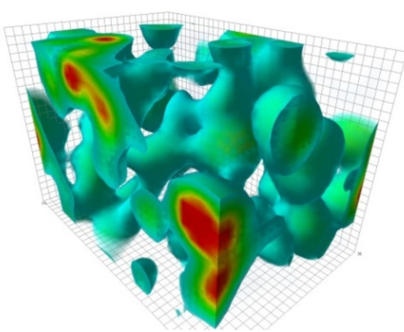
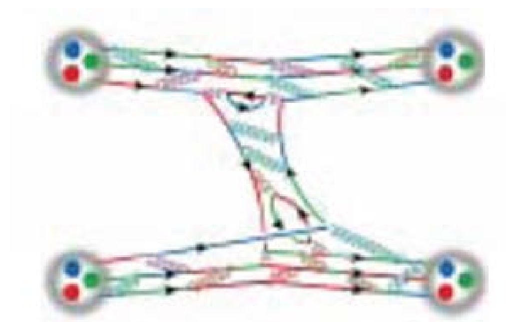
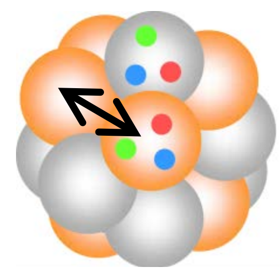
Most charming dibaryon near unitarity

Takumi Doi

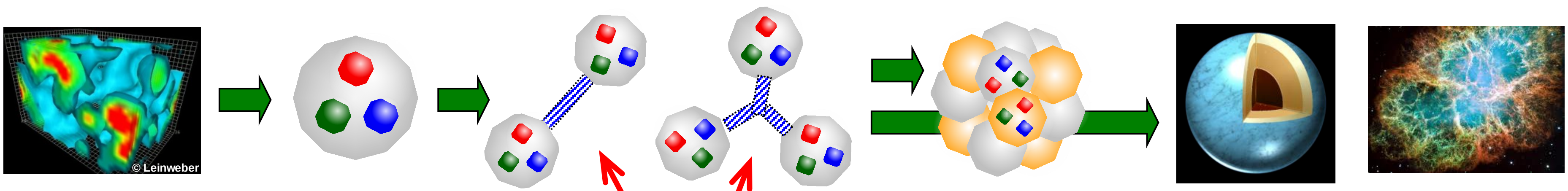
(RIKEN Nishina Center / iTHEMS)







The Odyssey from Quarks to Universe



QCD vacuum

Baryons

Nuclei

Neutron Stars / Supernovae

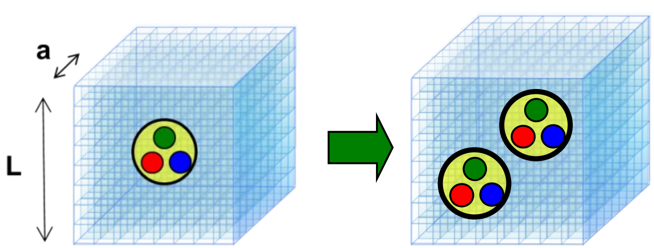
Nucleosynthesis

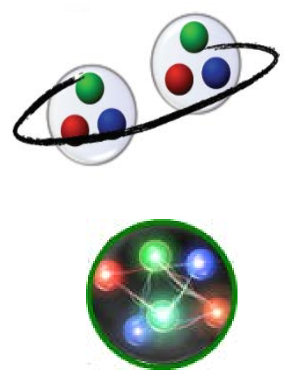
QCD

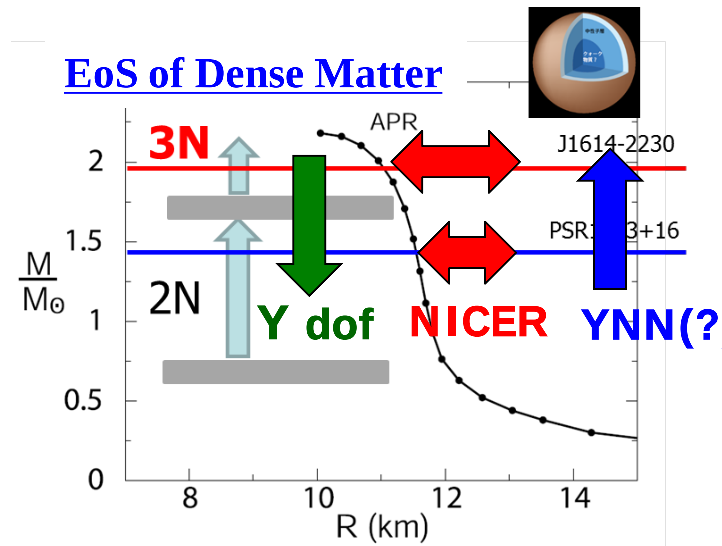
Hadron Forces

ab-initio nuclear calc.

1st-principle Lattice QCD





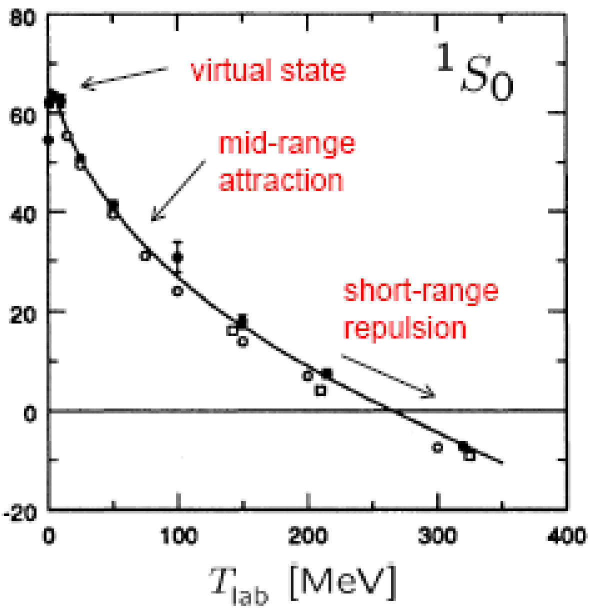


Nuclear Physics directly based on QCD

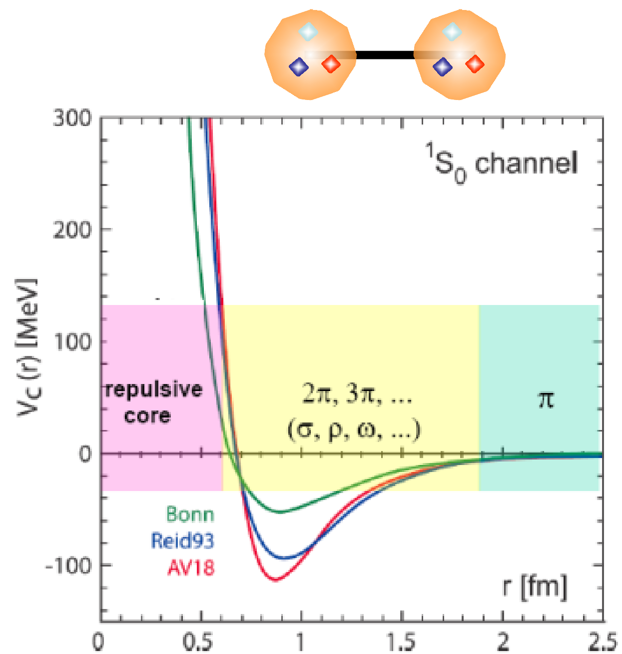
What kind of particle can exist in nature?

What kind of dense matter can exist in nature?

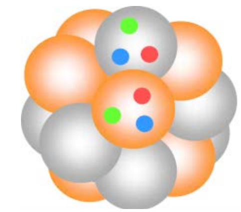
Nuclear Forces: Foundation of nuclear physics



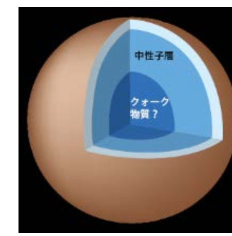
NN phase shifts
from experiments



Phenomenological
Nuclear Forces



Nuclei



Neutron Stars

Super Novae

Various applications

- ***Nuclear Forces* play *crucial roles***
  - ***Yet, no clear connection to QCD so far***

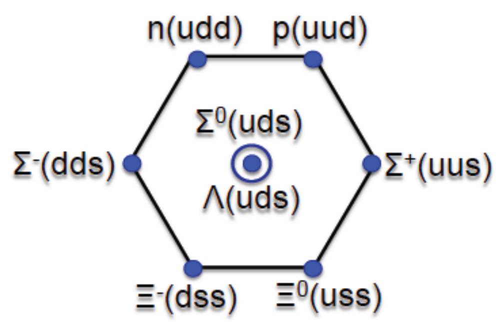
Phen. NN potentials: #params = 30~40

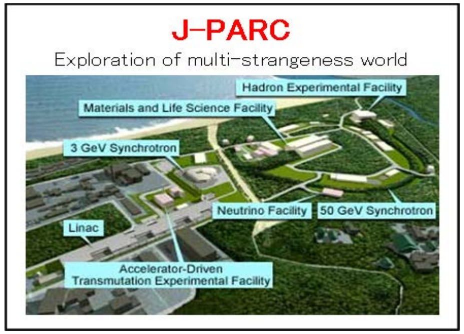
←→ QCD: #inputs = 4 : quark masses (m_u, m_d, m_s) & coupling α_s

Nuclear Forces → Baryon Forces (incl. Hyperons)

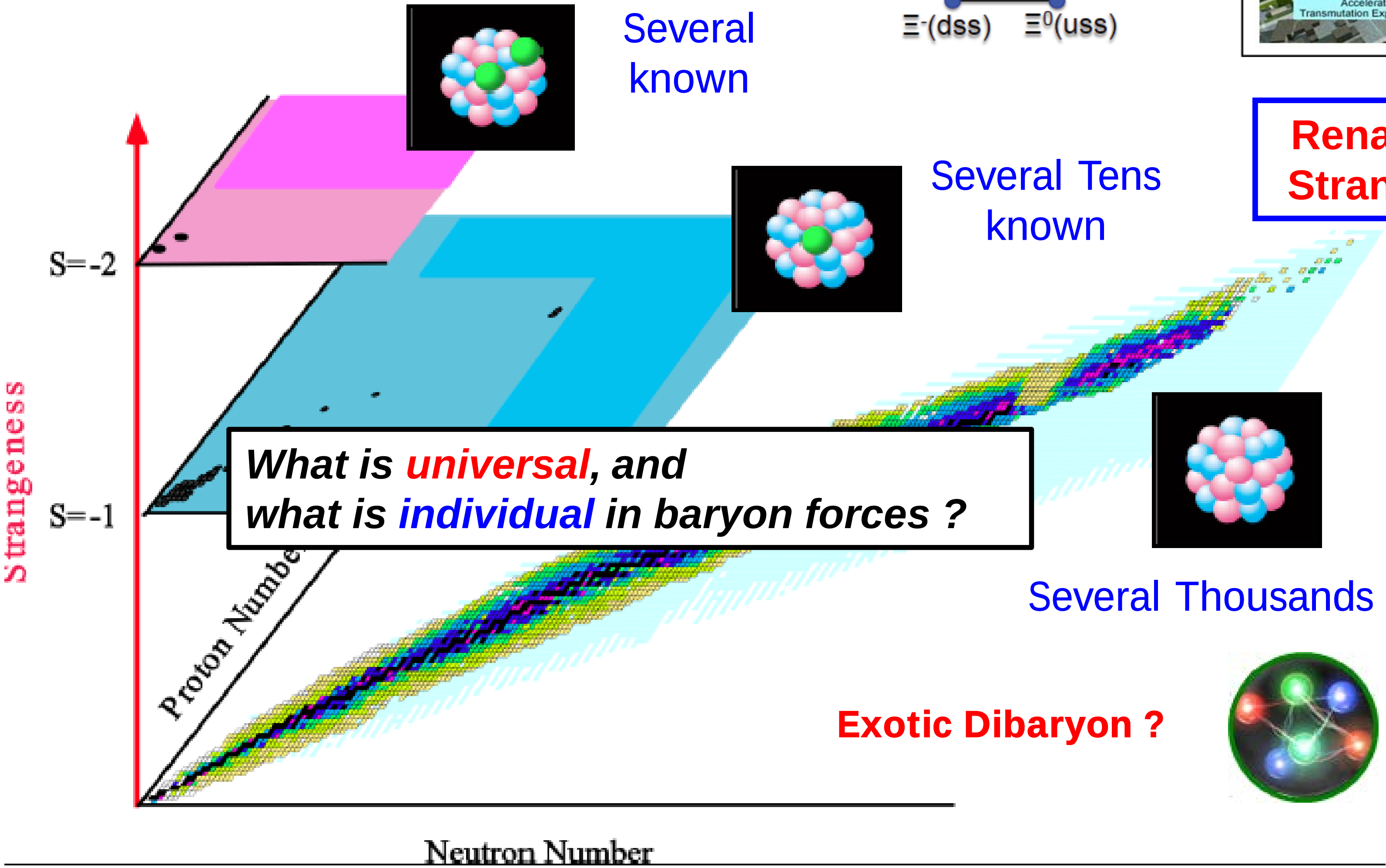
3D Nuclear Chart

Nucleons : u, d quarks
Hyperons : u, d, s quarks





Renaissance in Strange World !



Several known

Several Tens known

What is universal, and what is individual in baryon forces ?

Several Thousands known

Exotic Dibaryon ?

Nuclear/Hyperon Forces → Charmed Forces

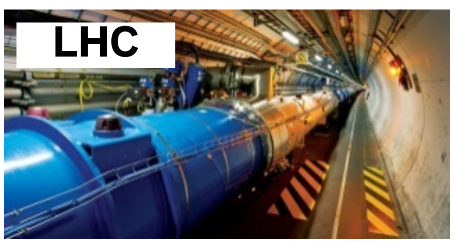
light	~ a few MeV	~100 MeV	~300 MeV	~1.3 GeV	~4.2 GeV	heavy
	up,down quarks	strange quark	Λ_{QCD}	charm quark	bottom quark	top

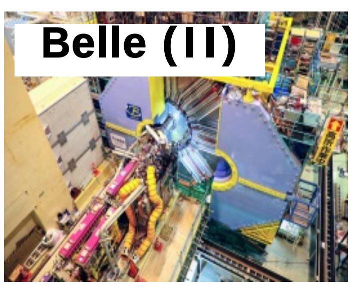
chiral symmetry

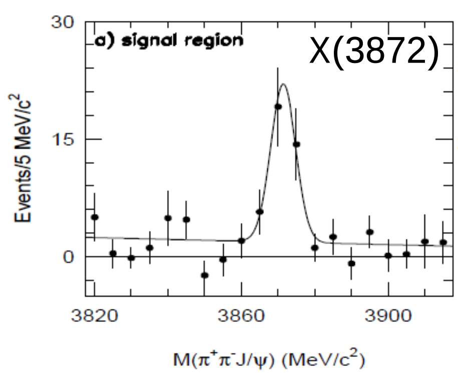
heavy quark symmetry

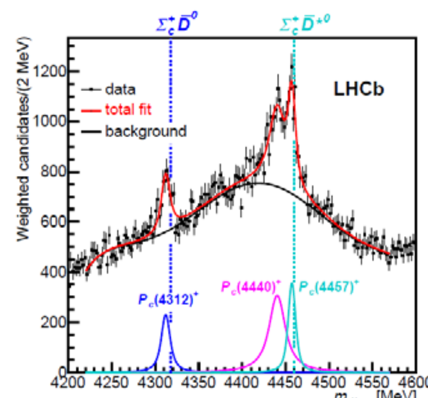
Heavy quarks: New doorway to the mysteries of QCD

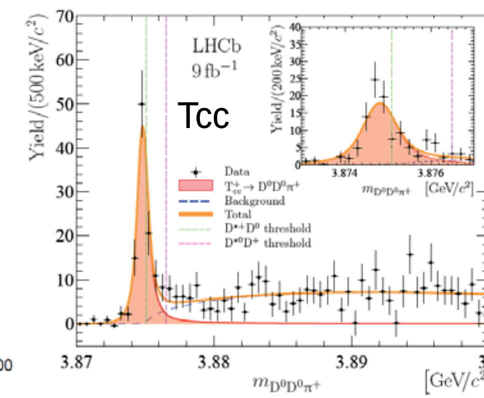
Many new exotic particles being reported!









