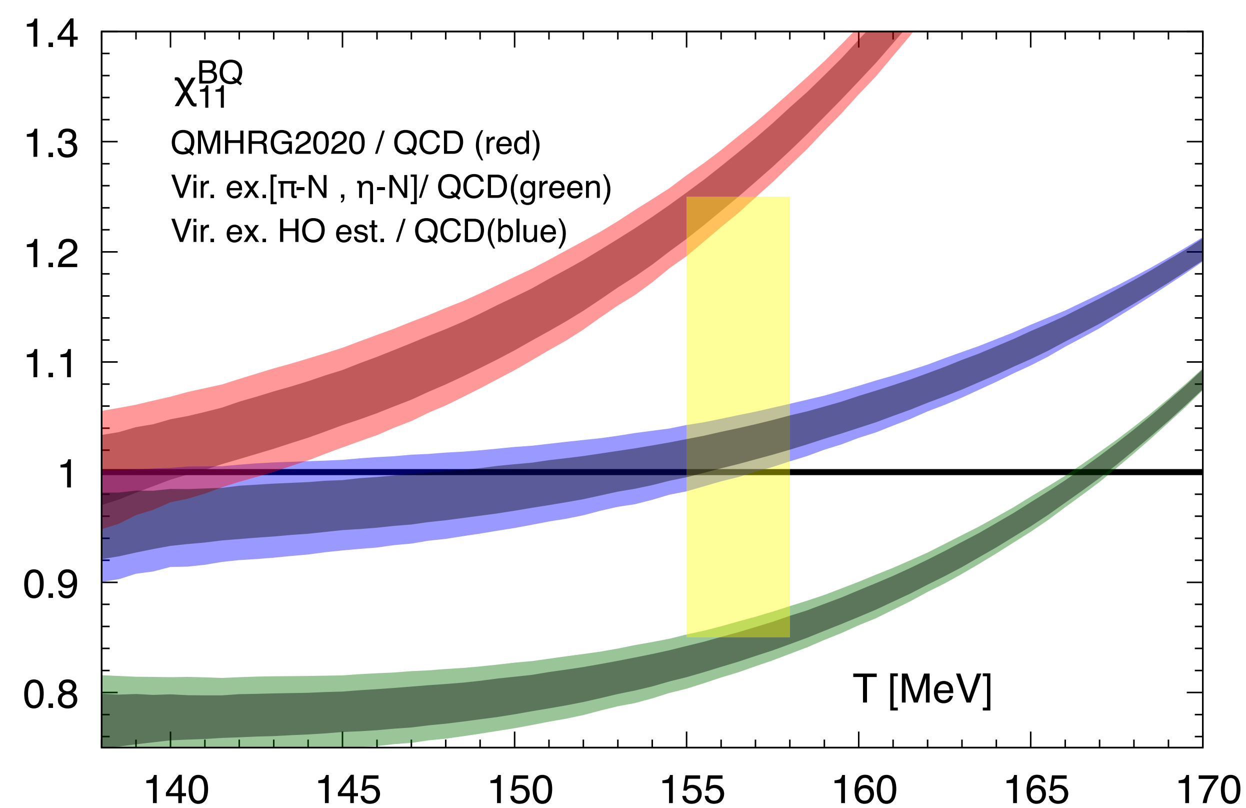
LO virial expansion falls below the LQCD data.

Higher order contributions currently modelled using Lattice QCD. A λ_3 parameter has been used to specify the strength of an effective $'\pi\pi N + higher\ order'$ interaction

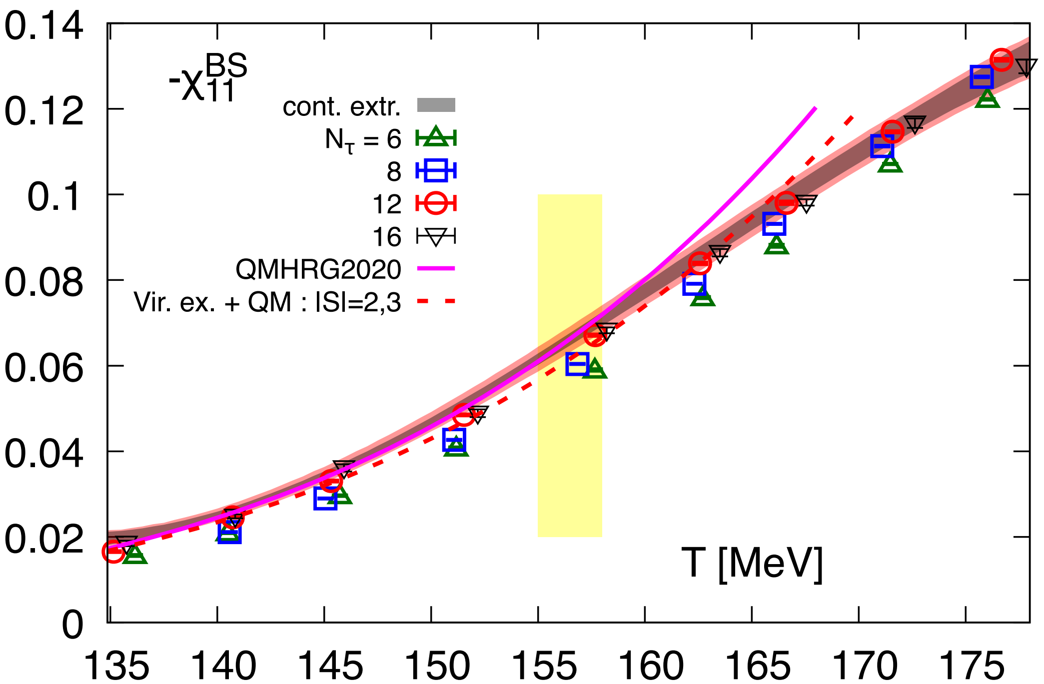
A. Andronic, et al. Phys. Lett. B 792, 304 (2019) , P.M LO, K. Redlich CPOD 2020

We update the value of λ_3 in the temperature range

$T \in [135 : 156], \lambda_3 \sim 3 - 1.5\ \mathrm{MeV}^{-2}$



Virial expansion + Missing strange particles

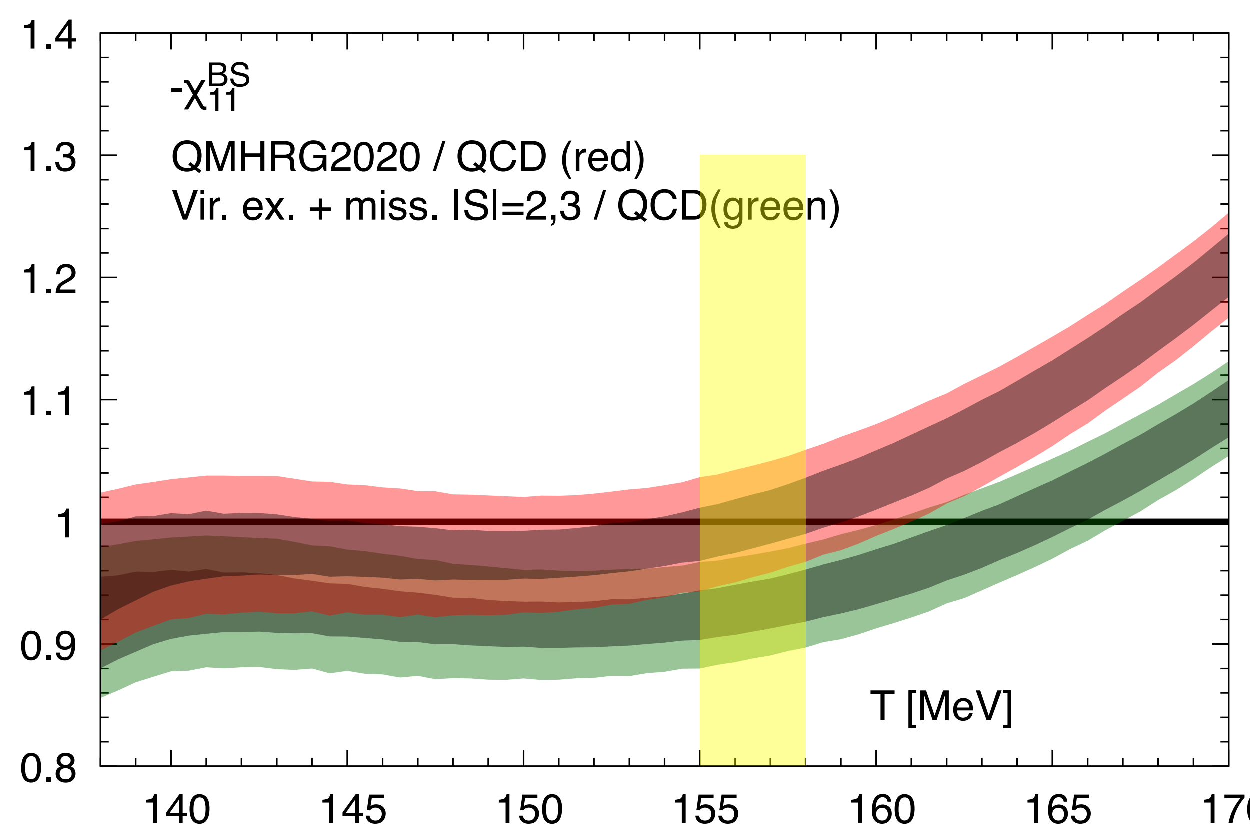


Virial expansion for BS channel works quite well.(**Ramírez et al**, Phys. Rev. C 98, 044910 (2018))

