Anomaly matching in QCD thermal phase transition

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Based on

- [1706.06104] with Hiroyuki Shimizu
- [1901.08188]

Introduction

QCD phase transition is important for cosmology:
Axiom abundance etc.

Most radical scenario: [Witten,1984]

If the phase transition is first order, the dark matter might be produced purely by QCD phase transition.
(Several other conditions need to be satisfied.)

The dark matter might be explained by the standard model!

Introduction

Lattice simulations suggest that QCD phase transition is cross-over (i.e. no definite phase transition).

But it is not completely settled yet, especially in the limit of small quark masses.

Therefore, it is desirable to study it by methods which do not rely on numerical simulations.

Introduction

A rough version of my claim

(I will explain more precise technical result later.)

If

- Small quark mass approximation is good,
- Large N expansion is good,

then

- QCD phase transition may be naturally first order.

Introduction

Both small quark mass approximation and
large N expansion are qualitatively very good
in QCD at zero temperature.

- Chiral perturbation theory,…
- Most mesons as $q\bar{q}$ (rather than $qq\bar{q}\bar{q}$), OZI rule,
- Simulation for pure Yang-Mills, AdS/CFT,…
  ($N_c = 3 \simeq \infty$)

Crossover phase transition may be in tension with those good concepts of QCD and the argument I discuss later.

Contents

't Hooft anomaly

't Hooft anomaly matching

UV: gauge fields + fermions with global symmetry F

↓ confinement

IR: ???

Anomaly of F in UV = Anomaly of F in IR

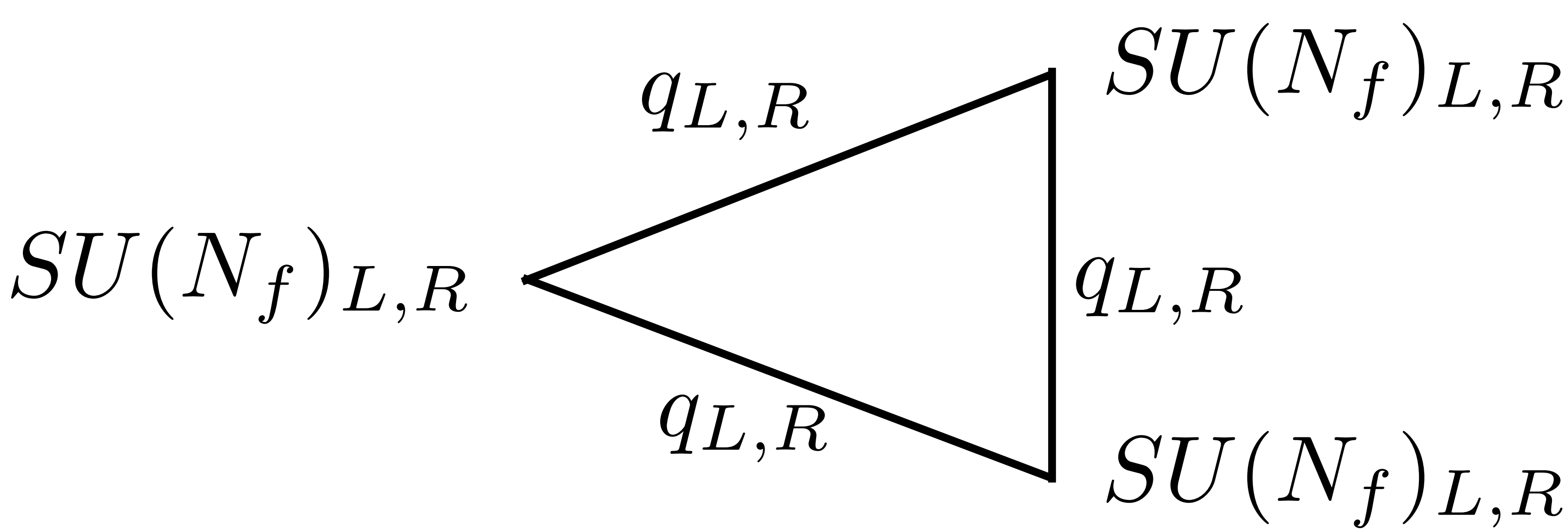
't Hooft anomaly in QCD

't Hooft anomaly matching in QCD at zero temperature

In QCD, there exist perturbative triangle anomalies of chiral symmetry $SU(N_f)_L \times SU(N_f)_R$

q_L : left-handed quarks, rotated by $SU(N_f)_L$

q_R : right-handed quarks, rotated by $SU(N_f)_R$



’t Hooft anomaly in QCD

UV: The quarks have the ’t Hooft anomaly

↓ confinement

IR: If there is no chiral fermion,
the chiral symmetry must be spontaneously broken.

’t Hooft anomaly in QCD

’t Hooft anomaly matching gives an important relation between the two most important concepts in QCD:

Confinement ↔ Chiral symmetry breaking

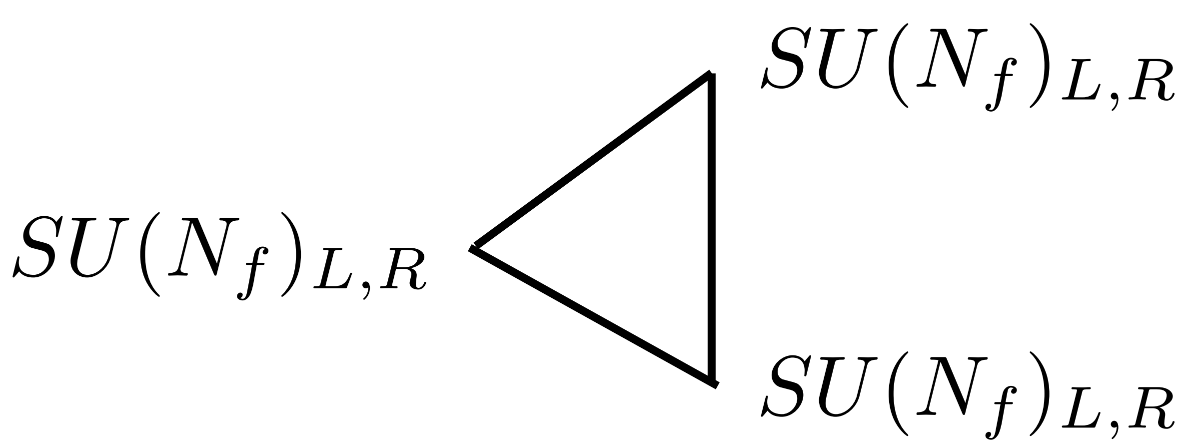
How about finite temperature?