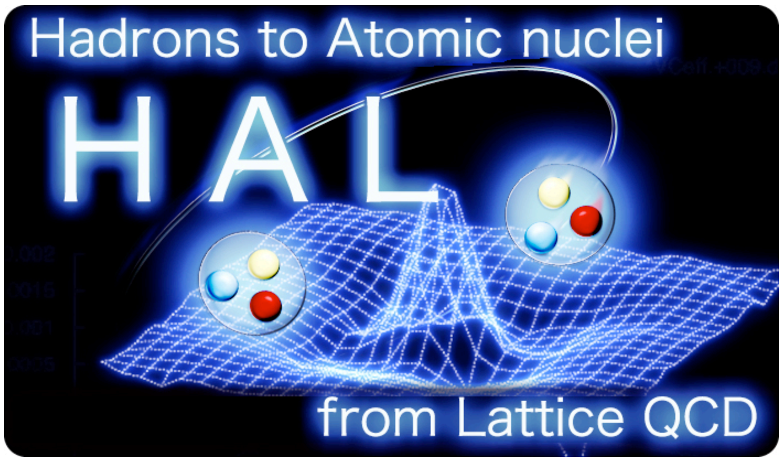


Hadron interactions crucial to understand these "signals" !

e.g., Zc(3900) from HAL LQCD → threshold cusp

Y. Ikeda et al. (HAL), PRL117(2016)242001

Hadrons to Atomic nuclei from Lattice QCD (HAL QCD Collaboration)



Y. Akahoshi, S. Aoki,
K. Murakami, H. Nemura (YITP)
T. Aoyama (KEK)
T. Doi, T. Hatsuda, T. Miyamoto, T. Sugiura (RIKEN)
T. M. Doi, N. Ishii, K. Sasaki (Osaka Univ.)
F. Etminan (Univ. of Birjand)
Y. Ikeda (Kyushu Univ.)
T. Inoue (Nihon Univ.)
Y. Lyu (Peking Univ.)
H. Tong (Tianjin Normal Univ.)

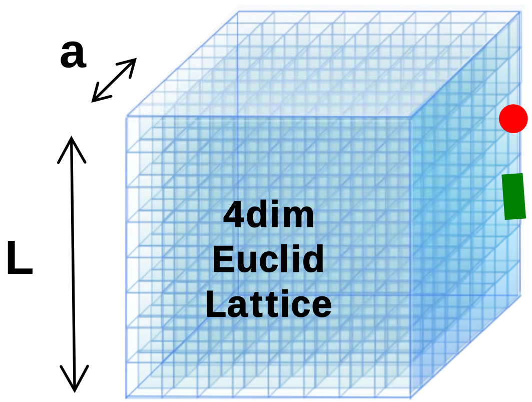
Y. Lyu, H. Tong et al.,
PRL127(2021)072003

「20XX年宇宙の旅」

from Quarks to Universe

Lattice QCD
First-principles calculation of QCD

$$Z = \int dU\, dq d\bar{q}\ e^{-S_E}$$



quarks on sites

$$q(x)$$

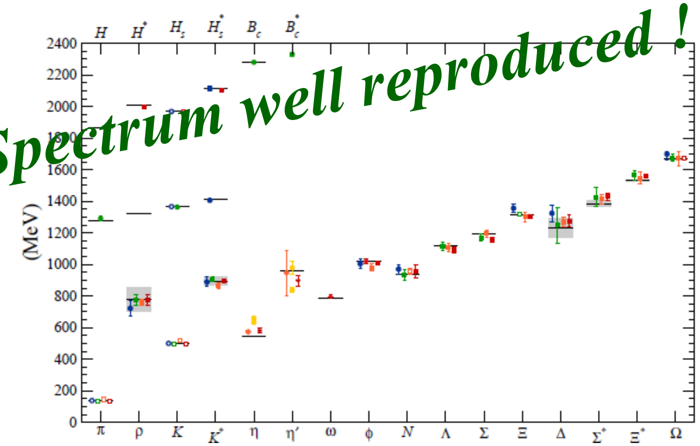
gluons on links

$$U_\mu(x, x+\mu) = \exp[-iaA_\mu]$$

K.G. Wilson
(1974)

- **Well-defined reguralized system**
- **Gauge-invariance manifest**
- **Fully-Nonperturbative**
- **DoF ~ 10^9 ➔ Monte-Carlo w/ Euclid time**

Inputs:
- **quark masses m_q**
- **coupling constant $\alpha_s = g^2/4\pi$**



Summary by Kronfeld, arXiv:1203.1204

Interactions on the Lattice

- Luscher's finite volume method
  - Phase shift & B.E. from temporal correlation in finite V

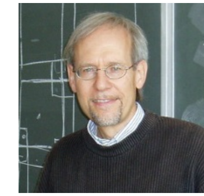
M.Luscher, CMP104(1986)177
CMP105(1986)153
NPB354(1991)531

- HAL QCD method
  - "Potential" from spacial (& temporal) correlation
  - Phase shift & B.E. by solving Schrodinger eq in infinite V

Ishii-Aoki-Hatsuda, PRL99(2007)022001, PTP123(2010)89
HAL QCD Coll., PTEP2012(2012)01A105

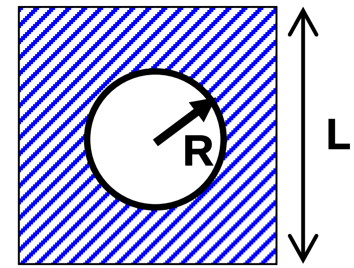
Luscher's formula: Scatterings on the lattice

- Consider Schrodinger eq at asymptotic region



$$(\nabla^2 + k^2)\psi_k(\boldsymbol{r}) = mV_k(\boldsymbol{r})\psi_k(\boldsymbol{r})$$

$$V_k(\boldsymbol{r}) = 0 \text{ for } r > R$$



  – (periodic) Boundary Condition in finite V
    ➔ constraint on energies of the system

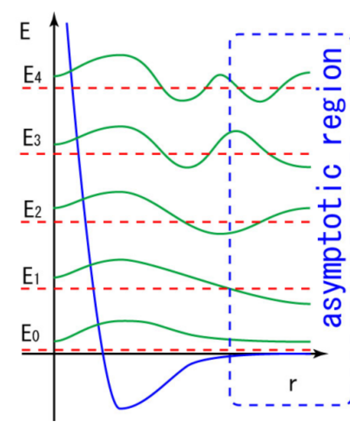
  – Energy E and phase shift (at E) are related

$$E = 2\sqrt{m^2 + k^2}$$

(QFT: $\psi_k(r) \to$ NBS w.f.)

$$k \cot \delta_{\mathbf{E}} = \frac{2}{\sqrt{\pi}L} Z_{00}(1; q^2), \quad q = \frac{kL}{2\pi}$$

$$Z_{00}(s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\boldsymbol{n} \in \boldsymbol{Z}^3} \frac{1}{(\boldsymbol{n}^2 - q^2)^s}$$



Large V expansion

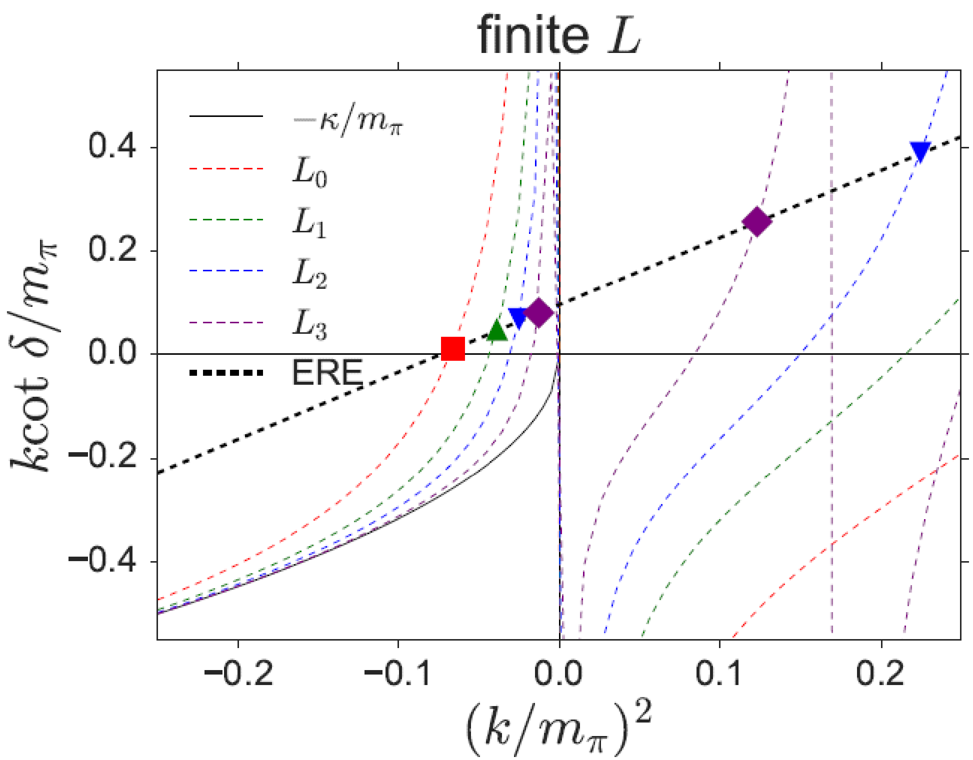
$$\Delta E = E - 2m = -\frac{4\pi \mathbf{a}}{mL^3}\left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2 + \mathcal{O}\left(\frac{1}{L^3}\right)\right]$$

c_1, c_2: geometric constants

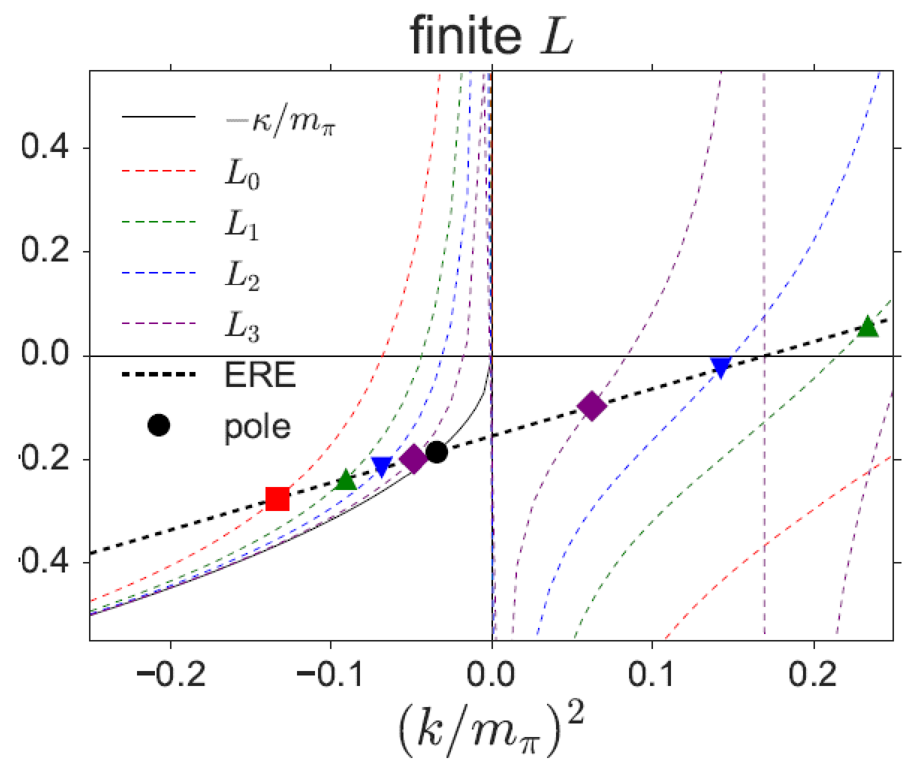
a: scattering length

Examples of Luscher's formula

Unbound example



Bound example



- **Effective Range Expansion (ERE)**

$$k \cot \delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2}\mathbf{r}\, k^2 + \cdots$$

  - "a" : scattering length, "r" : effective range

Unbound : 1/a > 0
Bound : 1/a < 0

Unitary limit (side remarks)

- **Effective Range Expansion (ERE)**

$$k \cot \delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2} \mathbf{r}\, k^2 + \cdots$$

In the limit of a = infinity → Just at the boundary of bound/unbound

(more precisely, r/a = 0 limit)

Phase shifts (S-wave) at low energies become $\delta_0 = \pi/2$

Scattering cross section $\sigma = \frac{4\pi}{k^2} \sin^2 \delta_0$ takes maximal value

Physics becomes independent of details of potential
only “a” controls the physics

Very active studies in various fields, e.g., cold atoms

As a dibaryon system, NN is known to be close to the unitary limit:
r/a (dineutron) = 0.15, r/a (deuteron) = -0.32

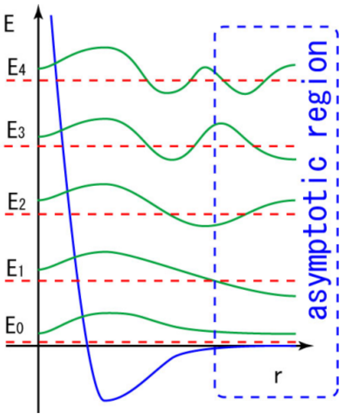
[HAL QCD method]

- "Potential" defined through phase shifts (S-matrix)

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r})N(\vec{x})|N(k)N(-k);W\rangle$$

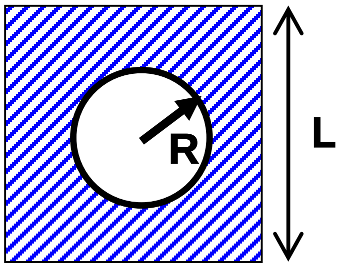
$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R \qquad W = 2\sqrt{m^2+k^2}$$



  – Wave function ←→ phase shifts

$$\psi(r) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

(below inelastic threshold)