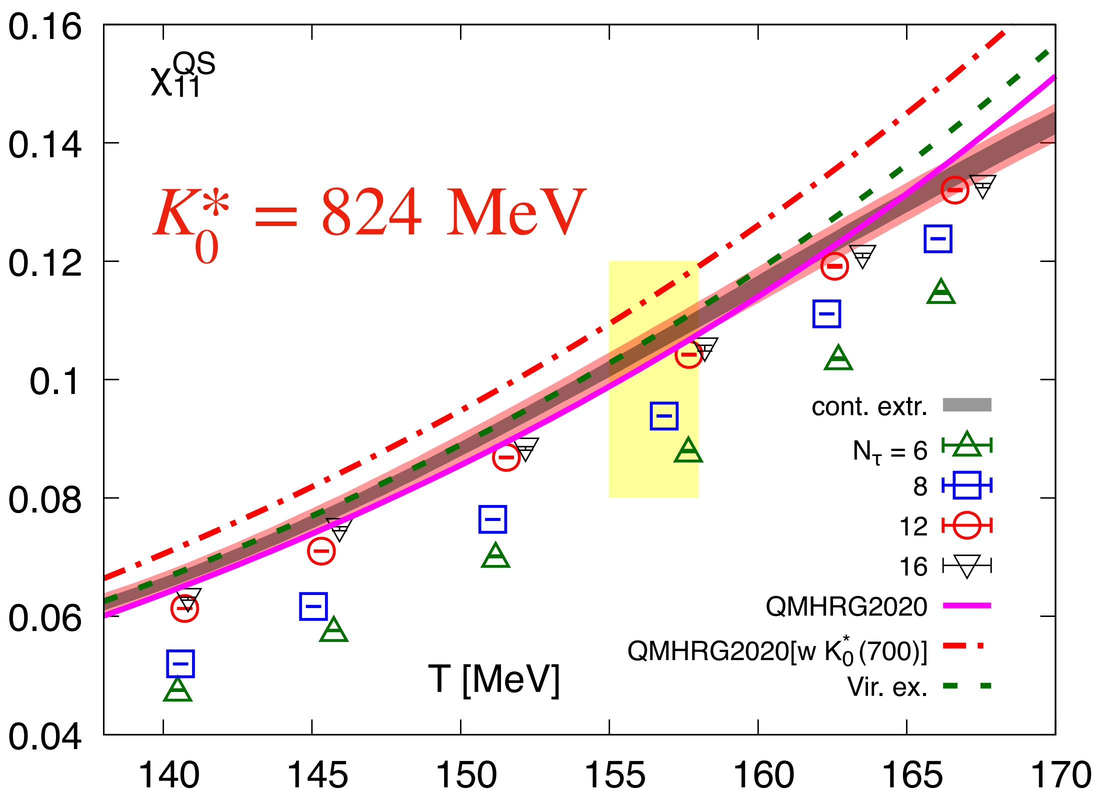
Multi-channel PWA by using the approximation of intermediate quasi two-body states.

Contributions from |S|=2,3 are still needed for agreement with QCD.

Virial expansion based treatment of $|S| = 2{,}3$ is not known !!



S-matrix based calculations of Kaons [K_0^*]



$$\chi_{1n}^{QS} \sim \sum_n Q_H S_H^n P_H$$

$$P_H \sim \exp(-m_H/T)$$

K_0^* does not contribute to the QCD thermodynamics as a point like non-interacting particle.

- K_0^* is not included in our QMHRG2020 list.
- At $T \sim 130$ MeV, the contribution of ground state kaon and its P-wave excitation $K^*(892)$ to χ_{11}^{QS} is more than 80 % .
- Contribution of $K_0^*(700)$ as a point like non-interacting resonances would change the HRG model result by almost 10 % .
- But the contribution is largely reduced in a virial expansion that makes use of information on scattering amplitudes in the S-wave $K - \pi$ channel.

B. Friman et al, Phys. Rev. D 92, 074003 (2015)

Close to pseudo-critical temperature $T = 156.5(1.5)\ \mathrm{MeV}$
the hadronic description fails.

Finite Chemical potential

Thermodynamics using Lattice QCD

The partition function of QCD:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}U \, \det[M(m_u, \mu_u)]^{1/4} \det[M(m_d, \mu_d)]^{1/4} \, \det[M(m_s, \mu_s)]^{1/4} \, e^{-S_G(U)}$$

Calculations at $\mu = 0$

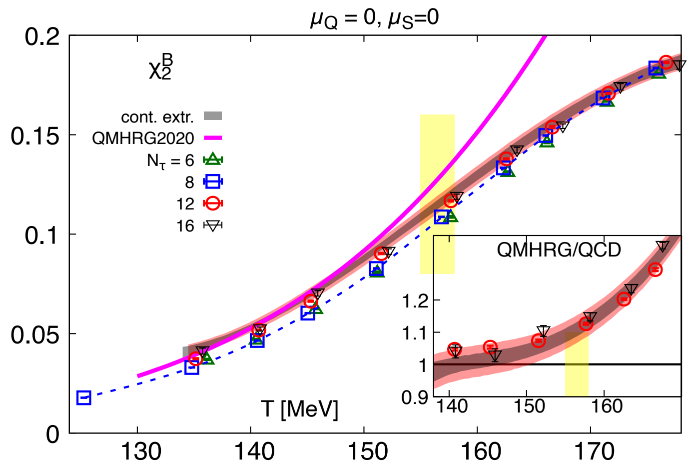
$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \ ,$$
$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \ ,$$
$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

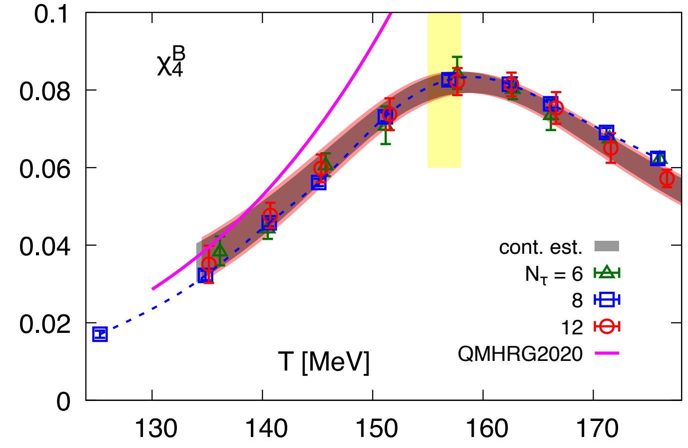
- The Taylor series of the QCD pressure at finite temperature and density: $\frac{P(T, \vec{\mu})}{T^4} = \frac{1}{VT^3} ln\mathcal{Z}_{QCD} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k \ , \ \hat{\mu} = \mu/T$

- Cumulants at vanishing chemical potential,

$$\chi_{ijk}^{BQS}(T,0) = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_X^{i,j,k}} \right|_{\mu_X=0} , \ X = B, Q, S$$