’t Hooft anomaly in QCD

Finite temperature?

spacetime: $R^4 \to R^3 \times S^1$ $\quad$ S^1 : Wick-rotated time direction

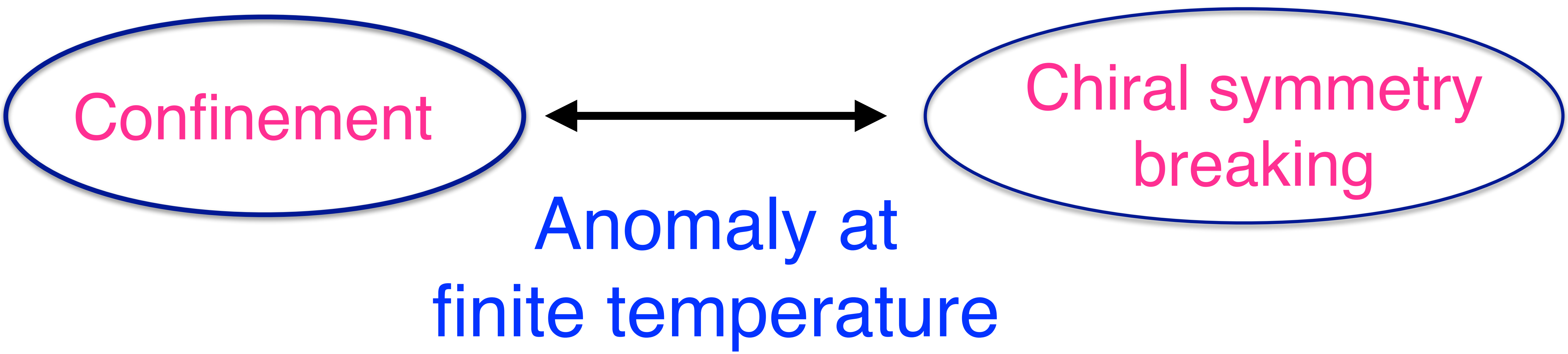
Unfortunately, the perturbative anomaly vanishes on R^3 :



$$\xrightarrow{S^1 \text{ reduction}} 0$$

Anomaly at finite temperature

I will argue the existence of a subtler anomaly at finite temperature if we include a small imaginary chemical potential.



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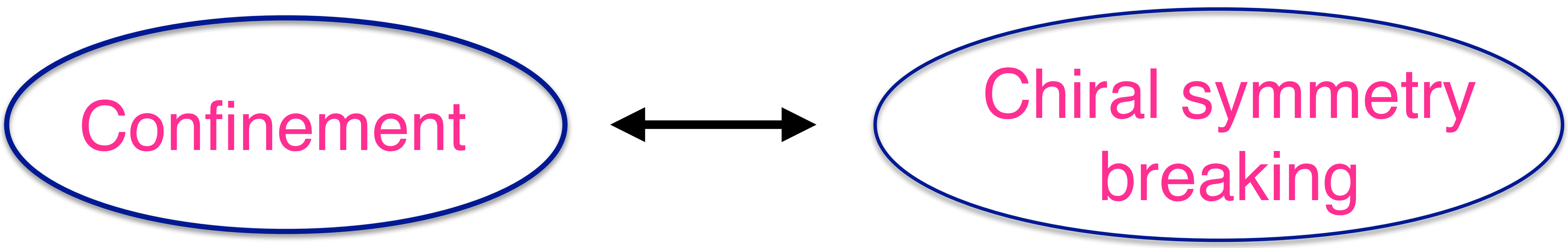
A problem in QCD



We want to study this relation at finite temperature.

However, a well-known problem is that “confinement” is not well-defined in finite temperature QCD because dynamical quarks can screen color fluxes.

Pure Yang-Mills

Let us recall how to define confinement in pure Yang-Mills.

Finite temperature: $Z = \mathrm{tr} e^{-\beta H} \longleftrightarrow R^3 \times S^1$