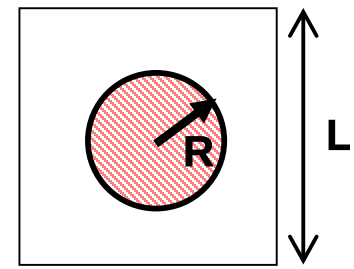
Extended to multi-particle systems

M.Luscher, NPB354(1991)531
C.-J.Lin et al., NPB619(2001)467
CP-PACS Coll., PRD71(2005)094504
Ishizuka, Pos LAT2009 (2009) 119
Aoki-Hatsuda-Ishii PTP123(2010)89
S.Aoki et al., PRD88(2013)014036

"Potential" as a representation of S-matrix

- Consider the wave function at "interacting region"

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m\int d\boldsymbol{r}'\, U(\boldsymbol{r},\boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$



  – U(r,r'): faithful to the phase shift by construction
    - U(r,r'): NOT an observable, but well defined
    - U(r,r'): E-independent, while non-local in general

      – "Proof of Existence": Explicit form can be given as

$$U(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{m}\sum_{n,n'}^{n_{\rm th}} (\nabla_{\boldsymbol{r}}^2 + k_n^2)\psi_n(\boldsymbol{r})\mathcal{N}^{-1}_{nn'}\psi^*_{n'}(\boldsymbol{r}') \qquad \mathcal{N}_{nn'} = \int d\boldsymbol{r}\,\psi^*_n(\boldsymbol{r})\psi_{n'}(\boldsymbol{r})$$

      – Non-locality ➔ derivative expansion

Okubo-Marshak(1958)

$$U(\vec{r},\vec{r}') = \left[V_c(r) + S_{12}V_T(r) + \vec{L}\cdot\vec{S}V_{LS}(r) + \mathcal{O}(\nabla^2)\right]\delta(\vec{r}-\vec{r}')$$

LO **LO** **NLO** **NNLO**

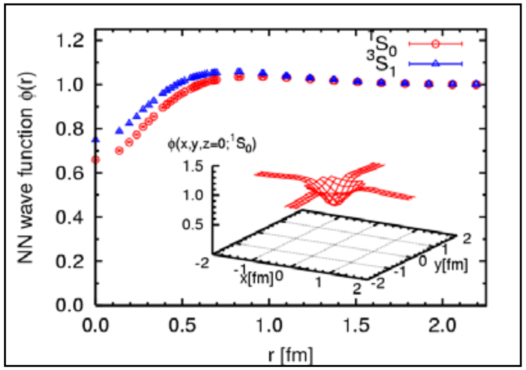
Aoki-Hatsuda-Ishii PTP123(2010)89

Check on convergence: K.Murano et al., PTP125(2011)1225

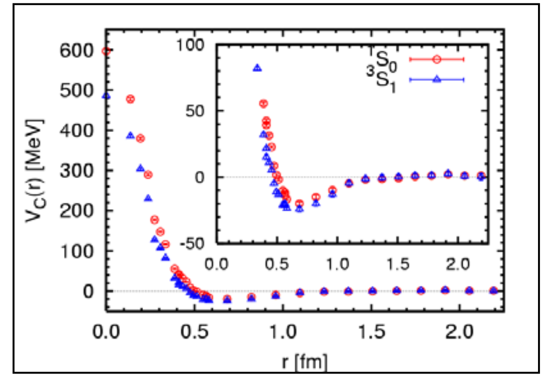
HAL QCD method

Lattice QCD

NBS wave func.



Lat Baryon Force



$$\psi_{NBS}(\vec{r}) = \langle 0|N(\vec{r})N(\vec{0})|N(\vec{k})N(-\vec{k}), in\rangle$$
$$\simeq A_k \sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

(at asymptotic region)

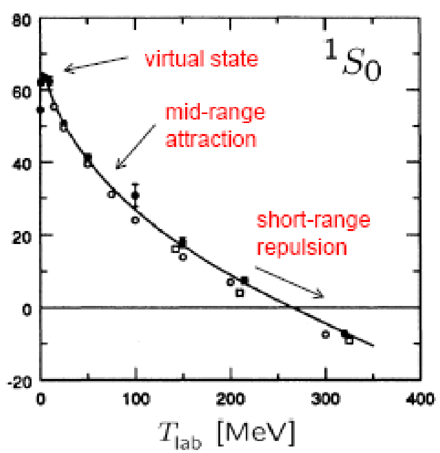
$$(k^2/m_N - H_0)\,\psi(\vec{r}) = \int d\vec{r}'\, U(\vec{r},\vec{r}')\psi(\vec{r}')$$

E-indep (& non-local) Potential:
Faithful to phase shifts

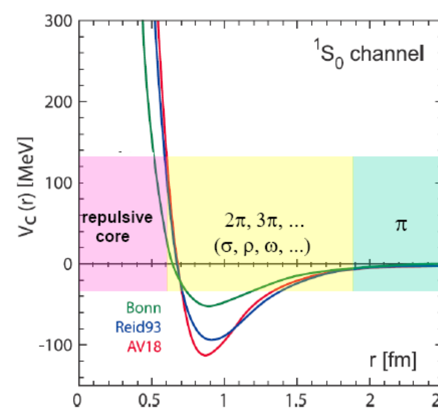
Analog to …

Scattering Exp.

Phase shifts



Phen. Potential



The Challenge in multi-baryons on the lattice

- Signal / Noise issue

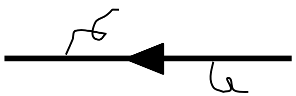
Parisi (‘84), Lepage (‘89)

  - G.S. saturation by t ➔ ∞ required in LQCD

$$G(r,t) = \langle 0|\mathcal{O}(r,t)\overline{\mathcal{O}}(0)|0|\rangle = \sum_n \alpha_n \psi_n(r) e^{-E_n t} \xrightarrow[t\to\infty]{} \alpha_0 \psi_0(r) e^{-E_0 t}$$

quark

each (dressed) quark propagator carries info of pions, nucleons, ...

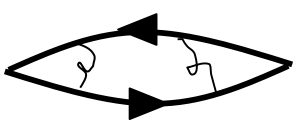


$$\sim \exp(-1/2\,\mathbf{m}_\pi \mathbf{t}) + \exp(-1/3\,\mathbf{m}_\mathbf{N} \mathbf{t}) + \cdots$$

a la D. Kaplan (via A. Walker-Loud)

pion

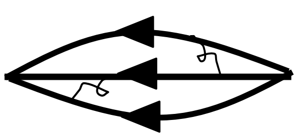
signal from the lowest (=dominant) mode



$$\frac{\text{Signal}}{\text{Noise}} \sim \frac{\langle \pi(t)\pi(0)\rangle}{\sqrt{\langle \pi\pi(t)\pi\pi(0)\rangle}} \sim \frac{\exp(-\mathbf{m}_\pi \mathbf{t})}{\sqrt{\exp(-2\mathbf{m}_\pi \mathbf{t})}} \sim \text{const.}$$

nucleon

small signal after the cancellation of dominant modes



$$\frac{\text{Signal}}{\text{Noise}} \sim \frac{\langle N^\mathbf{A}(t)\bar{N}^\mathbf{A}(0)\rangle}{\sqrt{\langle |N^\mathbf{A}(t)\bar{N}^\mathbf{A}(0)|^2\rangle}} \sim \frac{\exp(-\mathbf{A}\mathbf{m}_\mathbf{N}\mathbf{t})}{\sqrt{\exp(-3\mathbf{A}\mathbf{m}_\pi \mathbf{t})}}$$

$$\to \exp[-\mathbf{A}(\mathbf{m}_\mathbf{N} - \mathbf{3/2m}_\pi)\mathbf{t}]$$

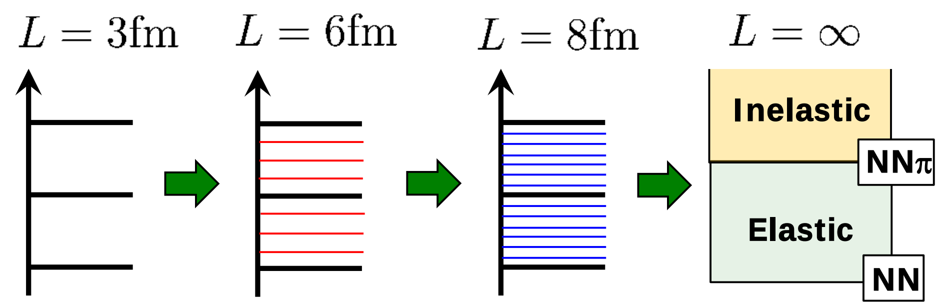
(A: mass number)

The Challenge in multi-baryons on the lattice

Existence of elastic scatt. states

➔ (almost) No Excitation Energy

➔ LQCD method based on G.S. saturation impossible



Signal/Noise issue

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

Parisi ('84), Lepage('89)

L=8fm @ physical point $(E_1 - E_0) \simeq 25\text{MeV} \Longrightarrow t > 10\text{fm}$

$$S/N \sim 10^{-32}$$

Naïve plateau fitting at t ~ 1fm is unreliable ("mirage" of true signal)

T. Iritani et al. (HAL) JHEP1610(2016)101
T. Iritani et al. (HAL) PRD96(2017)034521

Time-dependent HAL method

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

They are ***not contaminations***, ***but signals***

Original (t-indep) HAL method

$$G_{NN}(\vec{r},t) = \langle 0 | N(\vec{r},t) N(\vec{0},t) \overline{\mathcal{J}_{\rm src}(t_0)} | 0 \rangle$$

$$R(\boldsymbol{r},t) \equiv G_{NN}(\boldsymbol{r},t)/G_N(t)^2 = \sum A_{W_i} \psi_{W_i}(\boldsymbol{r}) e^{-(W_i - 2m)t}$$

← Many states contribute

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') \underline{\psi_{W_0}(\boldsymbol{r}')} = (\underline{E_{W_0}} - H_0) \underline{\psi_{W_0}(\boldsymbol{r})}$$

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') \underline{\psi_{W_1}(\boldsymbol{r}')} = (\underline{E_{W_1}} - H_0) \underline{\psi_{W_1}(\boldsymbol{r})}$$

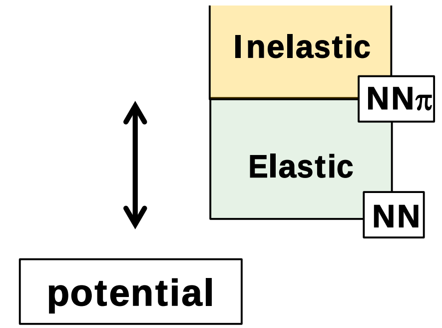
...

New t-dep HAL method

Coupled Channel formalism ←→ above inelastic threshold

All equations can be combined as

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') \underline{R(\boldsymbol{r}',t)} = \left(\underline{-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2}} - H_0\right) \underline{R(\boldsymbol{r},t)}$$



~~G.S. saturation~~ ➔ "Elastic state" saturation

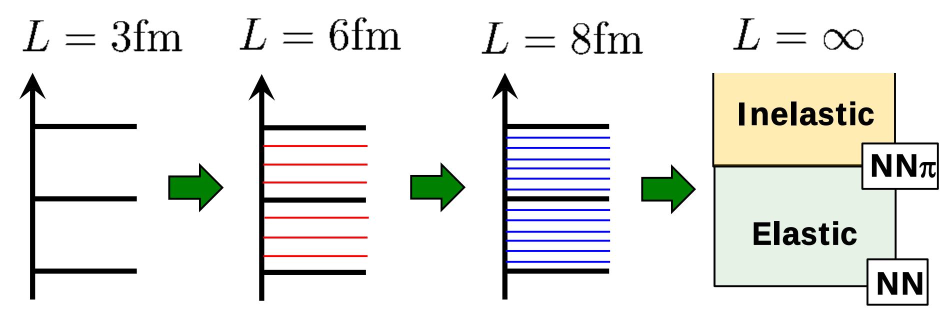
[Exponential Improvement]

The Challenge in multi-baryons on the lattice

Existence of elastic scatt. states

➔ (almost) No Excitation Energy

➔ LQCD method based on G.S. saturation impossible



Signal/Noise issue

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

Parisi ('84), Lepage('89)

	HAL QCD method	Direct method
Ground state	**Signal**	**Signal**
Excited states (elastic)	**Signal**	**Noise**
Excited states (inelastic)	**Noise**	**Noise**

c.f. Direct method [= Plateau fitting w/ GS saturation + Luscher's formula]

Reliability test of LQCD methods

T. Iritani et al. (HAL) JHEP10(2016)101, PRD96(2017)034521, PRD99(2019)014514, JHE03(2019)007

NN @ heavy quark masses

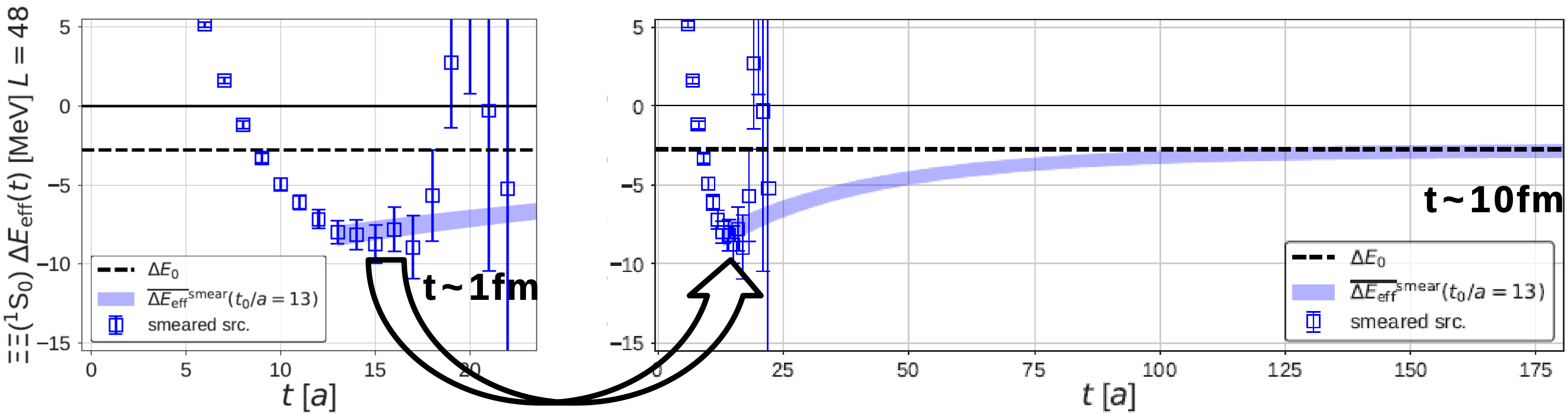
HAL method (HAL) : **unbound**

Direct method (PACS-CS (Yamazaki et al.)/NPL/CalLat): **bound**

Semi-improved calc w/ Luscher's formula (Mainz2019) : **unbound**

Variational calc w/ Luscher's formula (CalLat2020) : **unbound**

Variational calc w/ Luscher's formula (NPL2021) : **(unbound)**



"Pseudo-Plateau" by excited states

HAL QCD pot = Luscher's formula w/ Eigenstate projection

≠ Direct method w/ naïve plateau fitting

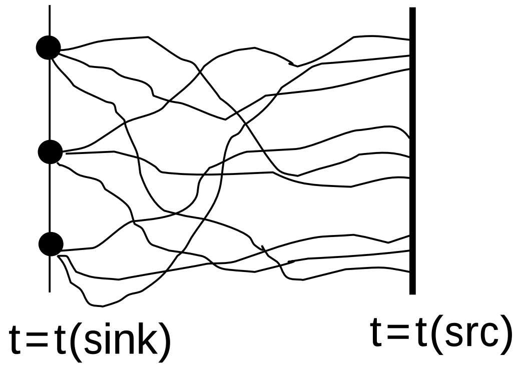
Computational Challenge

- **Enormous comput. cost for multi-baryon correlators**
  - Wick contraction (permutations)

$$\sim [\,(\tfrac{3}{2}A)!\,]^2$$ (A: mass number)