Expansion coefficients of the Taylor series

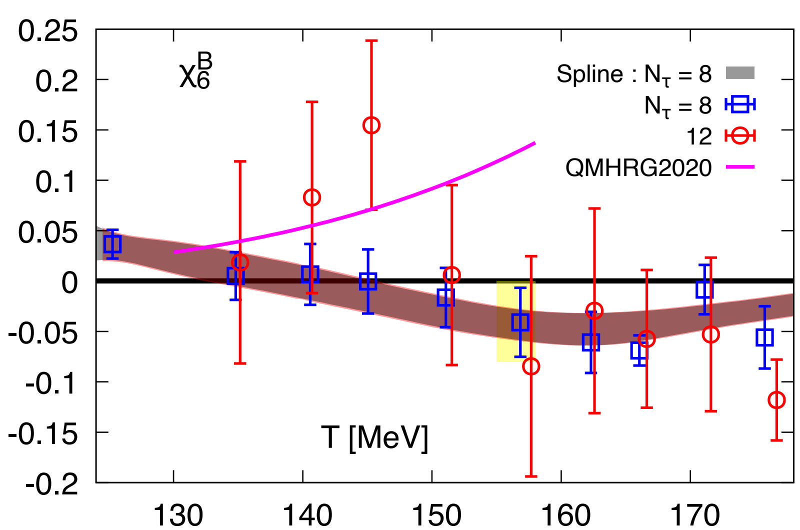


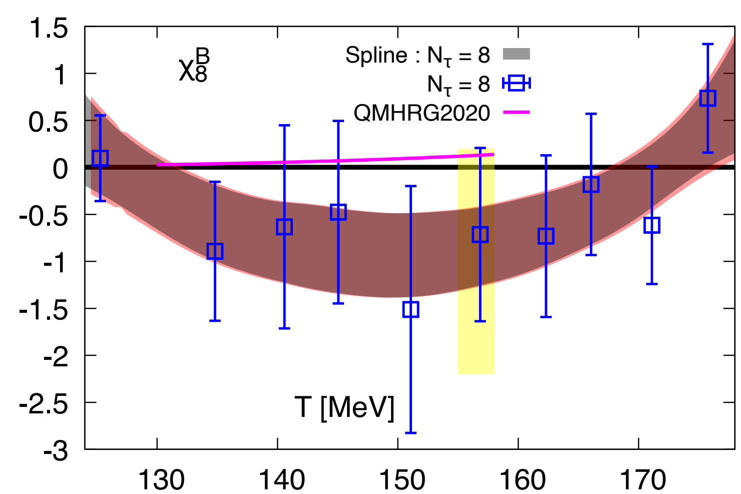


For simplicity we will consider the case, $\mu_B \neq 0,\ \mu_Q = \mu_S = 0$

Strictly positive for all temperatures.

- We show the continuum extrapolated results based on $N_\tau \in \{6,8,12,16\}$ and $\{6,8,12\}$ lattices respectively for the first two leading order coefficients.
- For the sixth and the eighth order expansion coefficients we have only used the $N_\tau = 8$ dataset, where we have generated ~ 1.5 million gauge configurations per T.
- Temperature dependence of the expansion coefficients depicts that deviations from the thermodynamics of a non interacting HRG rapidly become large for higher order cumulants at non-zero μ_B.





χ_8^B is strictly negative for temperature range, $T \in [135 : 165]$ MeV.

Radius of convergence of Taylor series upto $\hat{\mu}_B^8$

Taylor series can be constructed from the expansion co-efficients,

$$\chi_0^B(T,\mu_B) \equiv \frac{P(T,\mu_B) - P(T,0)}{T^4} = \sum_{k=1}^{4} P_{2k}(T)\hat{\mu}_B^{2k}, \text{ where, } P_{2k} = \frac{\chi_{2k}^B}{2k!}$$

$$\chi_1^B(T,\hat{\mu}_B) \equiv \frac{n_B(T,\mu_B)}{T^3} = \sum_{k=1}^{4} N_{2k-1}^B(T)\hat{\mu}_B^{2k-1}, \text{ where, } N_{2k-1}^B = \frac{\chi_{2k}^B}{(2k-1)!}$$

- The convergence will slow down for higher order cumulant as the highest expansion coefficient (χ_8^B) will be divide by smaller factorial.

- Range of reliability of the Taylor expansion will be smaller for the number density than the pressure series.

- The usual ratio estimator of radius of convergence can be described as, $r_{2k}^P = |P_{2k-2}/P_{2k}|^{1/2}$ and $r_{2k}^{nB} = |N_{2k-3}^B/N_{2k-1}^B|^{1/2}$ for pressure and number density respectively.

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$$\chi_0^B(T,\mu_B) \equiv \frac{P(T,\mu_B)-P(T,0)}{T^4} = \sum_{k=1}^{4} P_{2k}(T)\hat{\mu}_B^{2k}, \text{ Where, } P_{2k} = \frac{\chi_{2k}^B}{2k!}$$

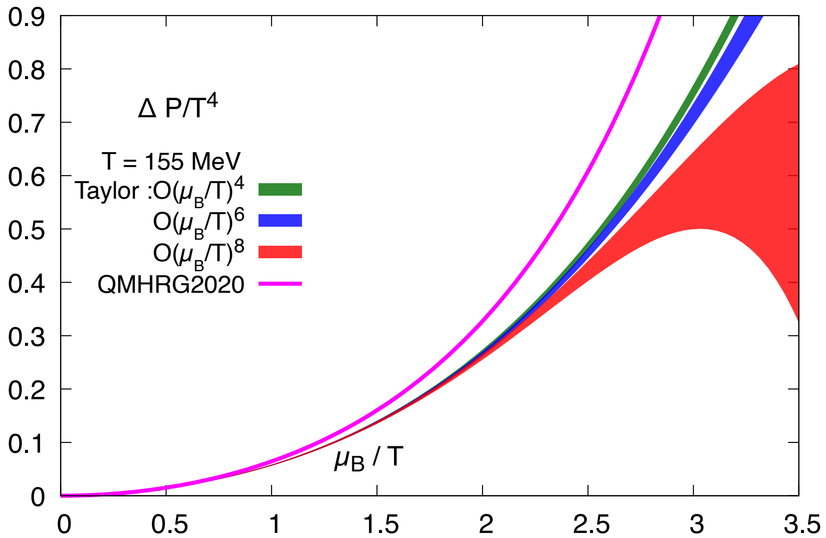
$$\chi_1^B(T,\hat{\mu}_B) \equiv \frac{n_B(T,\mu_B)}{T^3} = \sum_{k=1}^{4} N_{2k-1}^B(T)\hat{\mu}_B^{2k-1}, \text{ Where, } N_{2k-1}^B = \frac{\chi_{2k}^B}{(2k-1)!}$$

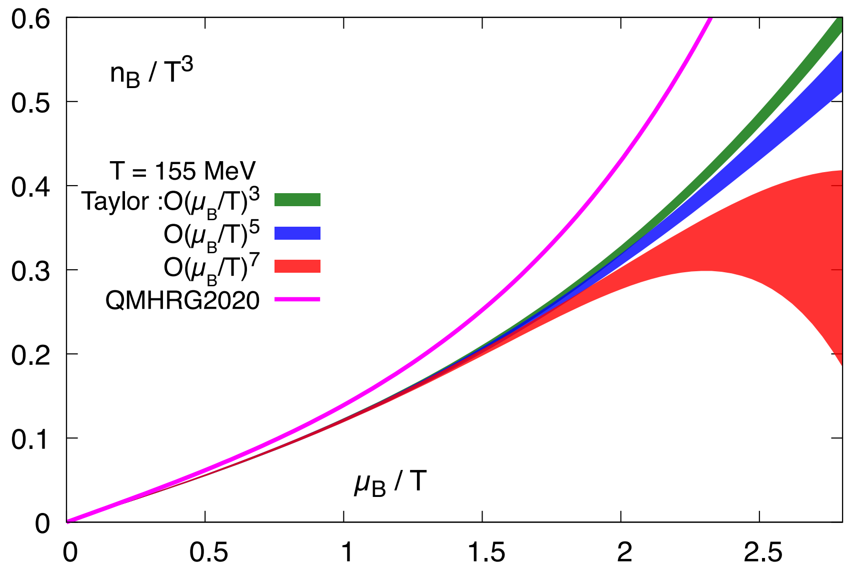
- The true radius of convergence can be written as ,
  $r_c^{True} = \lim_{k\to\infty} r_{2k}^P = \lim_{k\to\infty} r_{2k}^{nB}$.