$\beta = T^{-1}$: inverse temperature

Polyakov loop: $W = \mathrm{tr} P \exp(i \oint_{S^1} A_\mu dx^\mu)$

Wilson loop wrapping on the S^1

Pure Yang-Mills

Intuitively, the Polyakov loop behaves as

$$W \sim \exp(-\beta E_q)$$

E_q : energy of a single probe quark

Confinement : $E_q \to \infty \quad W = 0$

Deconfinement : $E_q < \infty \quad W \neq 0$

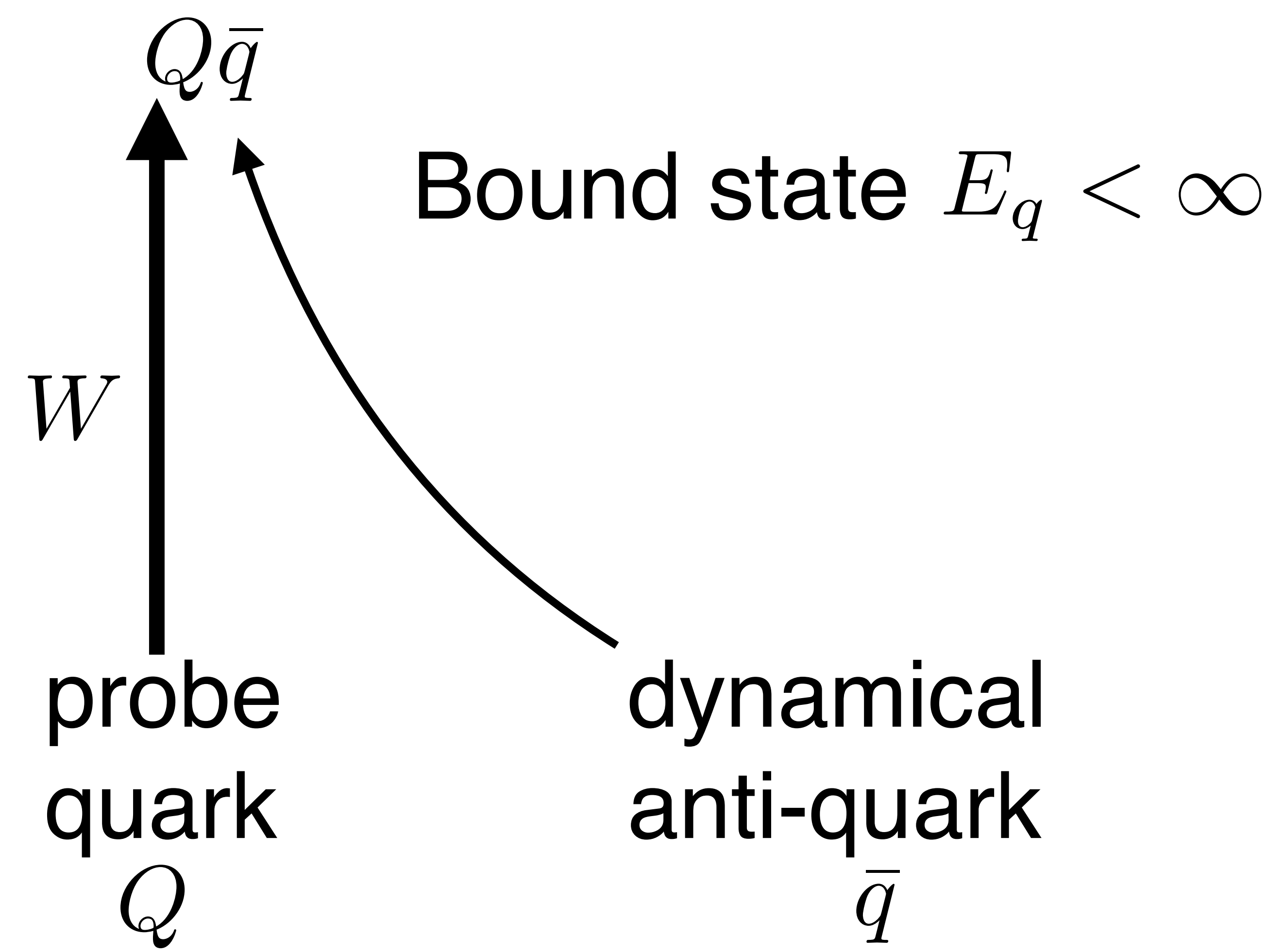
Order parameter

So the Polyakov loop can be regarded as an order parameter of confinement in pure-Yang-Mills.

How about QCD with dynamical quarks?

QCD

In QCD, the probe quark energy E_q is always finite.



The Polyakov loop W cannot be used to define confinement phase. Always $W \neq 0$

Imaginary chemical potential

To define confinement rigorously,
I slightly change the problem.

$$\mathrm{tr}\exp(-\beta H) \to \mathrm{tr}\exp(-\beta H + i\mu_B B)$$

B : baryon number charge
μ_B : baryon imaginary chemical potential

This changes the thermodynamics, but I will argue that the effect of the imaginary chemical potential is subleading in the large N_c expansion.

Imaginary chemical potential

I take

$$\mu_B = \pi$$

[Roberge-Weiss,1986]

All gauge invariant composites have integer $B \in \mathbb{Z}$

Mesons: $B = 0$ Baryons: $B = 1$

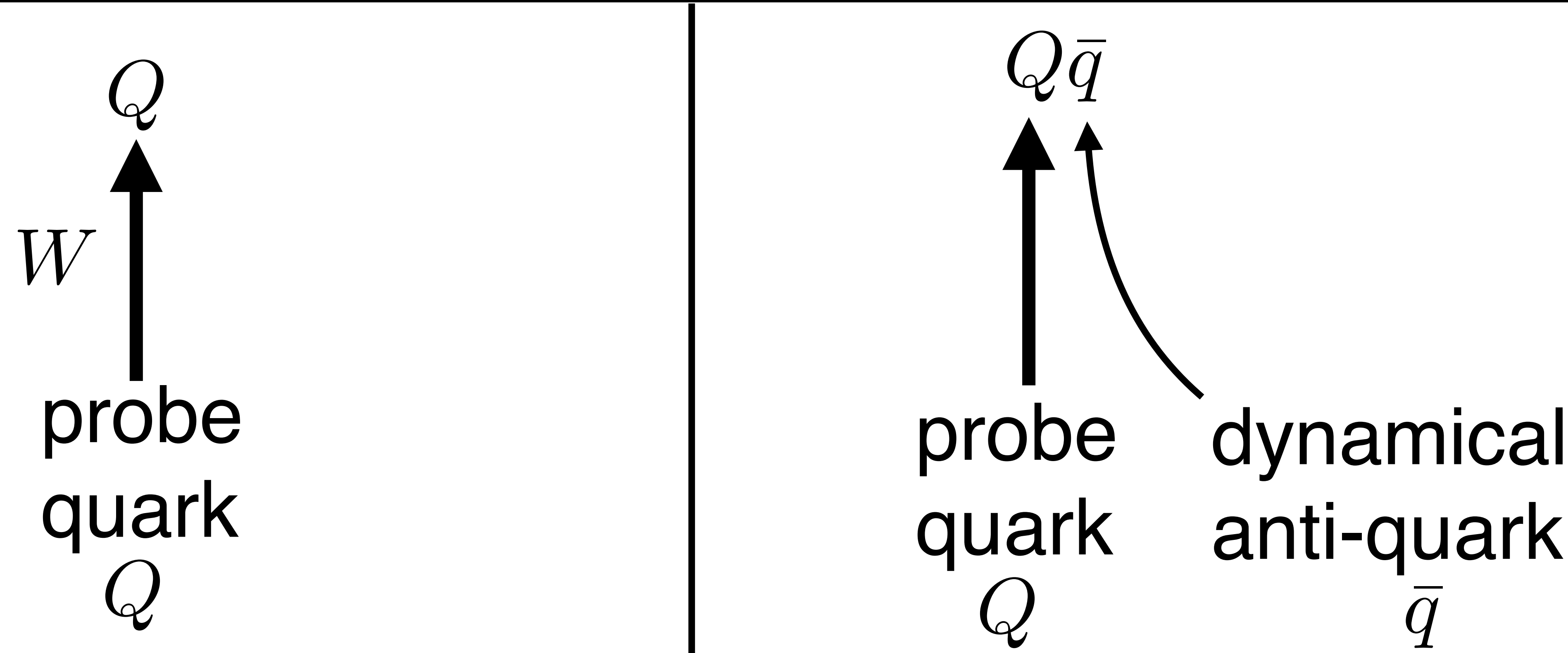
However, quarks have fractional baryon numbers.