  - color/spinor contractions

$$\sim 6^A \cdot 4^A \quad \text{or} \quad 6^A \cdot 2^A$$



See also T. Yamazaki et al., PRD81(2010)111504

  - **Unified Contraction Algorithm (UCA)**
    - A novel method which unifies two contractions

Permuted Sum

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

Sum over color/spinor unified list

Drastic Speedup

$\times 192$ for ^{3}H/^{3}He, $\times 20736$ for ^{4}He, $\times 10^{11}$ for ^{8}Be (x add'l. speedup)

TD and M.Endres, CPC184(2013)117

- Baryon Forces from LQCD — Ishii-Aoki-Hatsuda (2007)
- Exponentially better S/N — Ishii et al. (2012)
- Coupled channel systems — Aoki et al. (2011,13)

[Theory] = HAL QCD method

Baryon Interactions near the Physical Point

[Hardware]

= K-computer [11 PFlops]

+ HOKUSAI [(1+2.6) PFlops] @ RIKEN

+ HA-PACS [1 PFlops] @ Tsukuba

[Software]

= Unified Contraction Algorithm

- Exponential speedup — Doi-Endres (2013)

^{3}H/^{3}He : $\times 192$

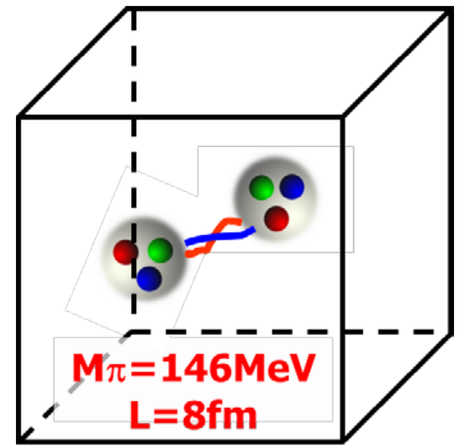
^{4}He : $\times 20736$

^{8}Be : $\times 10^{11}$

Lattice QCD Setup

- **Nf = 2 + 1 gauge configs**
  - clover fermion + Iwasaki gauge w/ stout smearing
  - **V=(8.1fm)^4**, a=0.085fm (1/a = 2.3 GeV)
  - **m(pi) ~= 146 MeV, m(K) ~= 525 MeV**
  - #traj ~= 2000 generated

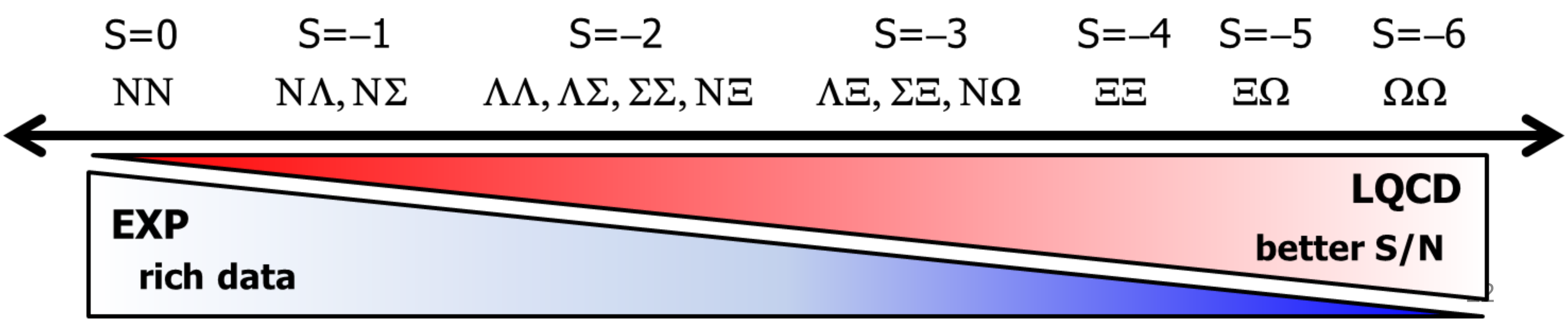
PACS Coll., PoS LAT2015, 075



  - (Quenched) Charm quark w/ RHQ action

Y. Namekawa (PACS), PoS LAT2016, 125

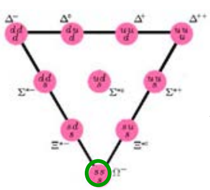
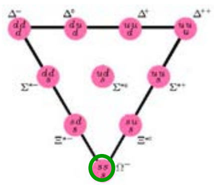
- Nuclear/Hyperon forces + Charmed forces in S, D-waves
  - Wall quark source ➔ LO potential in the derivative expansion



Candidates of di-baryons

Quark Pauli principle provides important guideline

M.Oka et al., NPA464(1987)700, T. Inoue et al. (HAL), NPA881(2012)28

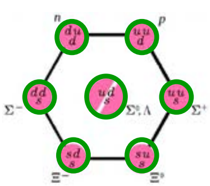
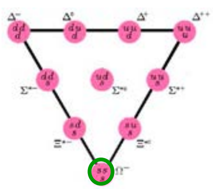
 X 

$$10 \times 10 = 28 + 27 + 10^* + 35$$

ΩΩ (J=0) ΔΔ (J=3)

Zhang et al. ('97)

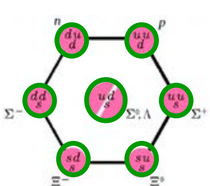
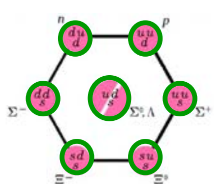
Dyson-Xuong ('64)
Kamae-Fujita ('77)
Oka-Yazaki ('80)

 X 

$$8 \times 10 = 35 + 8 + 10 + 27$$

NΩ (J=2)

Goldman et al. ('87)
Oka ('88)

 X 

$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8s$$

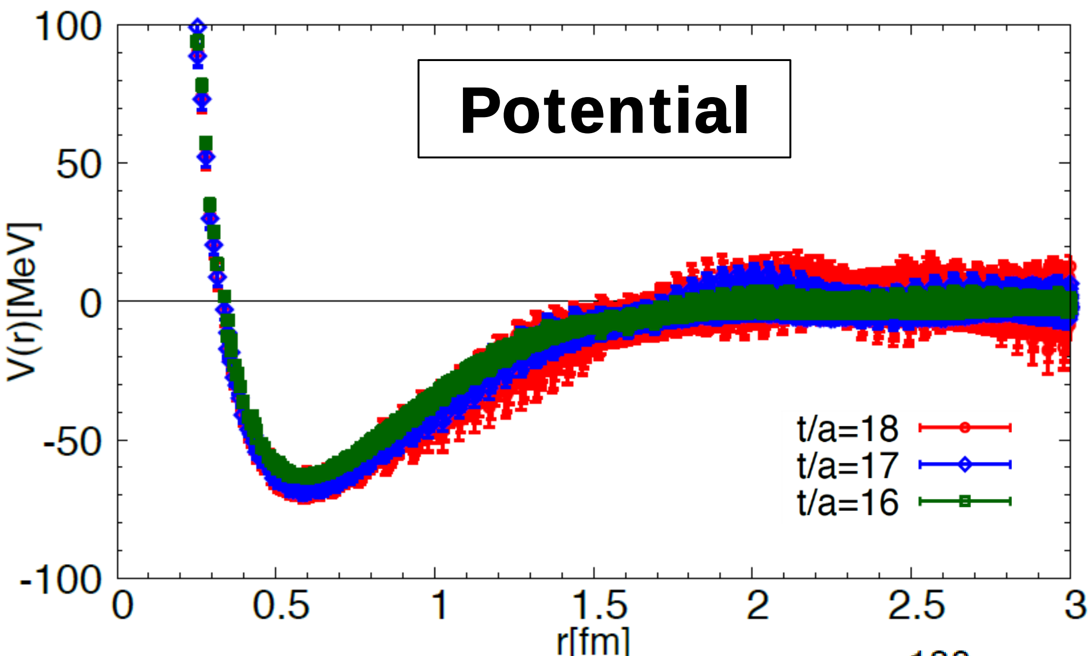
dineutron, ΞΞ etc. (J=0)

H-dibaryon (J=0)

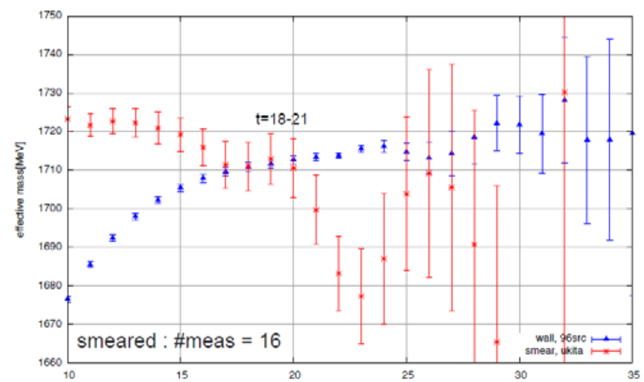
Deuteron (J=1)

ΩΩ system ($^1S_0$)

(The "most strange" dibaryon system)



m(eff) for single Ω



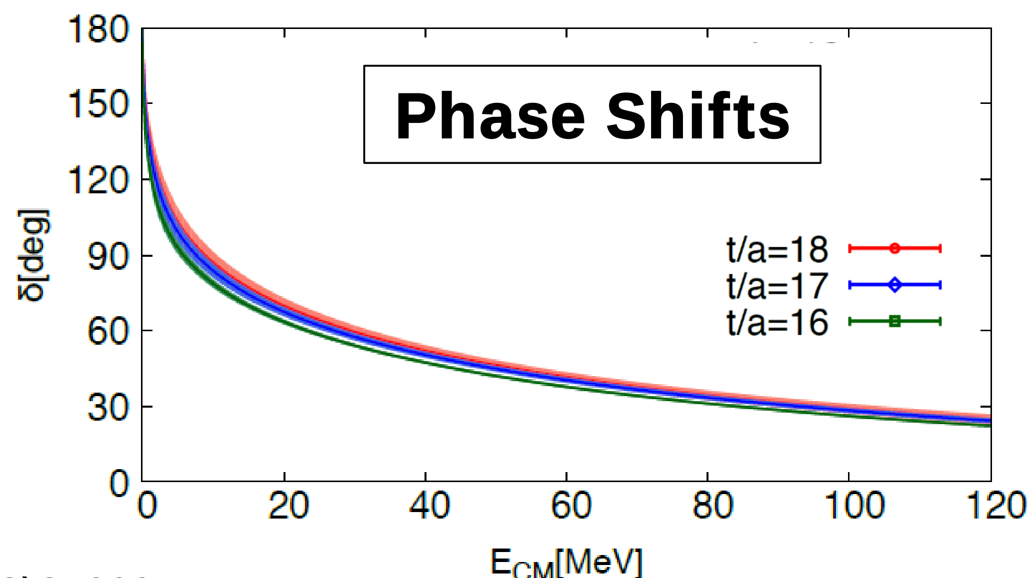
t = 18 : ~0.2-0.3% sys error

Attraction
+ (weak) repulsive core

(Pauli-allowed +
one-gluon exchange at small r)

➔ loosely bound state

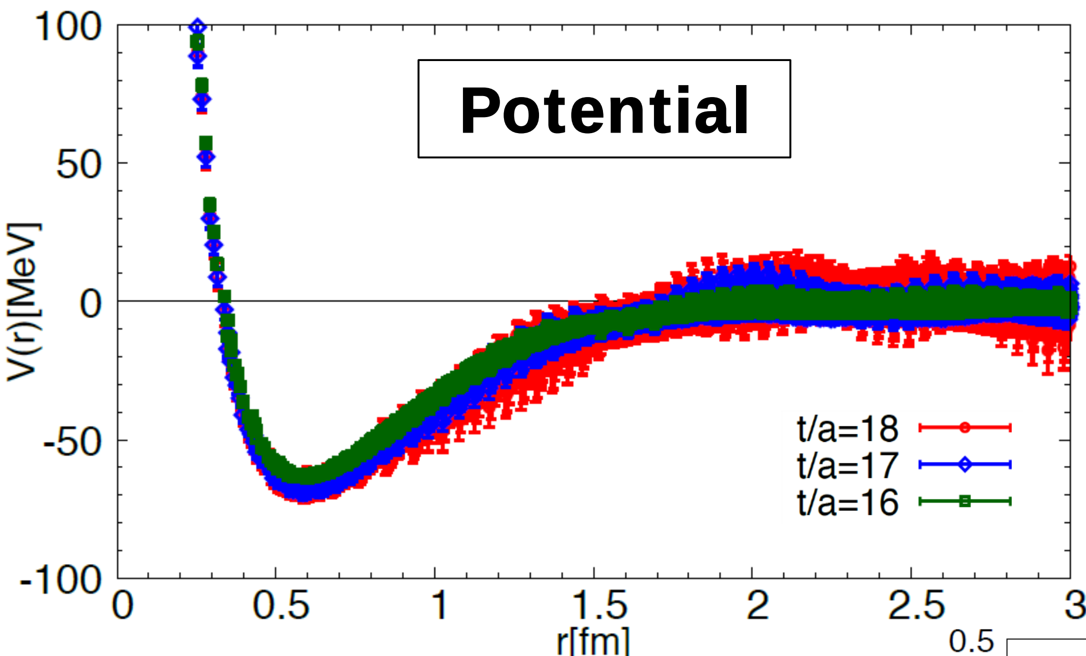
~ unitary limit

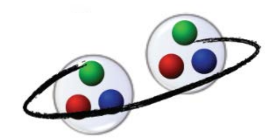


S. Gongyo et al. (HAL Coll.), PRL120(2018)212001

ΩΩ system ($^1S_0$)

(The "most strange" dibaryon system)





$$B_{\mathrm{QCD}} = 1.6(6)(^{+0.7}_{-0.6})\ \mathrm{MeV}$$

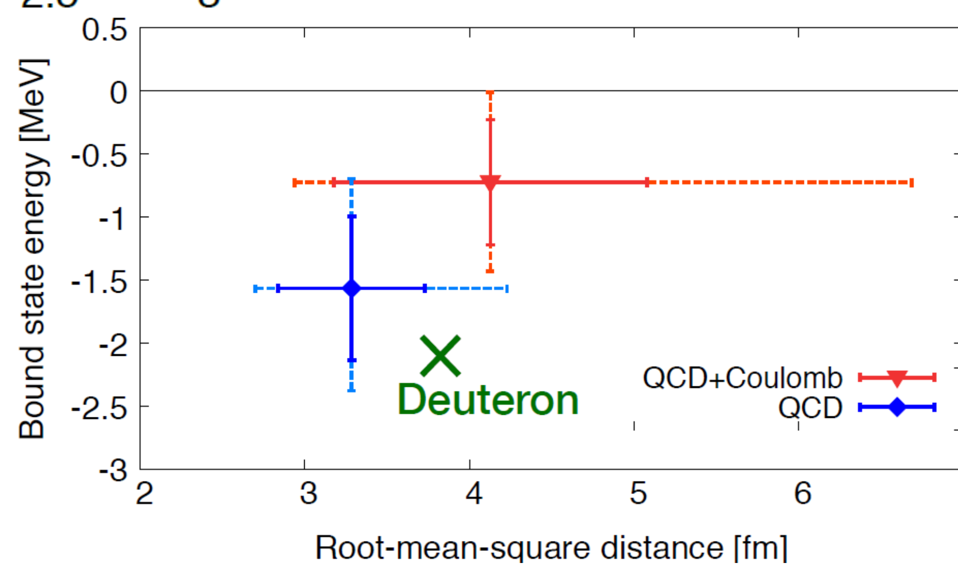
$$B_{\mathrm{QCD+Coul.}} = 0.7(5)(5)\ \mathrm{MeV}$$