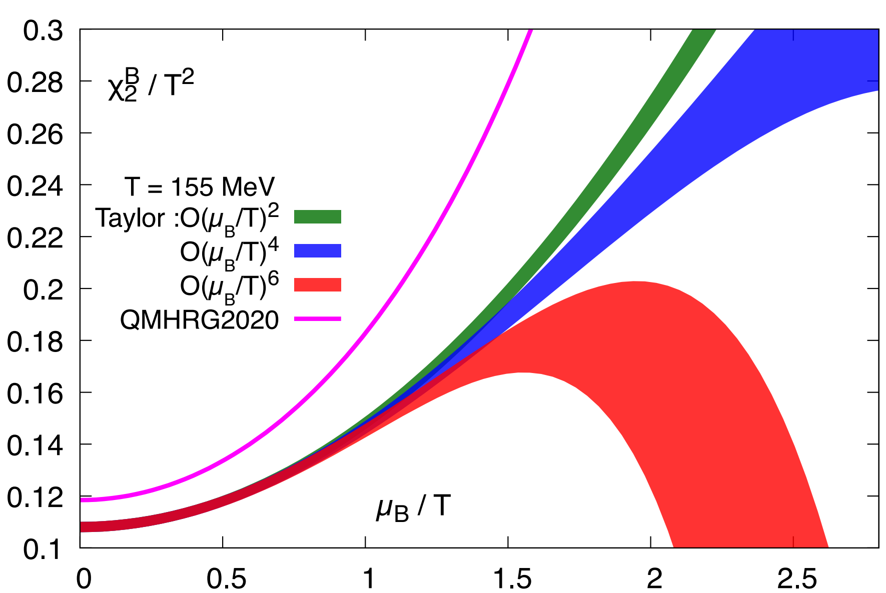
- The usual ratio estimator of radius of convergence can be described as,
  $r_{2k}^P = |P_{2k-2}/P_{2k}|^{1/2}$ and $r_{2k}^{nB} = |N_{2k-3}^B/N_{2k-1}^B|^{1/2}$ for pressure and number density respectively.

$$r_{2k}^P/r_{2k}^{nB} = \sqrt{[2k/(2k-2)]} = 1 + 1/2k + O(k^{-2})$$

Radius of convergence and reliability of the expansion







Reminder : χ_8^B is strictly negative for temperature range, $T \in [135 : 165]$ MeV.

- Taylor expansion of the pressure, net baryon-number density and the second order cumulant of net baryon-number fluctuations are shown for different orders.
- Agreement between subsequent orders shifted to smaller values of the chemical potential for higher order cumulants.
- The deviations from the thermodynamics of non interacting HRG also increase rapidly for higher order cumulants.

Searching for CEP using Padé approximants

In practice we work with finite number of coefficients!!

$$f(x) = \sum_{i=0}^{n} c_i x^i$$

- **Lee Yang** : Phase transitions are related to singularities of the Taylor series on the real axis.

- If all the expansion coefficients are of same sign, could be an indication that the singularity of the series is on the real axis and hence is an indication of a critical point.

- Alternatively, one could construct Padé approximants which are rational functions of the form, $f(x) = \dfrac{\sum_{i=0}^{a} c_i x^i}{1 + \sum_{j=0}^{b} d_j x^j}$, and evaluate its singularities.

- Furthermore, singularities in the complex plane can show some universal scaling behaviour related to LYEs close to a critical point (this will not be discussed here).

Padé approximants from $\hat{\mu}_B^8$ Taylor series

$$\Delta P/T^4 = \sum_{k=1}^{4} P_{2k}(T)\hat{\mu}_B^{2k} = (\bar{x}^2 + \bar{x}^4 + c_{6,2}\bar{x}^6 + c_{8,2}\bar{x}^8)P^2/P_4,\ \bar{x} = \sqrt{\frac{P_4}{P_2}}\ \hat{\mu}_B$$

Reminder : P_2 and P_4 are strictly positive for all temperatures.

- One can construct various $[m,2]$ and $[n,4]$ Padé's from the above series. $[m \in \{2,4,6\}$ and $n \in \{2,4\}]$

- The convergence of Padé approximants will be unaffected by a singularity in the complex plane contrary to the Taylor series.

- The poles of the Padé approximants closest to the origin determine the radius of convergence.

- The poles of a general $[m,2]$ and $[n,4]$ Padé's are the usual ratio estimator (r_c^n) and Mercer Roberts estimator (r_c^{MR}) of radius of convergence of the Taylor series.

$$r_c^{MR} = \frac{c_{n+2}\,c_{n-2} - c_n^2}{c_{n+4}\,c_n - c_{n+2}^2}\ ,\quad n \text{ even}$$

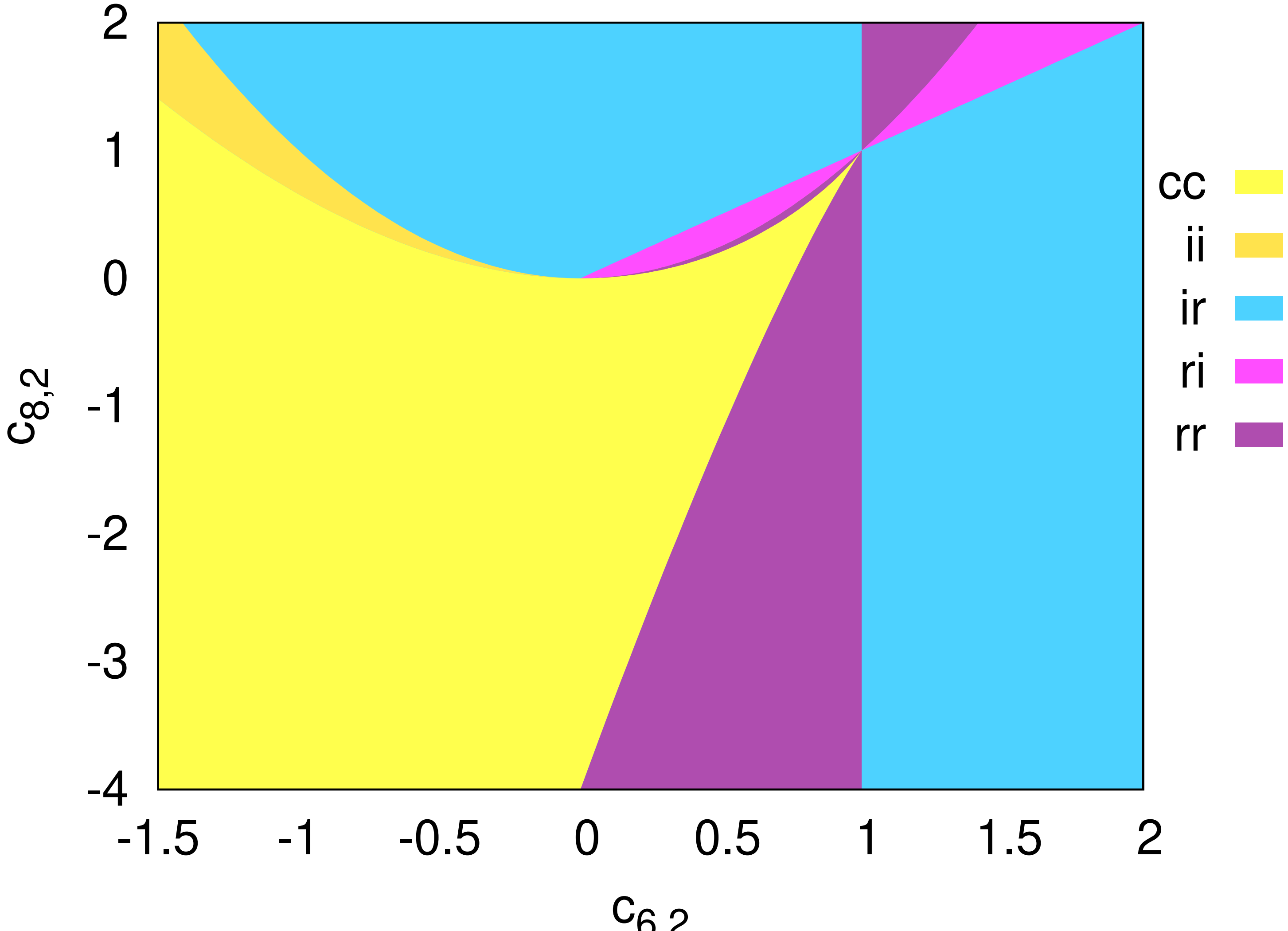
Constraints for a real pole of [4,4] Padé

Reminder : Critical points are related to the singularities on the real axis

$$P[4,4] = \frac{(1-c_{6,2})\bar{x}^2 + \left(1 - 2c_{6,2} + c_{8,2}\right)\bar{x}^4}{(1-c_{6,2}) + (c_{8,2} - c_{6,2})\bar{x}^2 + (c_{6,2}^2 - c_{8,2})\bar{x}^4}$$