Quarks: $B = 1/N_c$

$$\exp(i\pi B) = \begin{cases} \text{real for gauge invariant composites} \\ \text{imaginary for colored quarks} \end{cases}$$

Criterion for confinement

$$W \sim \exp(-\beta E_q + i\pi B)$$



$$\mathrm{Im}(W) \neq 0$$

deconfinement

$$\mathrm{Im}(W) = 0$$

confinement

$\mathbb{Z}_2$ symmetry for confinement

$$W = \mathrm{tr} P \exp(i \oint_{S^1} A_\mu dx^\mu)$$

By flipping the direction of integration on S^1 , we get

$$W \to W^*$$

This is a $\mathbb{Z}_2$ symmetry.

The order parameter of this $\mathbb{Z}_2$ is precisely $\mathrm{Im}(W)$

$$\mathbb{Z}_2 : \mathrm{Im}(W) \to -\mathrm{Im}(W)$$

Definition of confinement

We can summarize the above discussion as follows.

- There exists a $\mathbb{Z}_2$ symmetry (flipping the S^1 direction)
- The imaginary part of the Polyakov loop $\mathrm{Im}(W)$ is charged under the $\mathbb{Z}_2$
- Confinement and deconfinement are distinguished by

Deconfinement :	$\mathrm{Im}(W) \neq 0$	$\mathbb{Z}_2$ broken
Confinement :	$\mathrm{Im}(W) = 0$	$\mathbb{Z}_2$ unbroken

Remark on imaginary chemical

The effect of imaginary chemical potential is very suppressed in the large N expansion:

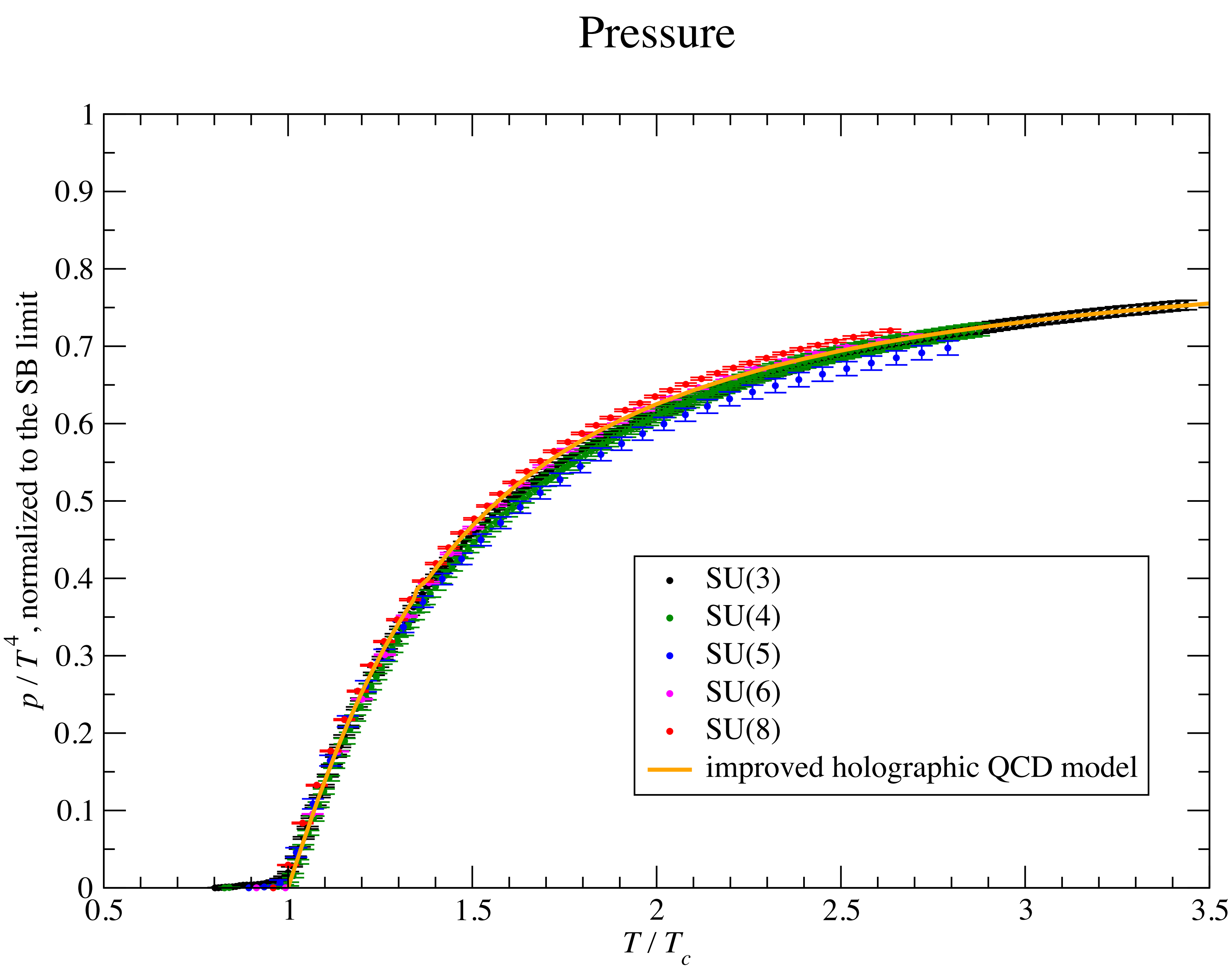
$$\frac{\text{effect of } \mu_B}{\text{total free energy}} \sim \frac{N_f}{N_c^3}$$

This follows from the fact that
the baryon charge of quarks is $1/N_c$

Therefore, the situation at $\mu_B = \pi$ should be similar to $\mu_B = 0$ as far as large N expansion is qualitatively good.