Attraction
+ (weak) repulsive core

(Pauli-allowed +
one-gluon exchange at small r)

➔ loosely bound state

~ unitary limit



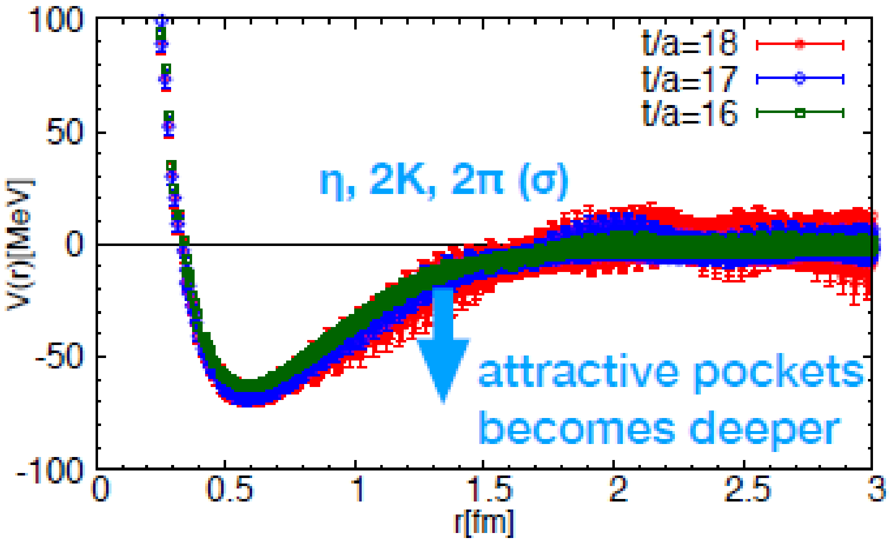
S. Gongyo et al. (HAL Coll.), PRL120(2018)212001

(Coulomb: Ω^- treated as a point particle)

Conservative estimate at exact phys. pt.

mπ=146 MeV -> 135 MeV, mΩ= 1712MeV -> 1672 MeV

B.E. ↑



V.S.

B.E. ↓

$$\mathcal{H} = -\frac{\nabla^2}{m_\Omega} + V_{\Omega\Omega}^{\mathrm{LQCD}}(r)$$

kinetic energy is increasing
-> B.E. is reduced

conservative estimate:
only change the mass of kinetic term

$$(B_{\Omega\Omega}^{(\mathrm{QCD})}, B_{\Omega\Omega}^{(\mathrm{QCD+Coulomb})}) = (1.6(6)\mathrm{MeV}, 0.7(5)\mathrm{MeV})$$
$$\rightarrow (1.3(5)\mathrm{MeV}, 0.5(5)\mathrm{MeV})$$

These changes are within errors

$\Omega_{ccc}\Omega_{ccc}$ system ($^1S_0$)

What happens if we replace strange quark with charm quark?

Y. Lyu, H. Tong et al., PRL127(2021)072003

c.f.

LHC: Ω_c, Ξ_{cc}	LHCb, PRL118('17), PRL119 ('17)
LQCD: Ω_{ccc}	Briceno et al. ('12), PACS-CS Coll. ('13), Z.S.Brown et al. ('14), C. Alexandrou et al. ('14), K. U. Can et al. ('15)
LQCD: $\Omega_c\Omega_{cc}$	P. Junnarkar et al., PRL123(2019)162003
Quark model: $\Omega_{ccc}\Omega_{ccc}$	H. X. Huang et al., arXiv:2011.00513

Lattice QCD Setup

- Nf = 2 + 1 gauge configs

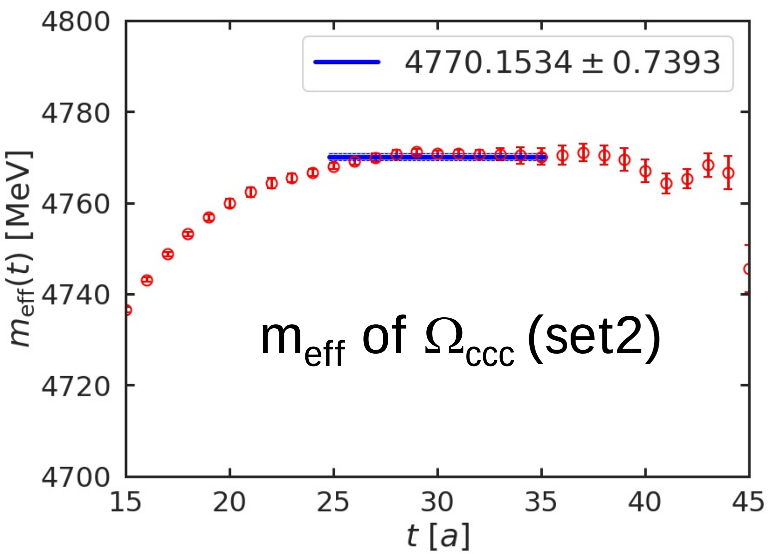
  PACS Coll., PoS LAT2015, 075

  – clover fermion + Iwasaki gauge w/ stout smearing near physical point
  – V=(8.1fm)^4, a=0.085fm (1/a = 2.3 GeV), m(pi) ~= 146 MeV, m(K) ~= 525 MeV

- Relativistic heavy quark (RHQ) action for charm quark
  – remove LO and NLO cutoff error in charm mass → remaining error $\mathcal{O}(\alpha_s^2 a\Lambda_{\rm QCD}, (a\Lambda_{\rm QCD})^2)$
  – RHQ parameters determined by dispersion relation and mass of 1S charmonium (Interpolation of two sets of params)

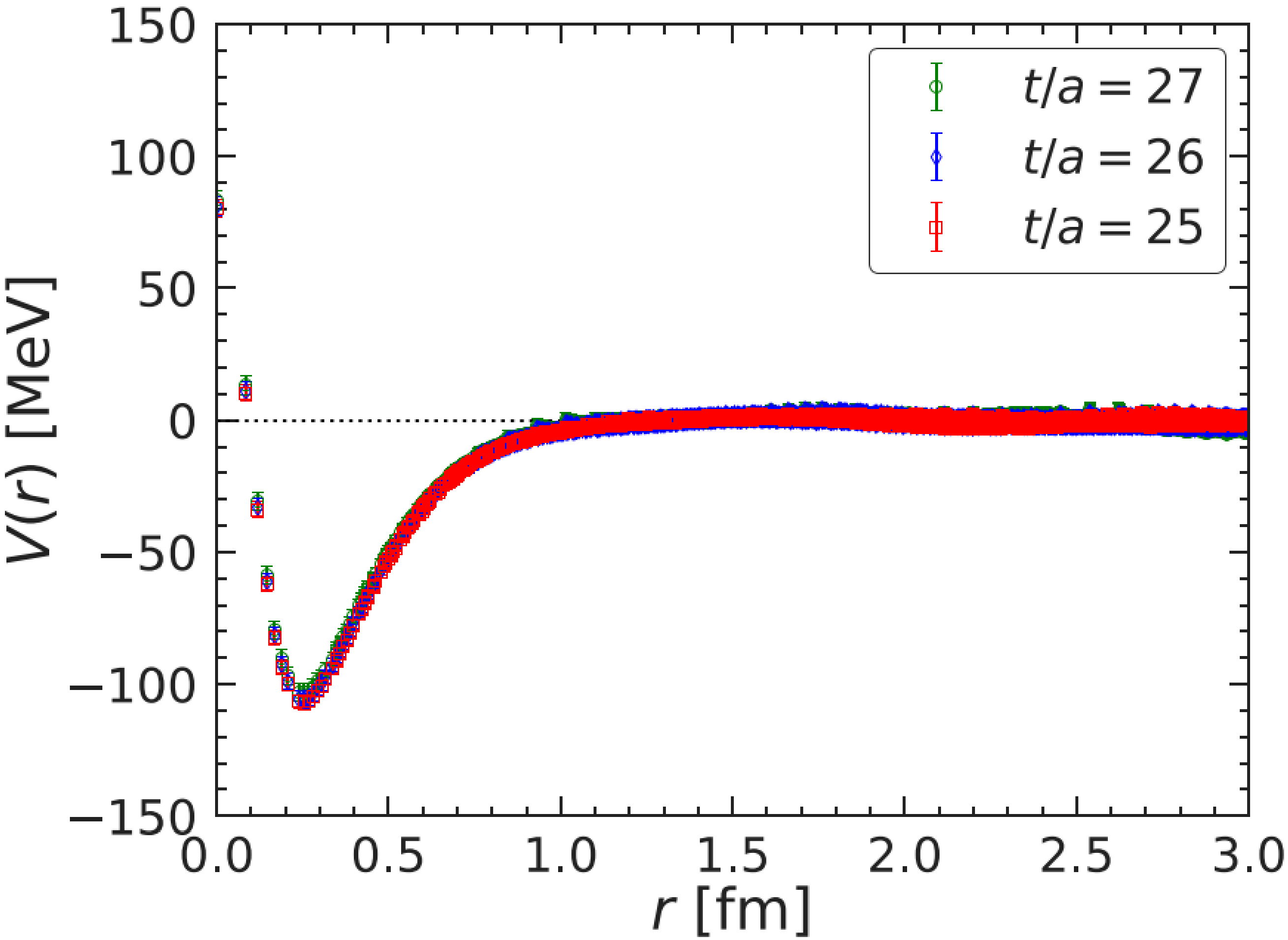
  Y. Namekawa (PACS), PoS LAT2016, 125

- Statistics : 896 meas = 112 conf x 4 src x 2 (fw/bw)



	$(m_{\eta_c} + 3m_{J/\Psi})/4$ [MeV]	$m_{\Omega_{ccc}}$ [MeV]
Set 1	3096.6(0.3)	4837.3(0.7)
Set 2	3051.4(0.3)	4770.2(0.7)
Interpolation	3068.5(0.3)	4795.6(0.7)
Experimental	3068.5(0.1)	···

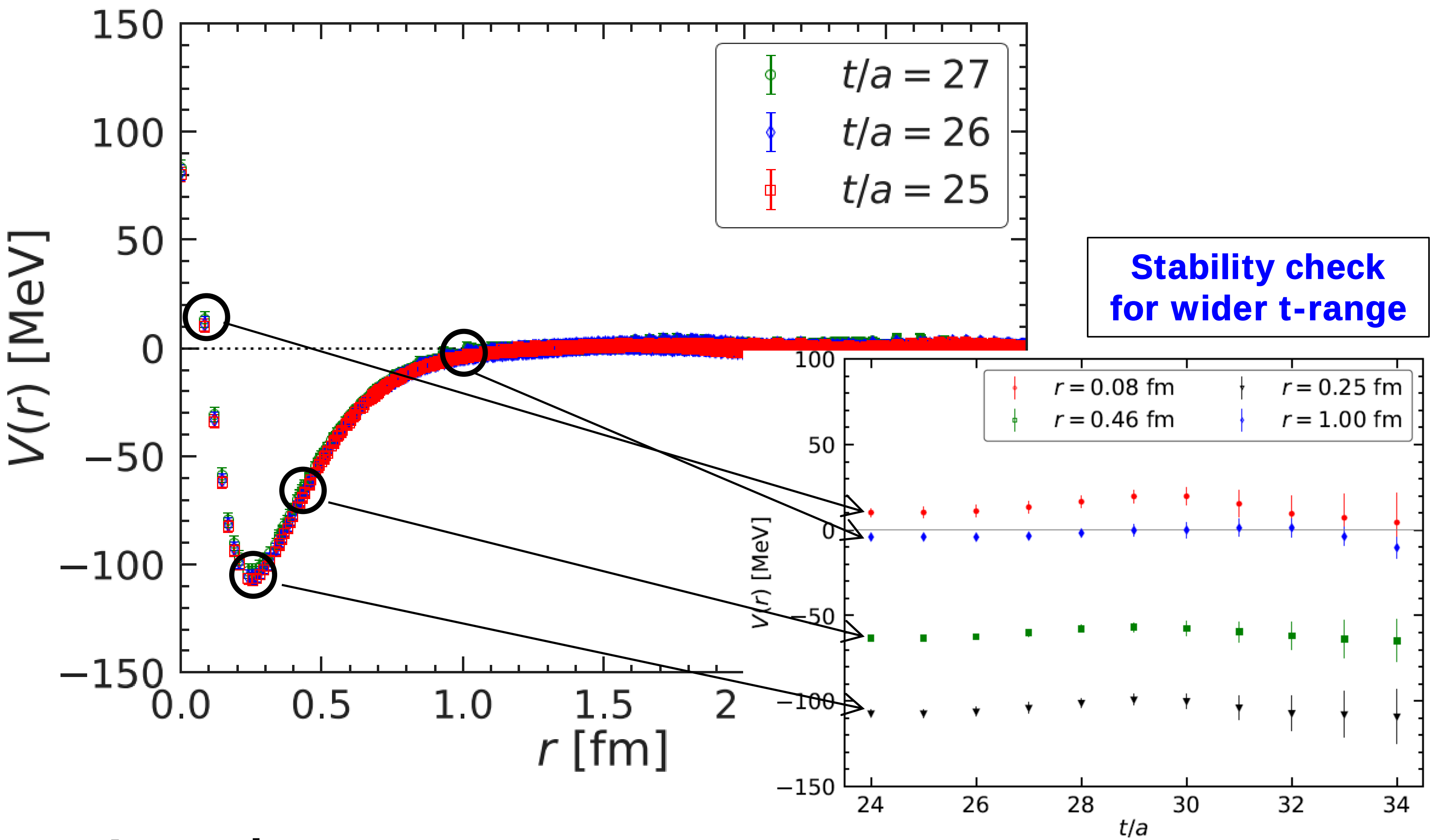
Potential of $\Omega_{ccc}\Omega_{ccc}$ ($^1S_0$)



Attraction
+ (weak) repulsive core

Pauli-allowed +
one-gluon exchange at small r

Potential of $\Omega_{ccc}\Omega_{ccc}$ ($^1S_0$)