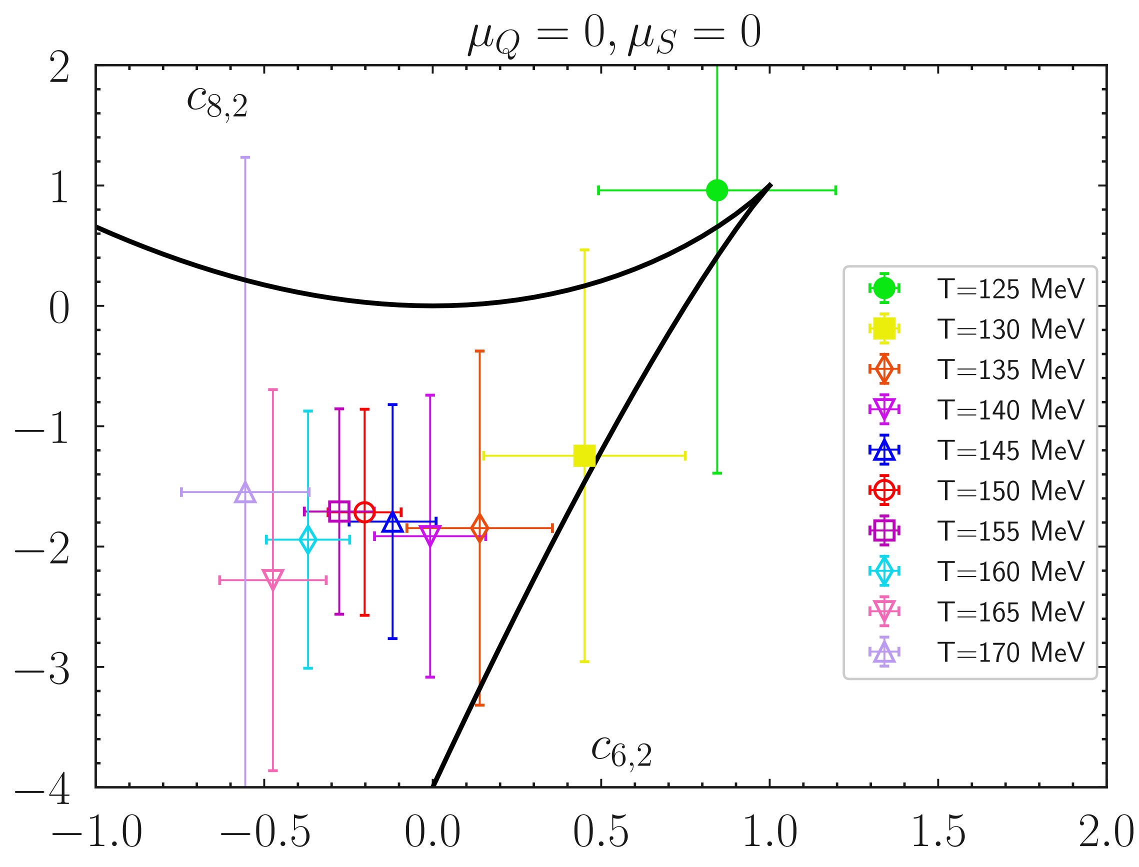
Poles can be written as,

$$z \equiv \bar{x} \qquad z^{\pm} = \frac{c_{8,2} - c_{6,2} \pm \sqrt{(c_{8,2} - c_{8,2}^{+})(c_{8,2} - c_{8,2}^{-})}}{2(c_{8,2} - c_{6,2}^2)}; \; c_{8,2}^{\pm} = -2 + 3c_{6,2} \pm 2(1 - c_{6,2})^{3/2}$$



Values(including sign) of $c_{6,2}$ and $c_{8,2}$ which are related to χ_6^B and χ_8^B are crucial to have a pole in the real axis.

Location of the poles from [4,4] Padé approximants for QCD



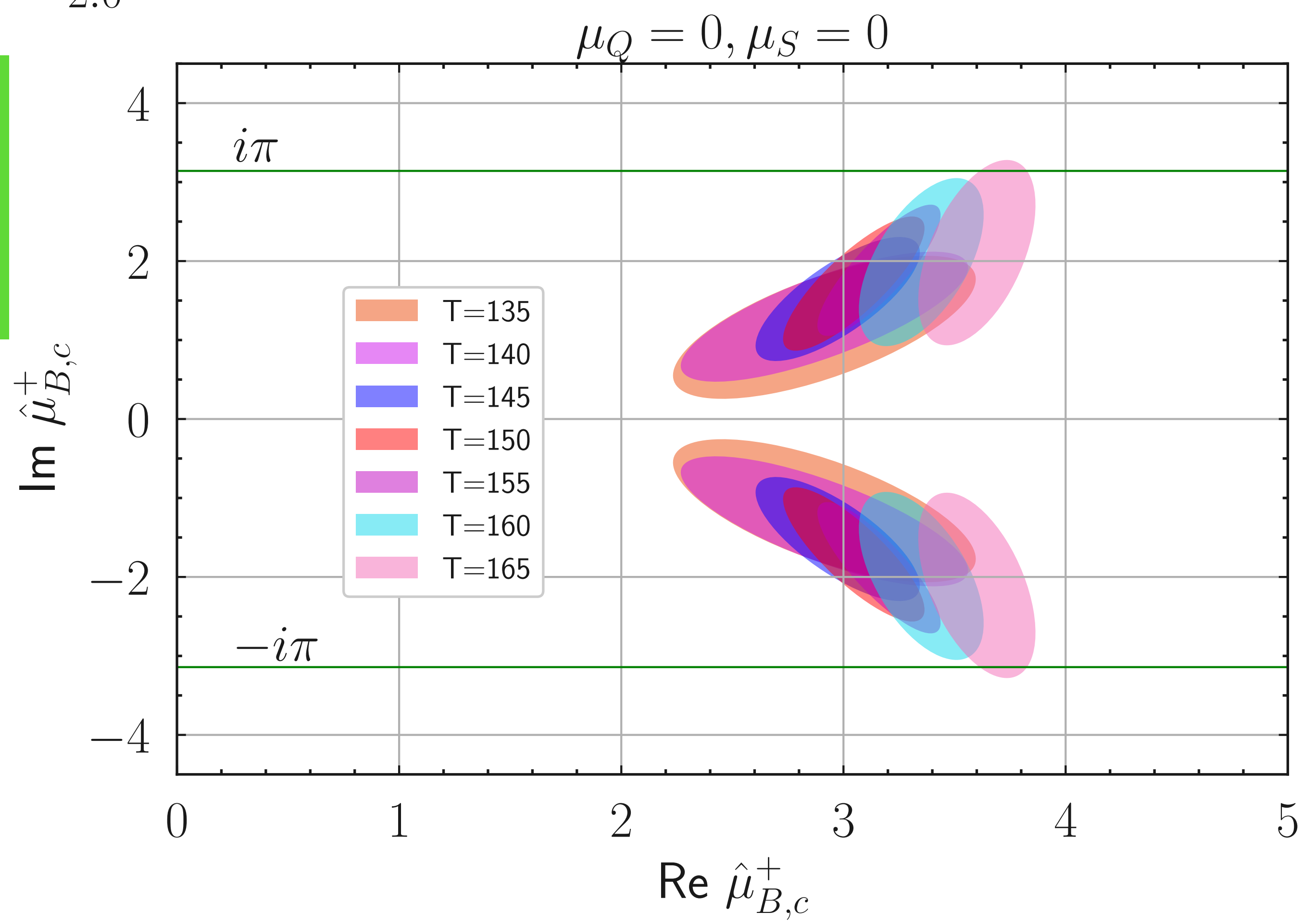
Poles are complex for the temperature range,

$T \in [135:165]$ MeV.

The possibility of occurrence of a real pole cannot be ruled out for $T < 135$ MeV

Only complex poles are shown.