Large N limit

At least for pure Yang-Mills, $3 \simeq \infty$



[Panero,2009]

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Symmetry and Anomaly

Massless QCD at finite temperature with imaginary chemical potential $\mu_B = \pi$ has (at least) two symmetries:

- Chiral symmetry $SU(N_f)_L \times SU(N_f)_R$
- $\mathbb{Z}_2$ symmetry

Result : (derivation later)

There exists a mixed 't Hooft anomaly between chiral symmetry and $\mathbb{Z}_2$ symmetry.

This is a parity anomaly in 3-dimensions.

Symmetry and Anomaly

Confinement ($\mathbb{Z}_2$ symmetry) ⟷ **Anomaly** ⟷ Chiral symmetry breaking

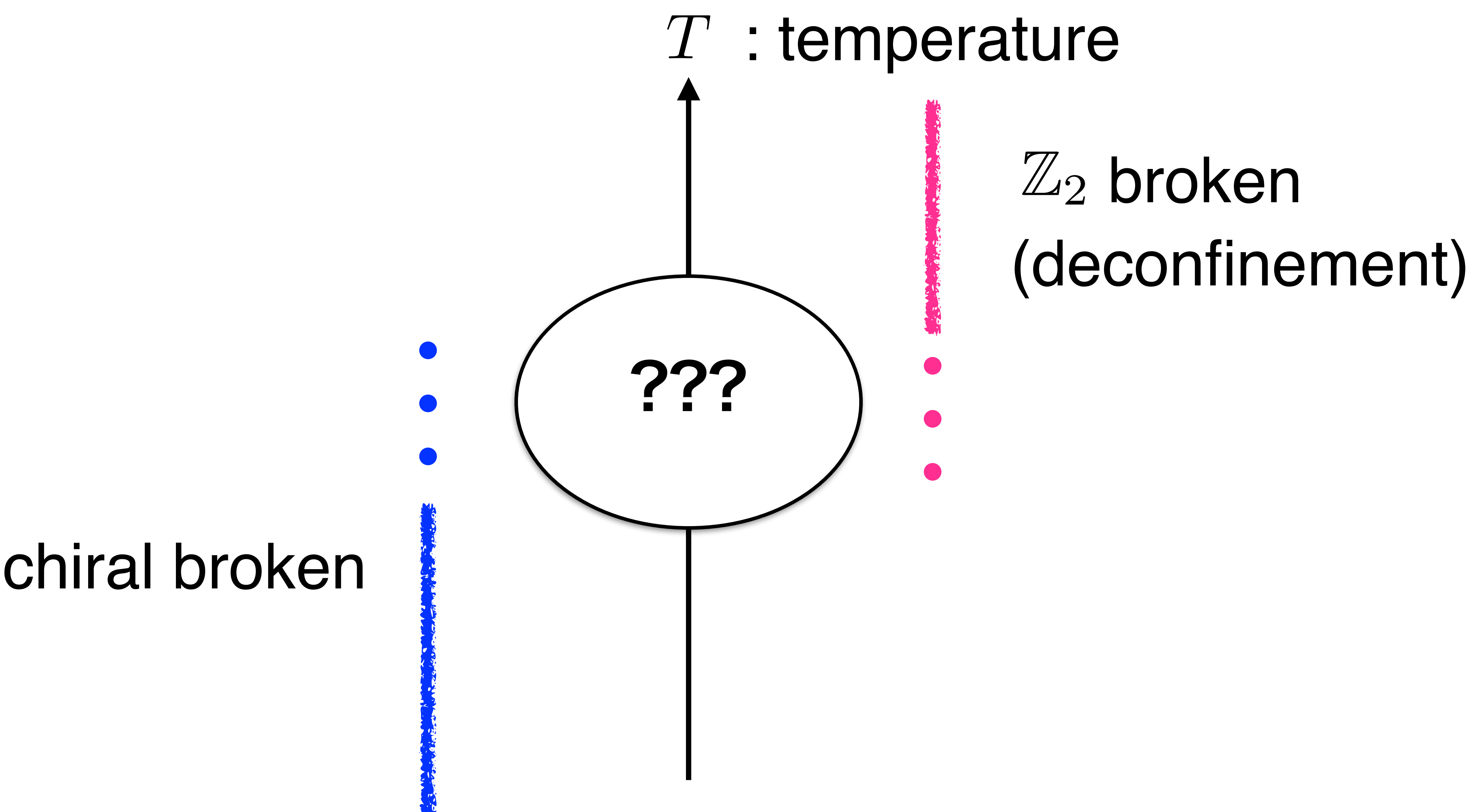
Result : (derivation later)

There exists a mixed 't Hooft anomaly between chiral symmetry and $\mathbb{Z}_2$ symmetry.

This is a parity anomaly in 3-dimensions.

Implications to phase transition

Let me discuss the implications of the anomaly to QCD phase transition.