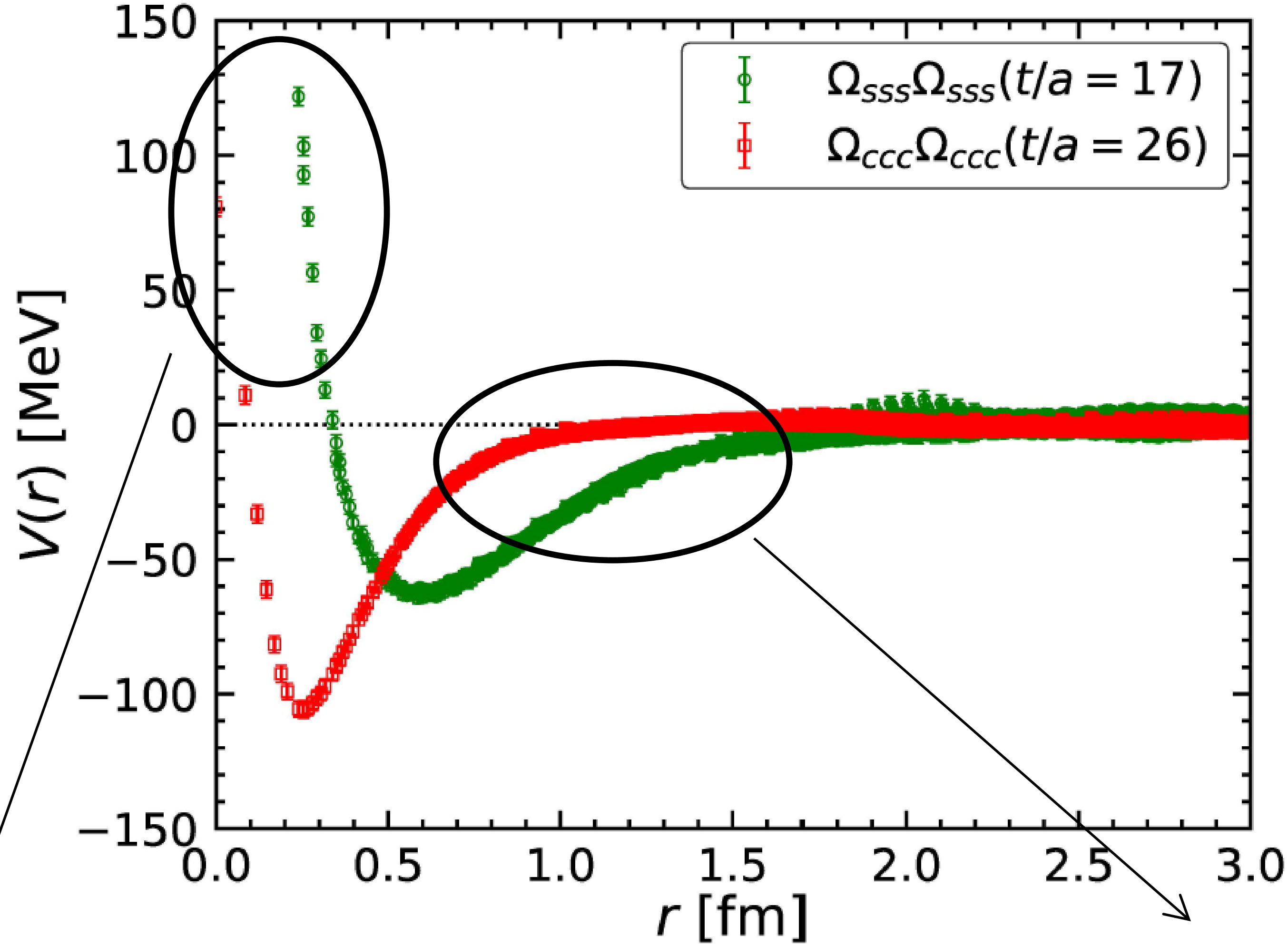


Attraction
+ (weak) repulsive core

Stable in t → sys err from inelastic states,
derivative expansion small

(more studies later)

Comparison of potentials



Color-Magnetic Int by one-gluon-exchange

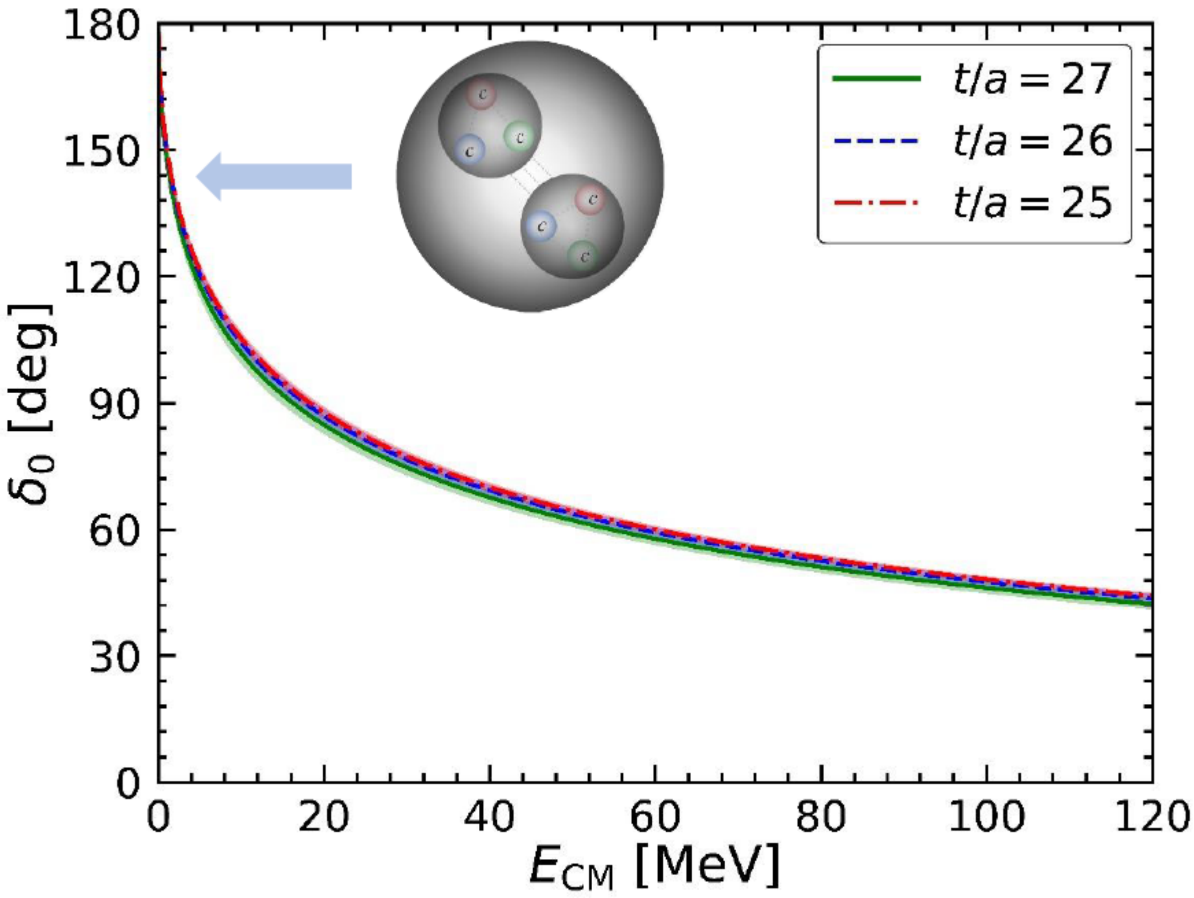
$$\sim \frac{(\vec{\lambda}\cdot\vec{\lambda})(\vec{\sigma}\cdot\vec{\sigma})}{m_i^* m_j^*}$$

Attraction short-ranged for heavier mass

Relative strength consistent!

$$V_{\rm CMI}^{cc}/V_{\rm CMI}^{ss} \simeq (m_s^*/m_c^*)^2 \sim (500/1500)^2 \sim 0.1$$

Phase shifts of $\Omega_{ccc}\Omega_{ccc}$ ($^1S_0$)



Scattering parameters, binding energy and root-mean-square distance

$$a_0 = 1.57(8)_{\text{sta.}} \left({}^{+12}_{-4}\right)_{\text{sys.}} \text{ fm}, \qquad r_{\text{eff}} = 0.57(2)_{\text{sta.}} \left({}^{+1}_{-0}\right)_{\text{sys.}} \text{ fm}$$

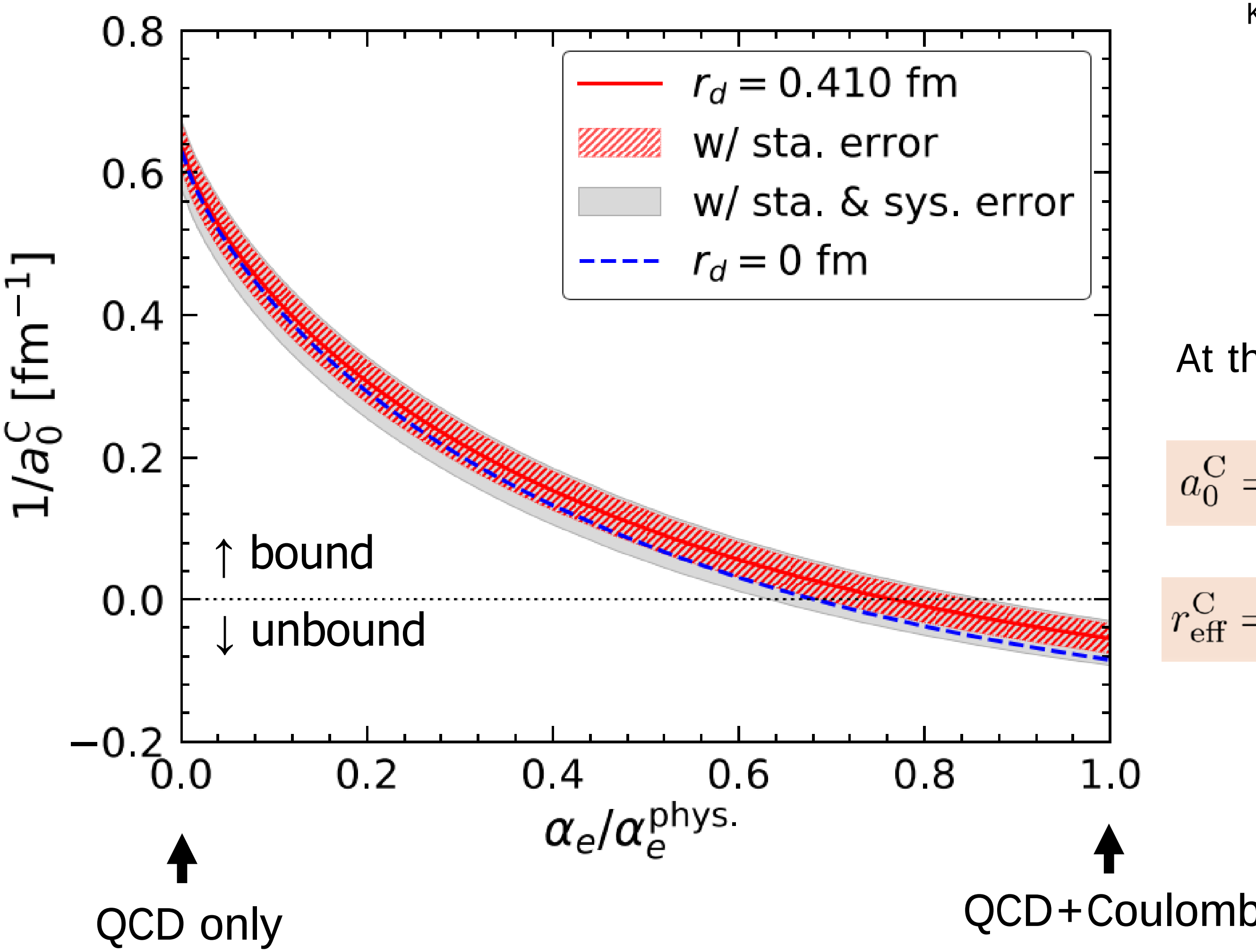
$$B = 5.68(77)_{\text{sta.}} \left({}^{+46}_{-102}\right)_{\text{sys.}} \text{ MeV}, \qquad \sqrt{\langle r^2 \rangle} = 1.13(6)_{\text{sta.}} \left({}^{+8}_{-3}\right)_{\text{sys.}} \text{ fm}$$

Effect of Coulomb repulsion

QCD + Coulomb $V^{\mathrm{QCD}} \to V^{\mathrm{QCD}} + V^{\mathrm{Coulomb}}, \quad V^{\mathrm{Coulomb}} = \frac{4\alpha_e}{r} F(r)$

F(r) represents effects of charge distribution of $\Omega_{ccc}{}^{++}$

K. Can et al., PRD92(2015)114515

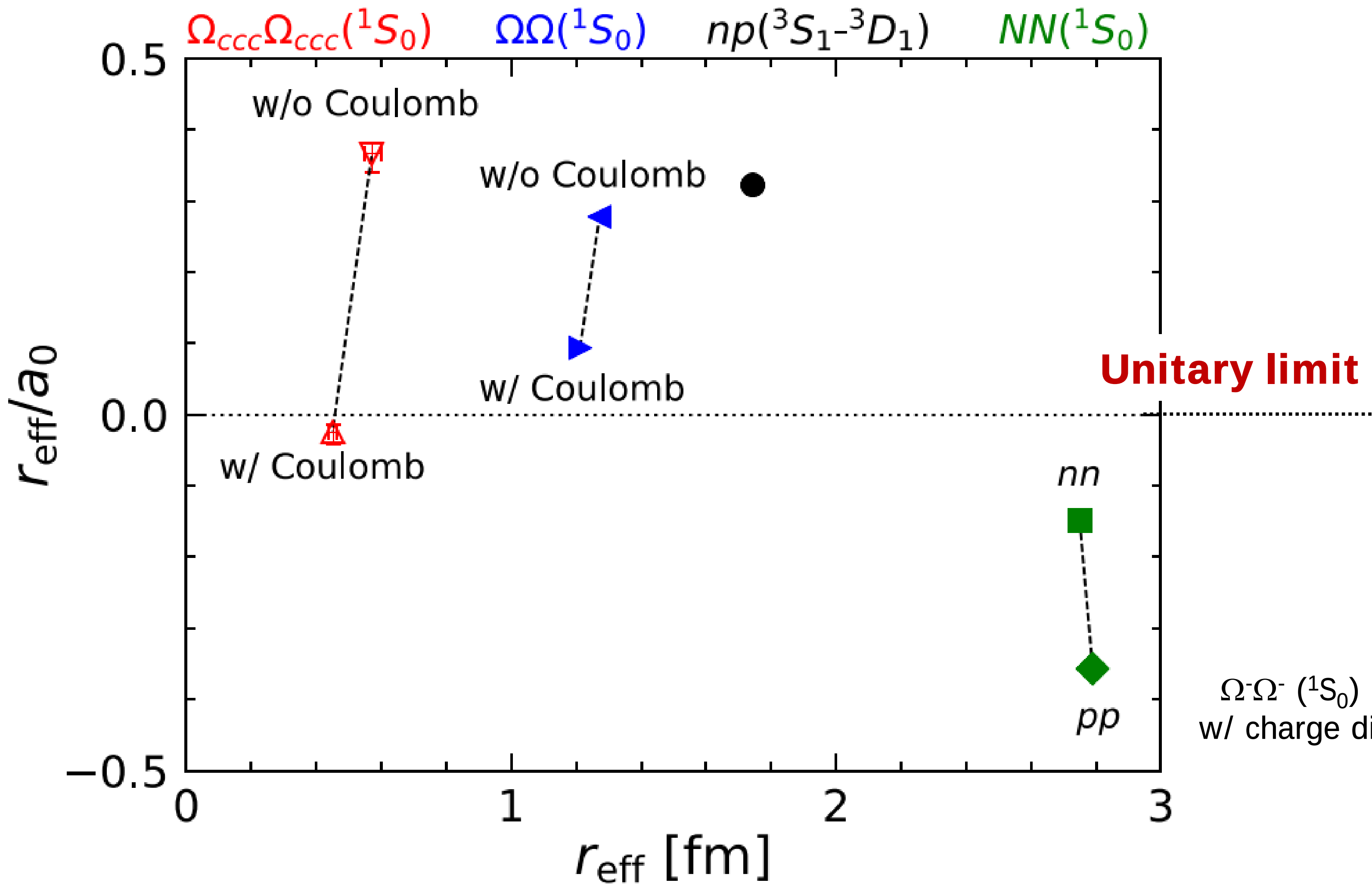


At the physical QED coupling,

$$a_0^{\mathrm{C}} = -19(7)_{\mathrm{sta.}} \left(^{+7}_{-6}\right)_{\mathrm{sys.}}\ \mathrm{fm},$$

$$r_{\mathrm{eff}}^{\mathrm{C}} = 0.45(1)_{\mathrm{sta.}} \left(^{+1}_{-0}\right)_{\mathrm{sys.}}\ \mathrm{fm}$$

Dibaryons near unitary limit



$\Omega^-\Omega^-$ ($^1S_0$) updated w/ charge distribution

$$\Omega_{ccc}^{++}\Omega_{ccc}^{++}(^1S_0): \quad r_{\text{eff}}^C/a_0^C = -0.024(0.010)(^{+0.006}_{-0.014})$$

→ closest to the unitarity!