The poles show a tendency to move to real axis for $T < 135$ MeV

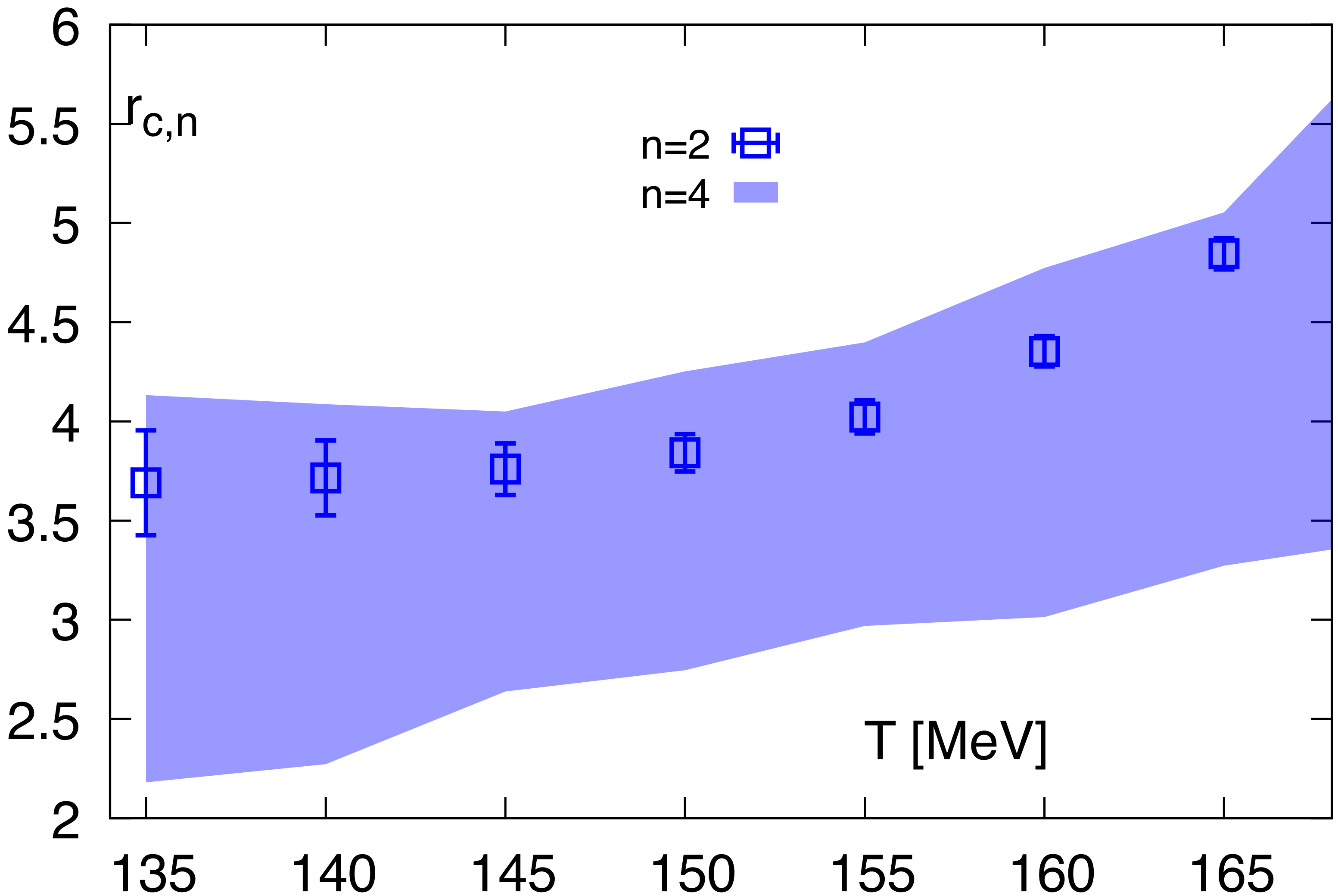


- Hence, the bound for CEP is, $T^{CEP} < 135$ MeV.
- Consistent with $T^{CEP} < T_c^{chiral}$ (~ 130 MeV)

Radius of convergence from diagonal Padé approximants

$$r_{c,2} = \sqrt{12\bar{\chi}_2^B/\bar{\chi}_4^B}$$

$$r_{c,4} = r_{c,2}\ |z^+z^-|^{1/4} = \sqrt{\frac{12\bar{\chi}_0^{B,2}}{\bar{\chi}_0^{B,4}}}\ \left|\frac{1-c_{6,2}}{c_{6,2}^2-c_{8,2}}\right|^{1/4}$$

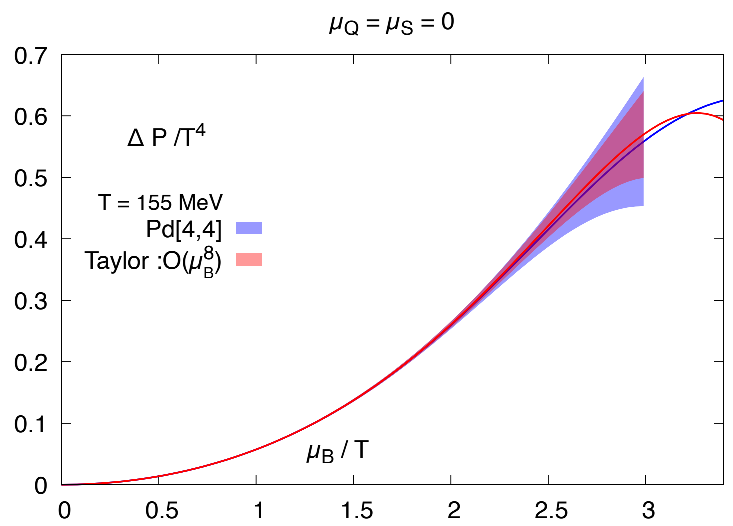


The radius of convergence of [2,2] and [4,4] Padé in the temperature range $T \in [135 : 165]$ MeV obtained as, $\left|\mu_B^c\right| \sim [2.5 : 4.5]$.

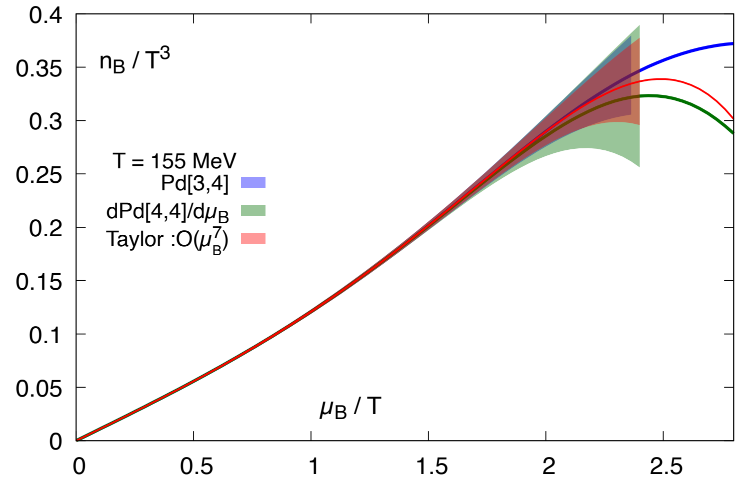
This is also the current updated estimate for the radius of convergence from a μ_B^8 Taylor series.

✦ Hence, the bound for CEP is, $\hat{\mu}_B^{CEP} > 2.5$.

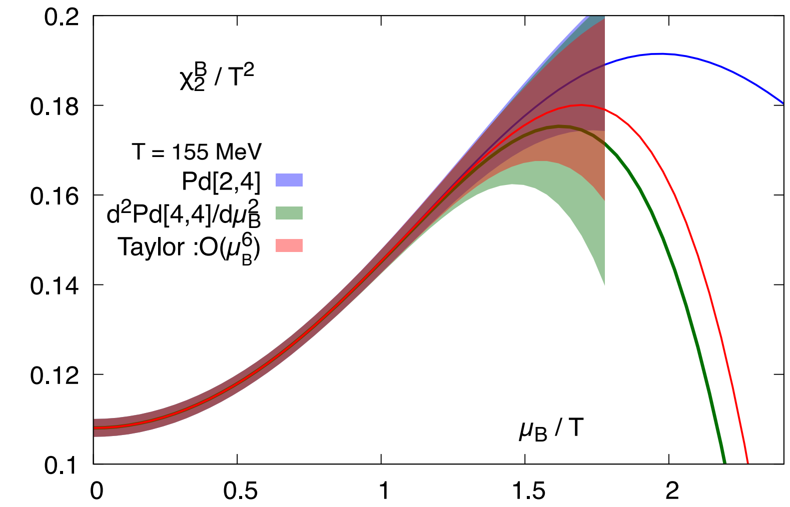
Radius of convergence and reliability of expansion



Padé approximant and Taylor series show good agreement for , $\hat{\mu}_B \leq 2.5$



Padé approximant and Taylor series show good agreement for , $\hat{\mu}_B \leq 2$



Padé approximant and Taylor series show good agreement for , $\hat{\mu}_B \leq 1.5$

- ★ The current range of reliability of the expansions are different for different observables. Which is , $\hat{\mu}_B/T \sim 2.5,\ 2$ and 1.5 for pressure, net-baryon number density and second order baryon number fluctuations close to the pseudo-critical temperature.
- ★ All the observables have same "true" radius of convergence. The current updated estimate is $\left|\hat{\mu}_B^c\right| \sim 3$, close to the pseudo-critical temperature.