Implications to phase transition

Two critical temperatures:

T_{chiral} : critical temperature for chiral symmetry

$T_{\text{deconfine}}$: critical temperature for $\mathbb{Z}_2$ symmetry

Let us consider possible scenarios. Either

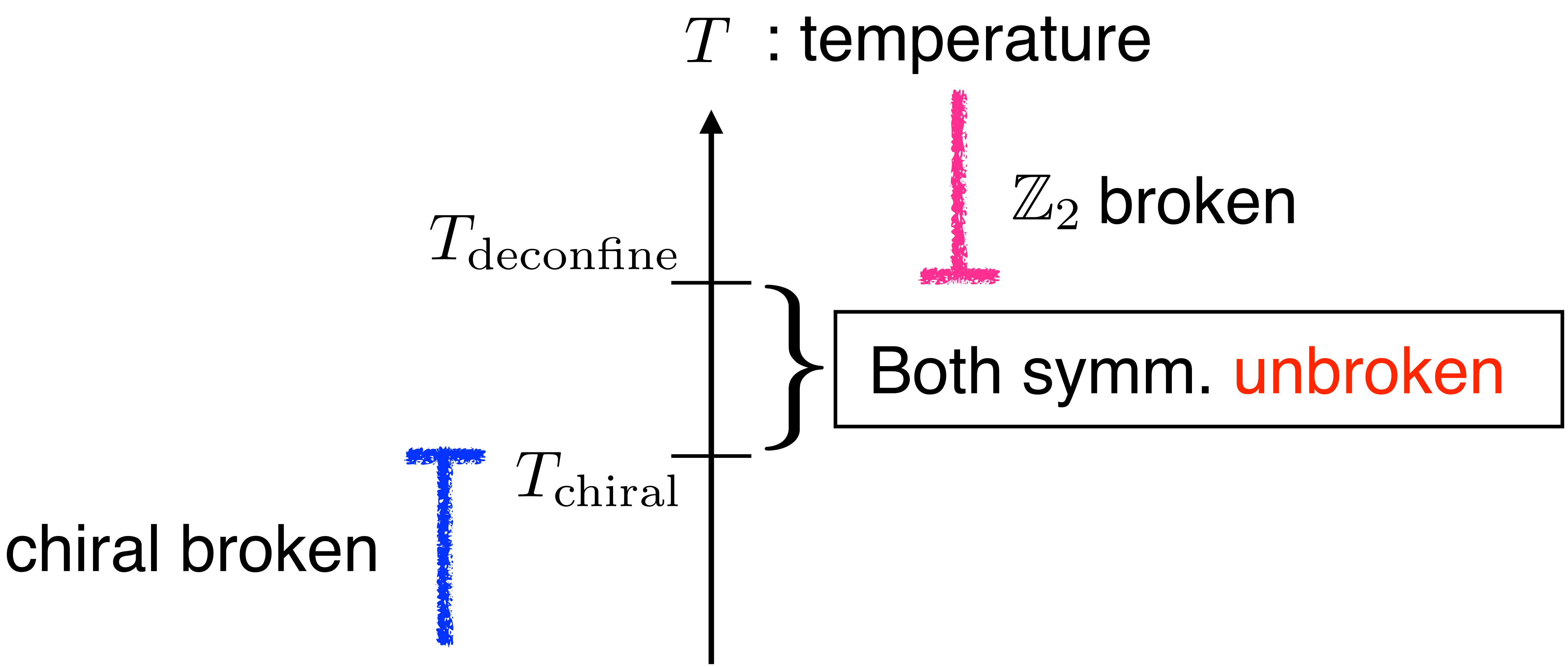
(1) $T_{\text{deconfine}} > T_{\text{chiral}}$

(2) $T_{\text{deconfine}} < T_{\text{chiral}}$

(3) $T_{\text{deconfine}} = T_{\text{chiral}}$

Scenario 1

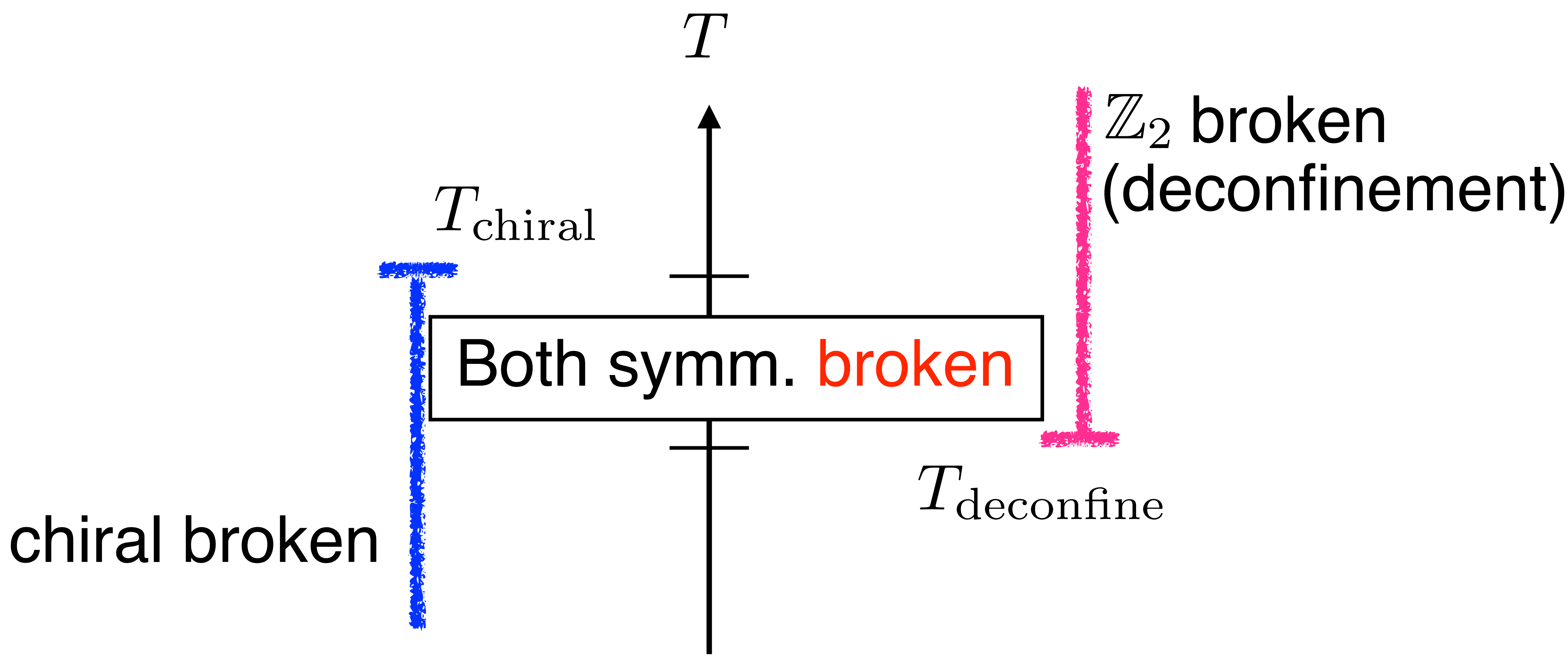
Scenario 1: $T_{\mathrm{deconfine}} > T_{\mathrm{chiral}}$



We need complicated massless degrees of freedom to match the anomaly.