Systematic errors

- Finite cutoff error
  - RHQ action for c-quark, non-perturbative O(a)-improvement for uds-quarks
    → Remaining error is $\mathcal{O}(\alpha_s^2 a\Lambda_{\rm QCD}, (a\Lambda_{\rm QCD})^2)$
    → expected to be only O(1)% error w/ 1/a=2.333GeV
- Errors in sea quark sector
  - Light sea quarks slightly heavy, (m(pi), m(K)) ~= (146, 525) MeV
    → expected to be small since they are rather irrelevant for $\Omega_{ccc}\Omega_{ccc}$
    (N.B.the range of potential is < 1fm, light quark dof irrelevant)
  - Charm quark loop is neglected (quenched)
    → suppressed by the heavy charm mass, typically O(1) %
- Another confirmation about these points
  - Our Ω_{ccc} mass is consistent with or has ~1% deviation at most from:
    Nf=2+1, phys point w/ finite a, Nf=2+1 w/ chiral & continuum extrapolation,
    Nf=2+1+1 w/ chial and continuum extrapolation

Briceno et al. (‘12), PACS-CS Coll. (‘13), Z.S.Brown et al. (‘14), C. Alexandrou et al. (‘14)

- Truncation err in derivative expansion in potential?

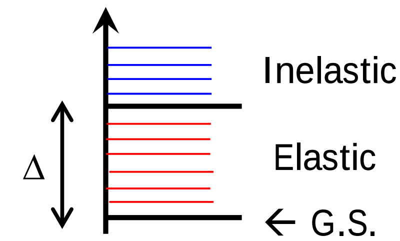
Systematic error in derivative expansion

Time-dependent HAL QCD method

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') R(\boldsymbol{r}',t) \;=\; \left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right) R(\boldsymbol{r},t)$$

Signal from (elastic) excited states

FV spectrum



Non-locality of U(r,r') ← derivative expansion

Okubo-Marshak(1958)

$$U(\boldsymbol{r},\boldsymbol{r}') = \textstyle\sum_n V_n(\boldsymbol{r}) \nabla^n \delta(\boldsymbol{r}-\boldsymbol{r}')$$

Expansion w.r.t. ∇/Λ, $(\Lambda \simeq \Lambda_{QCD}, \Delta)$

$$U(\boldsymbol{r},\boldsymbol{r}') = \left[V_c(r) + S_{12} V_T(r) + \boldsymbol{L}\cdot\boldsymbol{S} V_{LS}(r) + \mathcal{O}(\nabla^2) \right] \delta(\boldsymbol{r}-\boldsymbol{r}')$$

LO LO NLO NNLO

For phase shifts at energy region close to (far from) region which correlator couples to, truncation err of the expansion is expected to be small (large)

For $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$ (and many other cases), we calculate at LO

t-dep of potential is examined → if stable, small sys err

Better way to examine this systematics?

c.f. Explicit calc at N2LO
T. Iritani et al. (HAL),
PRD99(2019)014514

New method using FV spectrum w/ HAL pot

T. Iritani et al. (HAL), JHEP03(2019)007

- FV spectrum from temporal corr (w/ Luscher’s formula)
  - No issue on the derivative expansion of potential
  - Naïve plateau fitting is unreliable (“pseudo-plateau issue”)

T. Iritani et al., (HAL) (’16, ’17, ’19, ’19)

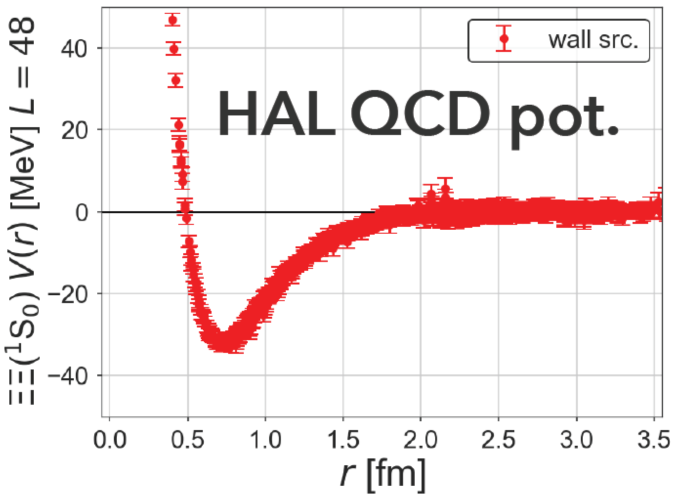
See Mainz(’19,’21), CallLat(’21) for variational study

[Utilize HAL pot to overcome the pseudo-plateau issue]

- Construct optimized op for each FV eigen state by HAL pot
  ➔ optimized temporal corr for each FV eigen state

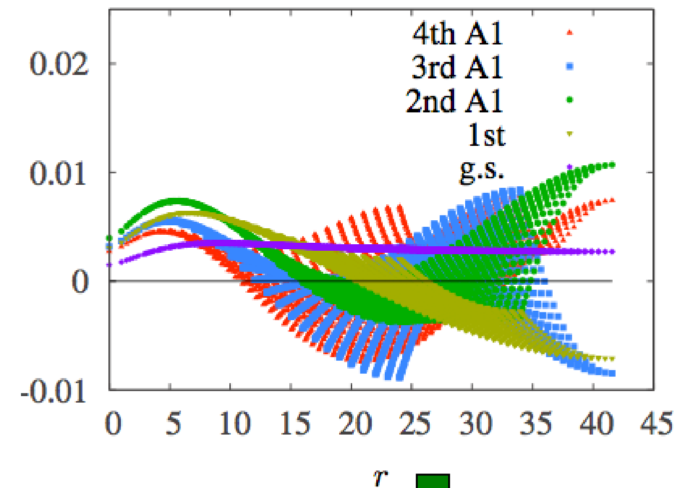
- Consistency check between
  (1) FV spectrum from optimized temporal corr
  (2) FV spectrum from HAL QCD pot
  - If consistent ➔ sys err in HAL pot is well under control

New method using FV spectrum w/ HAL pot



Solve Schrodinger eq. in Finite V

Eigen wave functions



Solve in FV

Eigen energies = FV spectrum

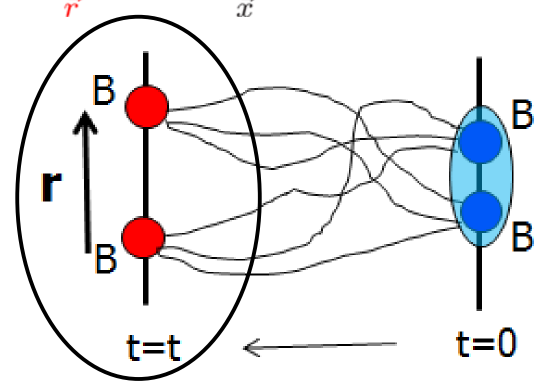
n-th A1	ΔE_n [MeV]
0	-2.58(1)
1	52.49(2)
2	112.08(2)
3	169.78(2)
4	224.73(1)

Consistency Check!

Optimized (sink) op

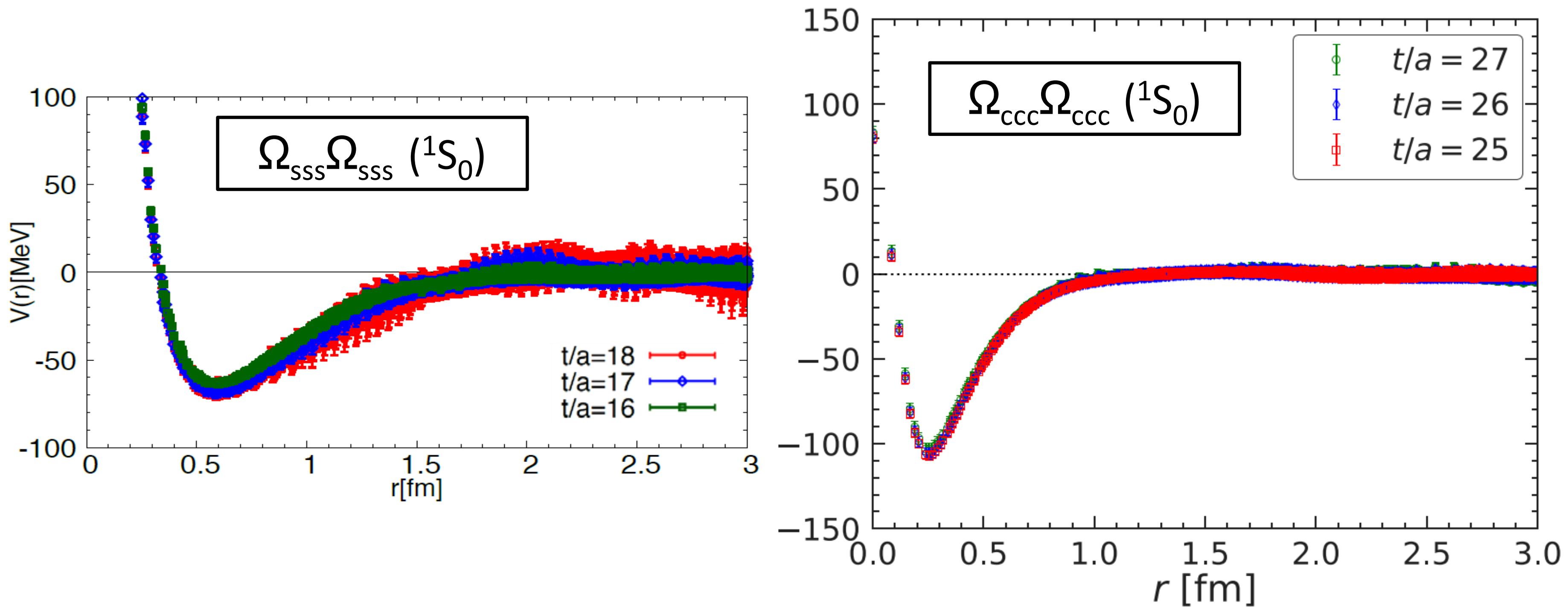
$$\mathcal{J}^{2B}_{\mathrm{sink}} = \sum_{\vec{r}} \psi^\dagger(r) \sum_{\vec{x}} B(\vec{r}+\vec{x})B(\vec{x})$$

FV spectrum from Optimized temporal corr



Eigen w.f. can be also used to analyze non-optimized temporal corr

HAL QCD Potential



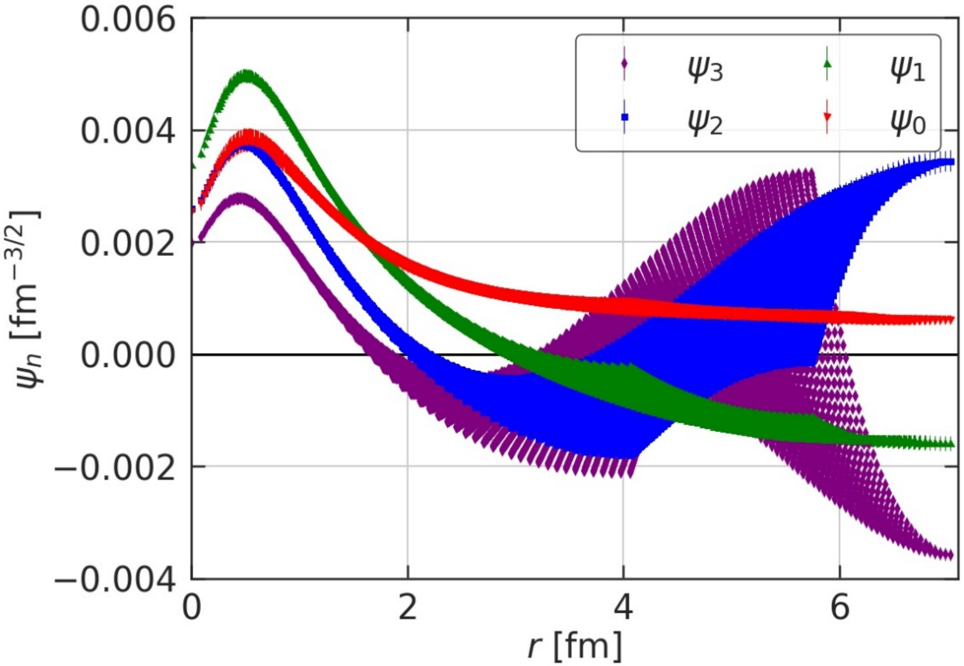
If we solve Schrodinger equation in infinite V, systems are bound

$$B^{(\mathrm{QCD})} = 1.6(0.6)\left({}^{+0.7}_{-0.6}\right)\ \mathrm{MeV} \qquad B^{(\mathrm{QCD})} = 5.68(0.77)\left({}^{+0.46}_{-1.02}\right)\ \mathrm{MeV}$$

$$\sqrt{\langle r^2\rangle}^{(\mathrm{QCD})} = 3.3(0.5)\left({}^{+0.8}_{-0.3}\right)\ \mathrm{fm} \qquad \sqrt{\langle r^2\rangle}^{(\mathrm{QCD})} = 1.13(0.06)\left({}^{+0.08}_{-0.03}\right)\ \mathrm{fm}$$