Similar qualitative results can be obtained for $n_S = 0, n_Q/n_B = 0.4$, the condition that is met in the relativistic Heavy Ion collision experiments

Pseudo-critical line from LQCD and Freeze out from RHIC

Thermal conditions at HIC :

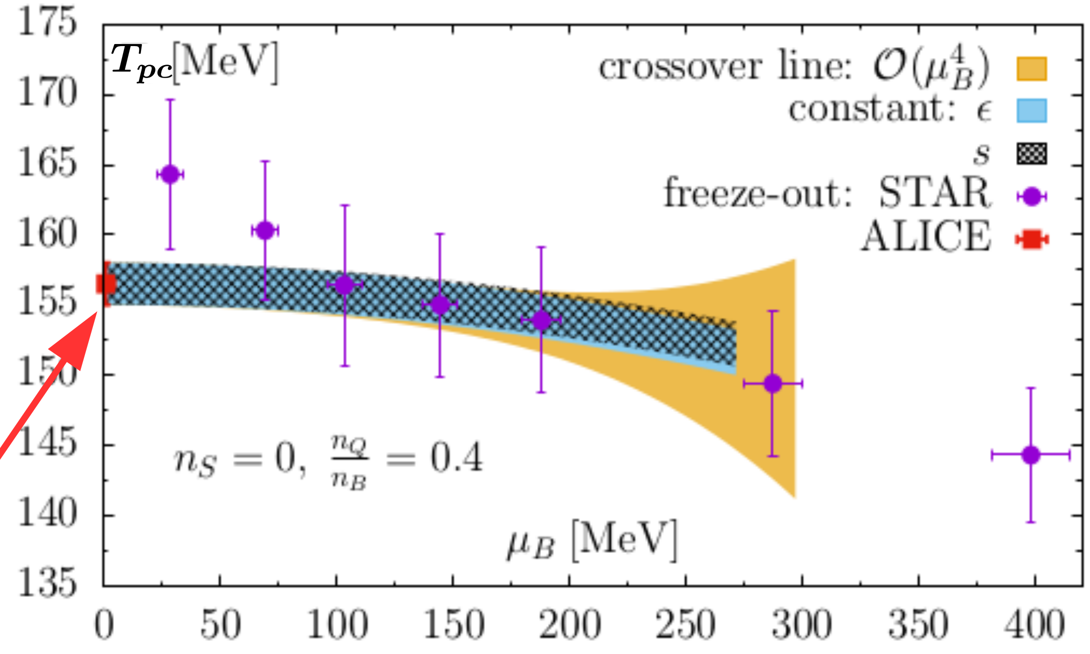
1. **Strangeness Neutrality :** $\langle N_S \rangle = 0 = \chi_1^S$
2. $$\frac{\langle N_Q \rangle}{\langle N_B \rangle} = 0.4 = \frac{\chi_1^Q}{\chi_1^B}$$

Modification in Taylor series,

$$\mu_S = s_1 \mu_B + s_3 \mu_B^3 + \ldots..$$

$$\mu_Q = q_1 \mu_B + q_3 \mu_B^3 + \ldots..$$

$$T_{pc}(\mu_B) = T_{pc} \left(1 - \kappa_2 \left(\frac{\mu_B}{T_c} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c} \right)^4 + \ldots \right)$$



STAR: arXiv:1701.07065

A. Andronic et al., Nature 561 (2018) 321

$T_{pc} = (156.5 \pm 1.5)$ MeV

$\kappa_2 = 0.012(4)$

$\kappa_4 = 0.000(4)$

A. Bazavov et al. [HotQCD], Phys. Lett. B795, 15 (2019), arXiv:1812.08235

$T_{pc} = (158.0 \pm 0.6)$ MeV

$\kappa_2 = 0.0153(18)$

$\kappa_4 = 0.00032(67)$

S. Borsanyi, et al, arXiv:2002.02821

Comparison of LQCD with experiments

$\mu_Q = \mu_S = 0$, for simiplicity

$$P(T,\mu_B) - P(T,0) = \chi_2^B \mu_B^2/2! + \chi_4^B \mu_B^4/4! + \chi_6^B \mu_B^6/6! + \chi_8^B \mu_B^8/8!$$

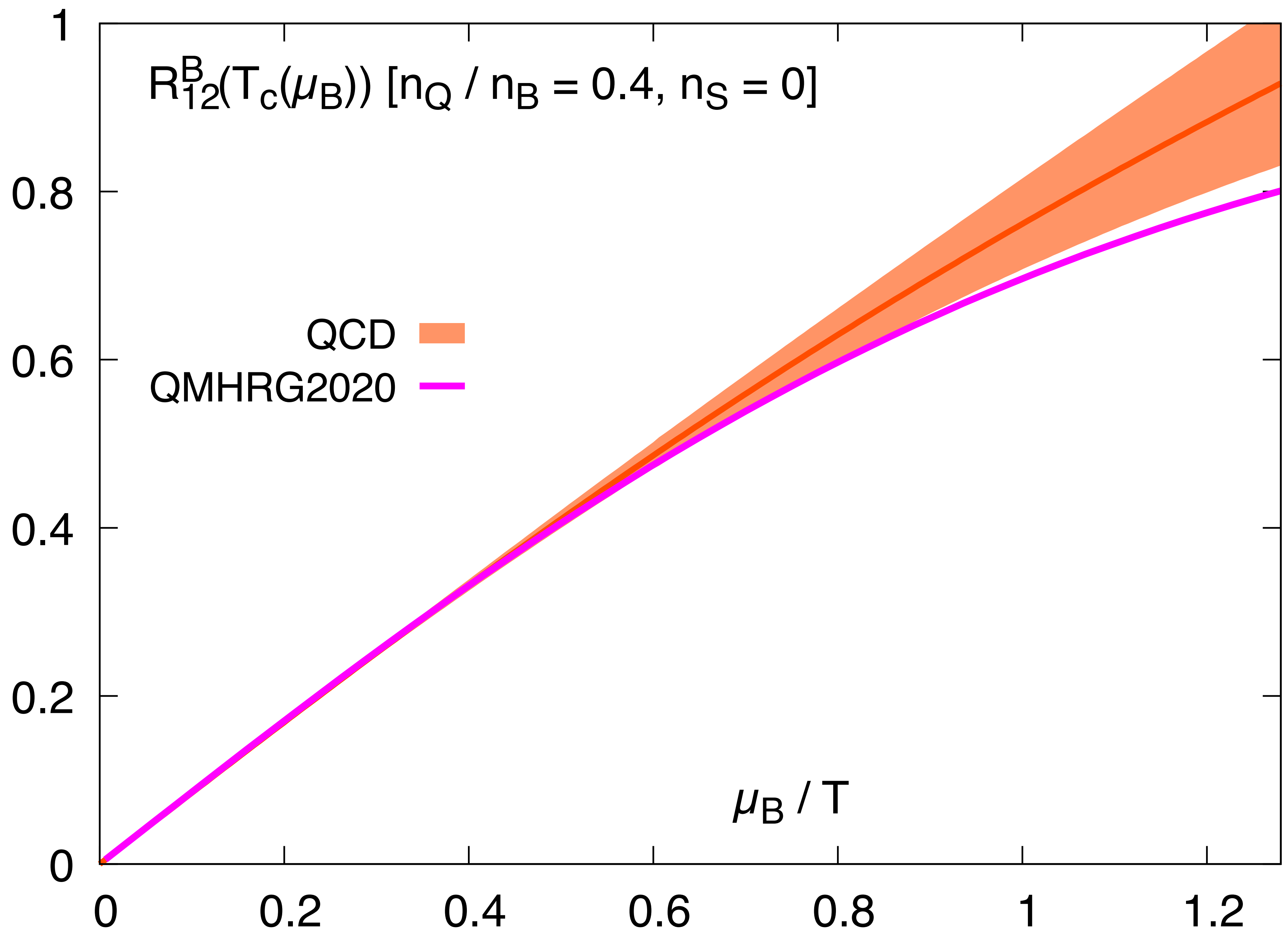
$$\text{Mean}: \chi_1^{\text{B}}(\text{T},\mu_{\text{B}}) = \chi_2^B \mu_B + \chi_4^B \mu_B^3/3! + \chi_6^B \mu_B^5/5! + \chi_8^B \mu_B^7/7! = \frac{1}{VT^3} M_B$$

$$\text{Variance}: \chi_2^{\text{B}}(\text{T},\mu_{\text{B}}) = \chi_2^B + \chi_4^B \mu_B^2/2! + \chi_6^B \mu_B^4/4! + \chi_8^B \mu_B^6/6! = \frac{1}{VT^3} \sigma_B^2$$