Scenario 2

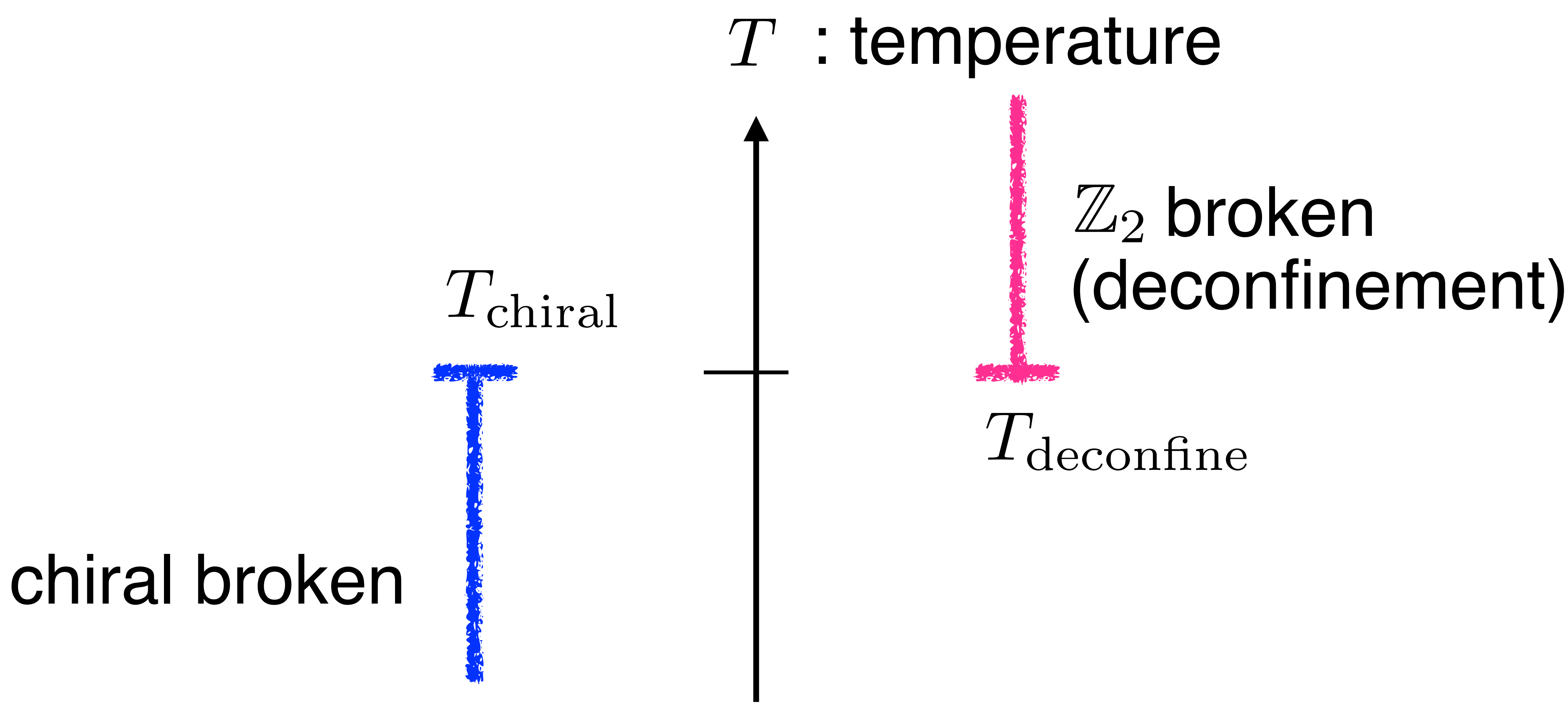
Scenario 2: $T_{\text{deconfine}} < T_{\text{chiral}}$



Chiral symmetry breaking ($q\bar{q}$ condensation) happens in deconfinement phase.

Scenario 3

Scenario 3: $T_{\text{deconfine}} = T_{\text{chiral}}$



It may be natural if the phase transition is first order
to avoid complicated d.o.f. at the critical temperature,

Natural scenario?

There are many logical possibilities, but
a first order transition at a single critical temperature
may be the simplest (most natural) scenario.

Otherwise, the 't Hooft anomaly requires either
of the following:

(1) Complicated massless d.o.f. for anomaly matching
(2) $q\bar{q}$ condensation in deconfinement phase
(3) Something more complicated

For more details, please see [KY,2019].

Implication for real QCD

Suppose the phase transition is first order for

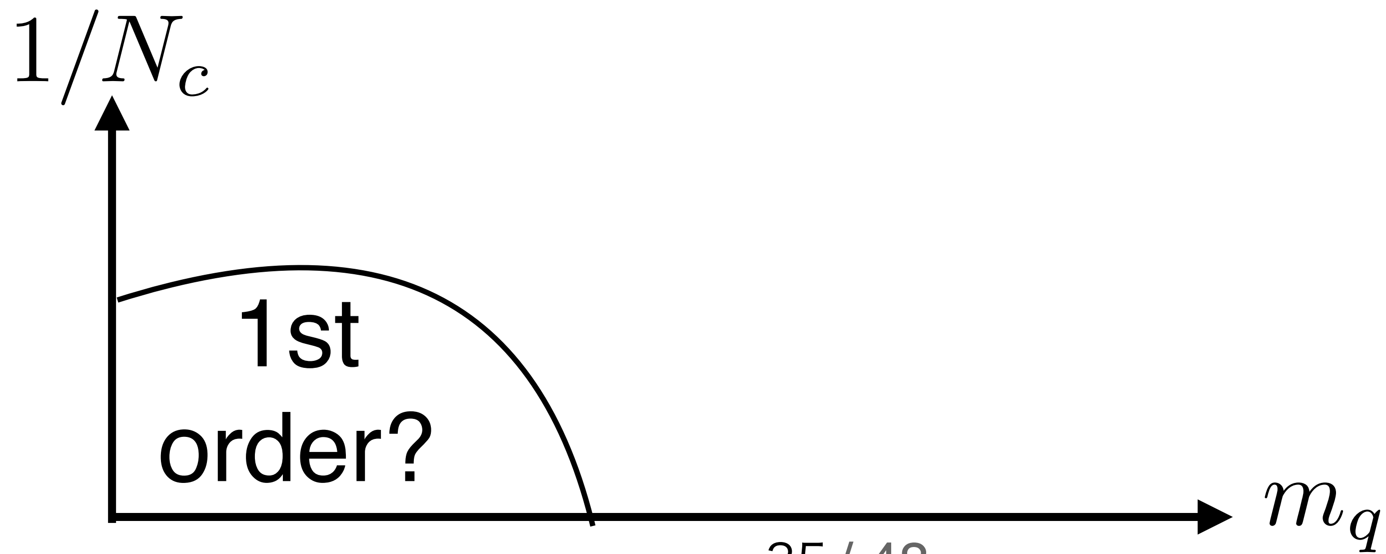
$$m_q = 0, \qquad \mu_B = \pi$$

Then it is expected to remain first order for

$$m_q \neq 0, \qquad \mu_B = 0$$

as far as

$$m_q \ll \Lambda, \qquad 1/N_c \ll 1$$



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Reduction from 4 to 3 dim.

Thermodynamics is described by compactification

spacetime: $R^4 \to R^3 \times S^1$

In the absence of gauge fields,
fermions have anti-periodic boundary condition.

$$\Psi(x, \tau + \beta) = -\Psi(x, \tau)$$

τ : coordinate of S^1

β : circumference of S^1

Boundary condition

Gauge fields effectively changes the boundary condition.

$$U = P\exp(i \oint A_\mu dx^\mu)$$

: Wilson line of gauge fields around S^1

In a gauge in which locally $A_4 = 0$,

$$\Psi(x, \tau + \beta) = -U\Psi(x, \tau)$$

Boundary condition

The gauge field A consists of

A_C : dynamical color $SU(N_c)$ gauge field

A_B : background baryon $U(1)_B$ gauge field

$$A = A_C + \frac{1}{N_c} I A_B$$

Imaginary baryon chemical potential :

$$\mu_B = \oint A_B$$

Boundary condition

$$U = P\exp(i\oint A_C + \frac{1}{N_c} I A_B)$$
$$= e^{i\mu_B/N_c} P\exp(i\oint A_C)$$

The determinant of U is

$$\det U = e^{i\mu_B}$$

Boundary condition

Focus attention to the case $\mu_B = \pi$

$$\det U = e^{i\mu_B} = -1$$

If U preserves the $\mathbb{Z}_2$ symmetry of flipping S^1,

$$U = \mathrm{diag}(\underbrace{-1, \cdots, -1}_{K}, \underbrace{+1, \cdots, +1}_{N_c - K})$$

$\det U = (-1)^K = -1$: K is odd.

Boundary condition

$$\Psi(x, \tau + \beta) = -U\Psi(x, \tau)$$

$$U = \mathrm{diag}(\underbrace{-1, \cdots, -1}_{K}, \underbrace{+1, \cdots, +1}_{N_c - K})$$

Among N_c color components,

K components: periodic condition

$N_c - K$ components: anti-periodic condition

This means that K = odd fermions are massless in 3-dim.

Massless fermion in 3-dim.

$\Psi = (\Psi_L, \Psi_R)$ in 4-dim.

↓ dimensional reduction on S^1

K massless $\psi = (\psi_L, \psi_R)$ in 3-dim.

ψ_L : fundamental of $SU(N_f)_L$

ψ_R : fundamental of $SU(N_f)_R$

KK modes are irrelevant for anomalies.

Parity anomaly in 3-dim.

The following fact is known in 3-dim. : parity anomaly

If there is K massless fermion coupled to an $SU(N_f)$ gauge field, the parity transformation

$$\vec{x} \to -\vec{x}$$ (3-dim. Euclidean sense)

is anomalous for odd K.

In other words, there is a mixed anomaly between

$SU(N_f)$ and Parity

Parity anomaly in 3-dim.

In our case:

K massless $\psi = (\psi_L, \psi_R)$ in 3-dim.

ψ_L : fundamental of $SU(N_f)_L$

ψ_R : fundamental of $SU(N_f)_R$

K is odd for $\mu_B = \pi$

Each chiral symmetry $SU(N_f)_{L,R}$ has parity anomaly.

Parity anomaly in 3-dim.

Parity $\vec{x} \to -\vec{x}$ in 3d comes from Lorentz symmetry in 4d $x^\mu \to -x^\mu$ which flips the S^1-direction.

This is the $\mathbb{Z}_2$-symmetry which I used for the definition of confinement.

$SU(N_f)_L \times SU(N_f)_R$ ⟷ $\mathbb{Z}_2$-symmetry

Contents

Summary

- There exists a subtle ’t Hooft anomaly in finite temperature QCD when an imaginary chemical potential is introduced.

$\mathbb{Z}_2$ symmetry
Confinement

Anomaly ⟷

$SU(N_f)_L \times SU(N_f)_R$
Chiral symmetry breaking

- A first order transition may be the most natural scenario of QCD phase transition if large N expansion and small quark mass approximation are qualitatively good.