Volume independent ratio of susceptibility,

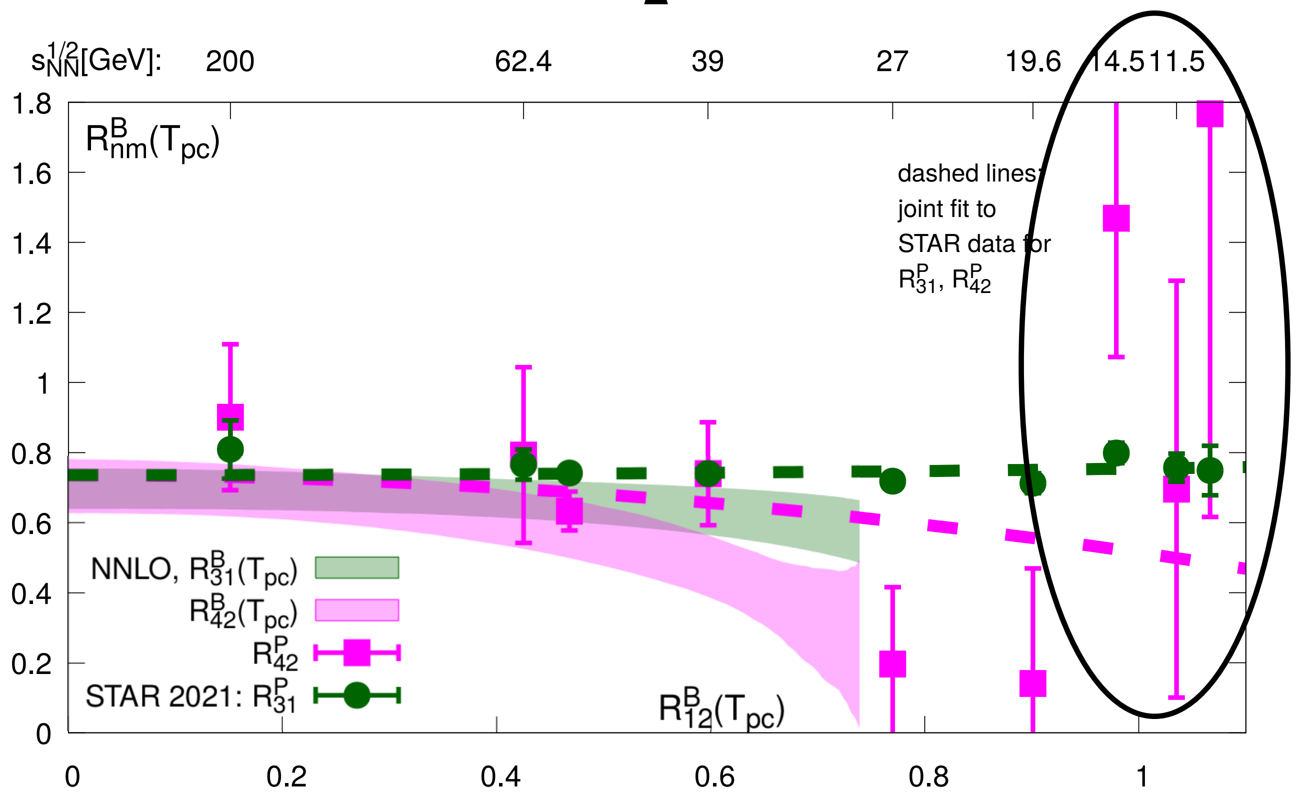
$$\frac{M_B}{\sigma_B^2} = \frac{\chi_1(T,\mu_B)}{\chi_2(T,\mu_B)}, \quad \frac{S_B \sigma_B^3}{M_B} = \frac{\chi_3(T,\mu_B)}{\chi_1(T,\mu_B)}, \quad \kappa_B \sigma_B^2 = \frac{\chi_4(T,\mu_B)}{\chi_2(T,\mu_B)}$$

Ratio of Mean and Variance in the pseudo-critical line



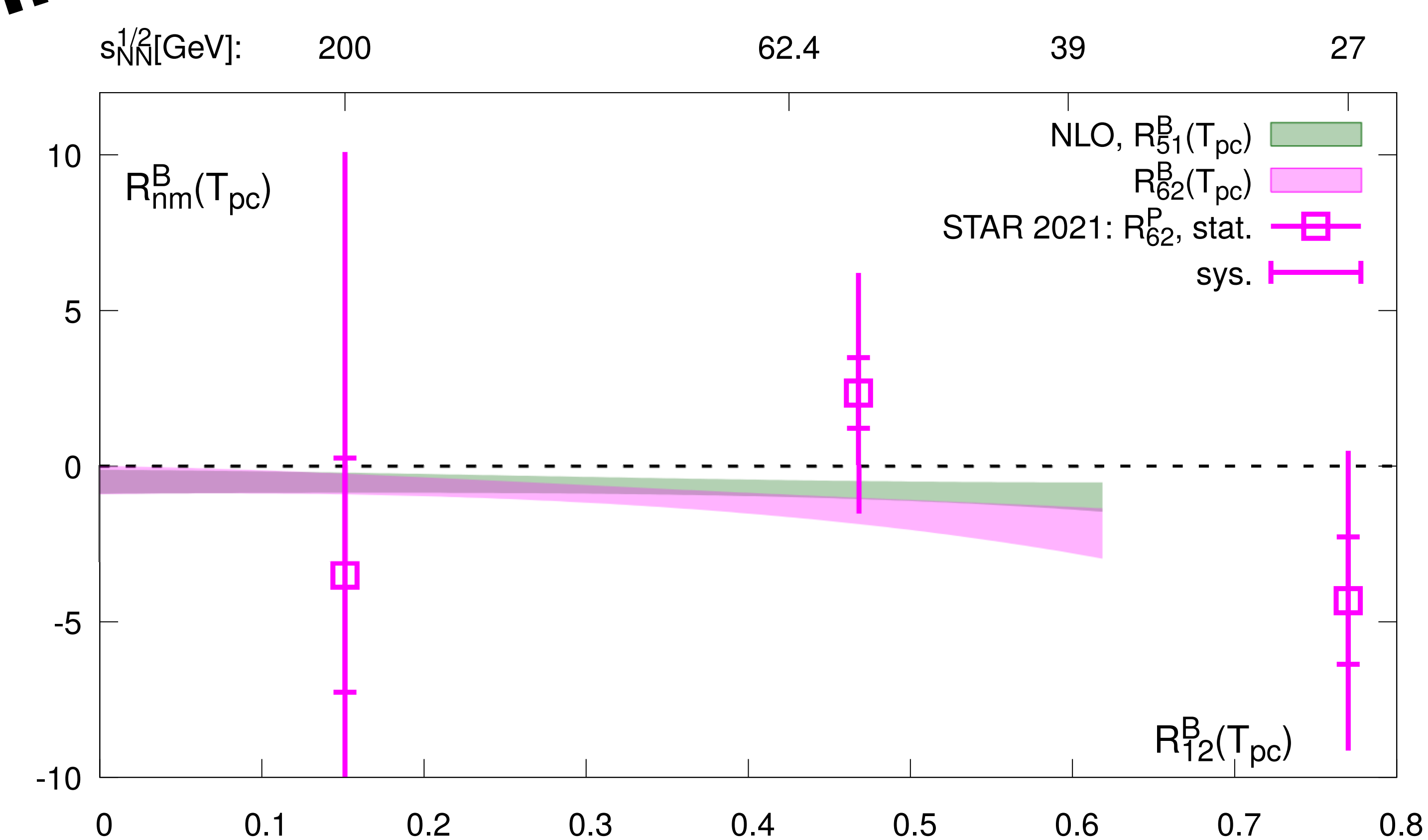
A. Bazavov et al, *Phys.Rev.D* 101 (2020) 7, 074502

STAR 2021 update



Indication on critical point ??

HotQCD Preliminary and STAR 2021!!



Constraint on CEP ,

$T_c^{CEP} < 135$ $\hat{\mu}_B^{CEP} > 2.5$