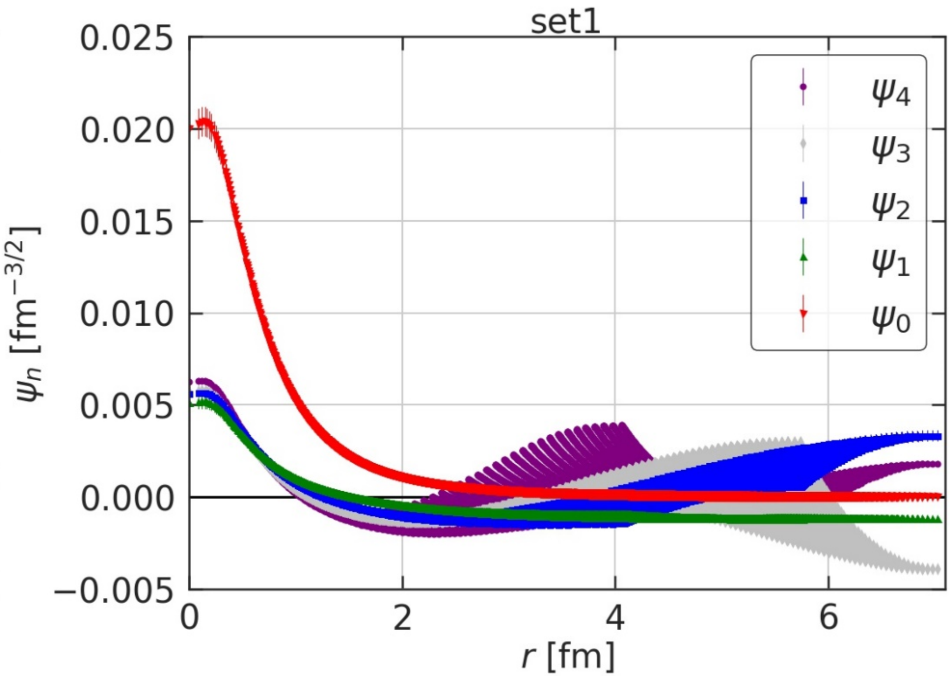
Eigen wave functions and energies in finite V

$\Omega_{sss}\Omega_{sss}$ ($^1S_0$)



Level	ΔE_n [MeV]
G.S.	-1.81(0.26)
1st	7.35(0.42)
2nd	22.55(0.54)
3rd	39.34(0.62)

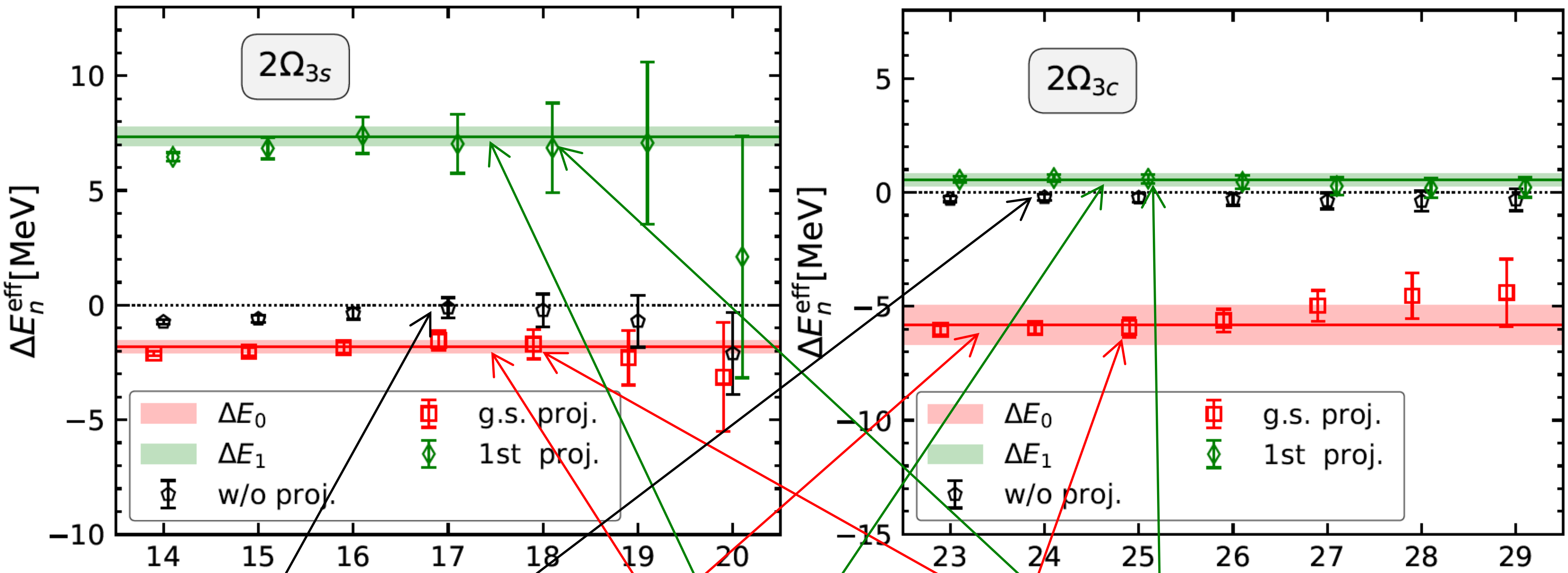
$\Omega_{ccc}\Omega_{ccc}$ ($^1S_0$)



Level	ΔE_n [MeV]
G.S.	-5.87(0.85)
1st	0.53(0.25)
2nd	6.58(0.29)
3rd	12.44(0.36)

Comparison of FV spectrum (effective energy shift)

$\Omega_{sss}\Omega_{sss}$ ($^1S_0$) $\Omega_{ccc}\Omega_{ccc}$ ($^1S_0$)



ΔE of g.s. from HAL pot ⟺ **ΔE from opt. t-corr for g.s.**

ΔE of 1st from HAL pot ⟺ **ΔE from opt. t-corr for 1st.**

Good agreement!

Sys err in HAL pot under control

ΔE from un-optimized t-corr
(usual one in direct method)

➔ Significant deviation from the correct ΔE of g.s.

Decomposition of correlator

In the HAL QCD method,

the following R-corr is used to obtain the potential

$$
\begin{aligned}
R(\boldsymbol{r},t) &= \langle 0|\mathrm{T}[B(\boldsymbol{r},t)B(\boldsymbol{0},t)\overline{\mathcal{J}_{\rm src}(0)}]|0\rangle/(G_{\rm 2pt}(t))^2 \\
&= \sum_n a_n \psi_n(\boldsymbol{r}) e^{-\Delta E_n t}
\end{aligned}
$$

Since we know ψ_n, ΔE_n from FV eigenmodes, a_n can be determined

In the direct method,

the following (non-optimized) temporal corr is usually used

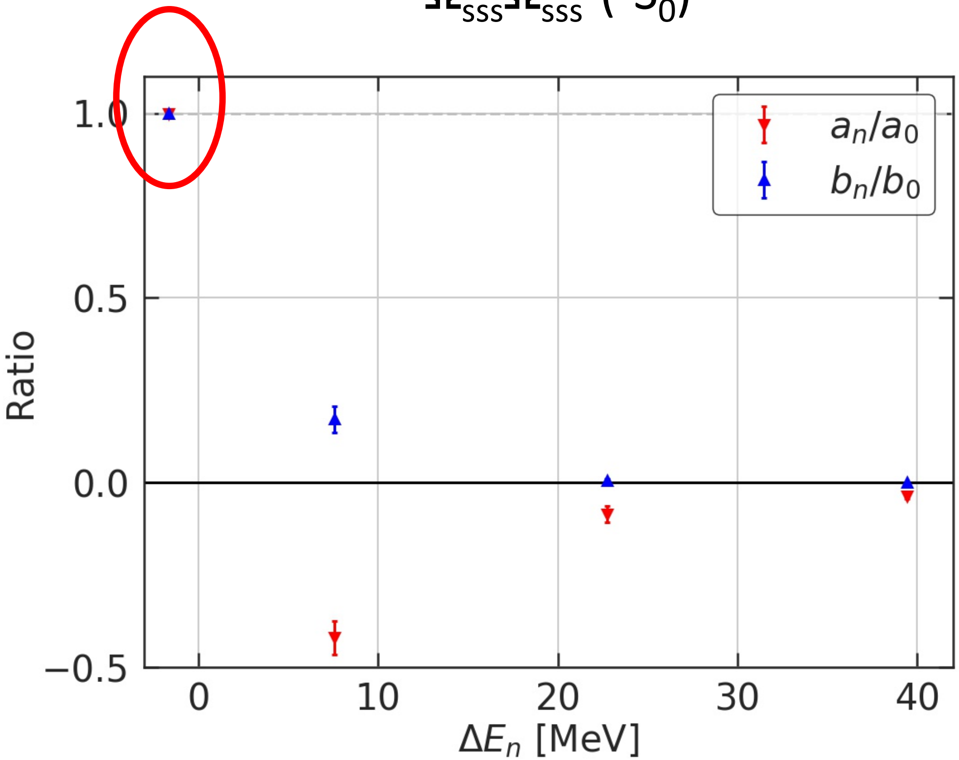
$$
R(\boldsymbol{r},t) \;=\; \sum_{\boldsymbol{r}} R(\boldsymbol{r},t) = \sum_n b_n e^{-\Delta E_n t}
$$

$$
b_n = \sum_{\boldsymbol{r}} a_n \psi_n(\boldsymbol{r})
$$

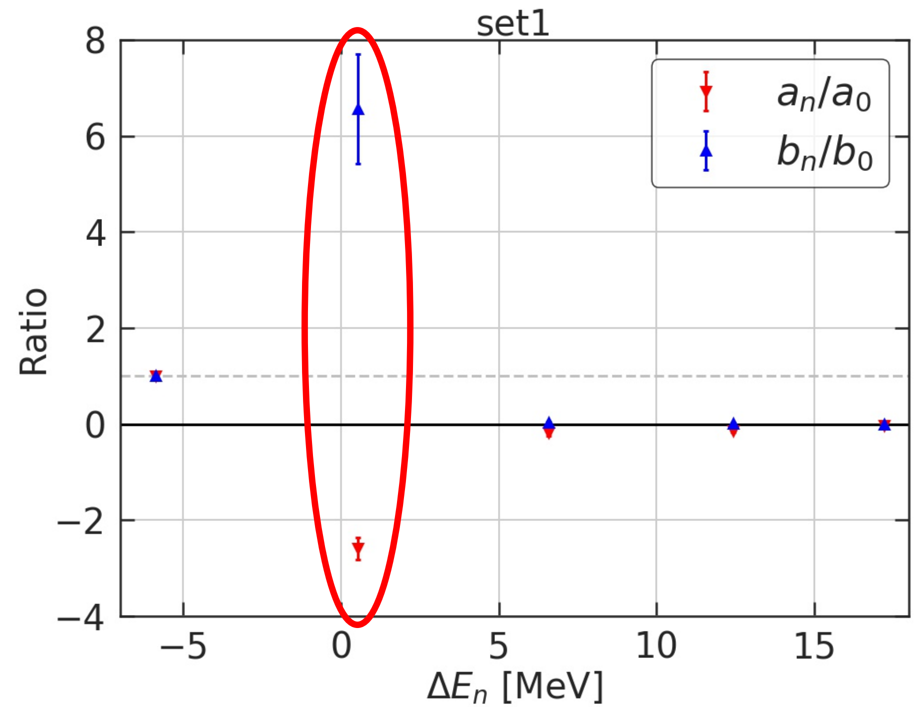
← Each baryon op @ sink is projected to zero-momentum

Decomposition of correlator

$\Omega_{sss}\Omega_{sss}$ $(^1S_0)$



$\Omega_{ccc}\Omega_{ccc}$ $(^1S_0)$



Both of R(r,t) and R(t): G.S. dominant

Both of R(r,t) and R(t): 1st. dominant

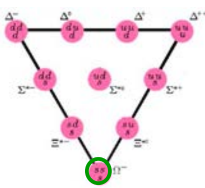
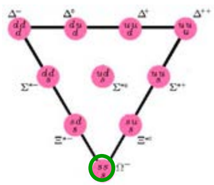
Potential can be reliably extracted regardless whether R-corr is dominated by G.S. or 1st excited state

Decomposition params can be also compared w/ output from variational study

Candidates of di-baryons

Quark Pauli principle provides important guideline

M.Oka et al., NPA464(1987)700, T. Inoue et al. (HAL), NPA881(2012)28

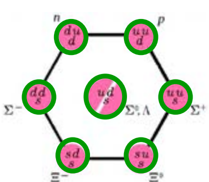
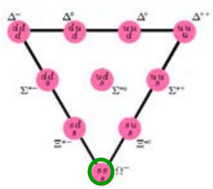
 X 

$10 \times 10 = 28 + 27 + 10^* + 35$

$\Omega\Omega$ (J=0) $\Delta\Delta$ (J=3)

Zhang et al. ('97)

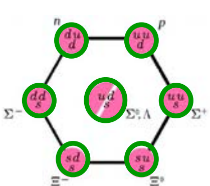
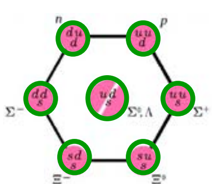
Dyson-Xuong ('64)
Kamae-Fujita ('77)
Oka-Yazaki ('80)

 X 

$8 \times 10 = 35 + 8 + 10 + 27$

$N\Omega$ (J=2)

Goldman et al. ('87)
Oka ('88)

 X 

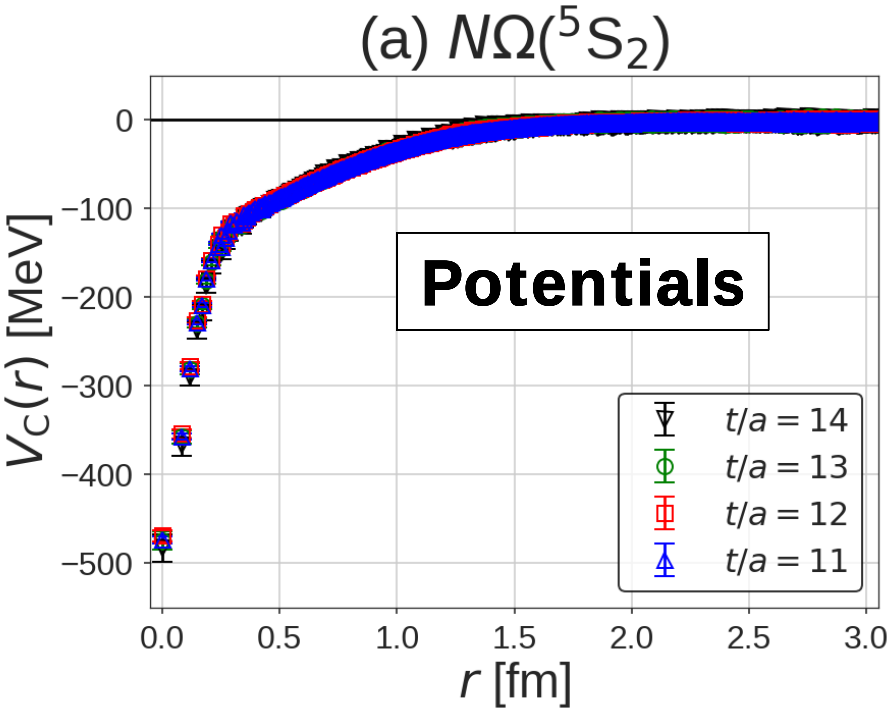
$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8s$

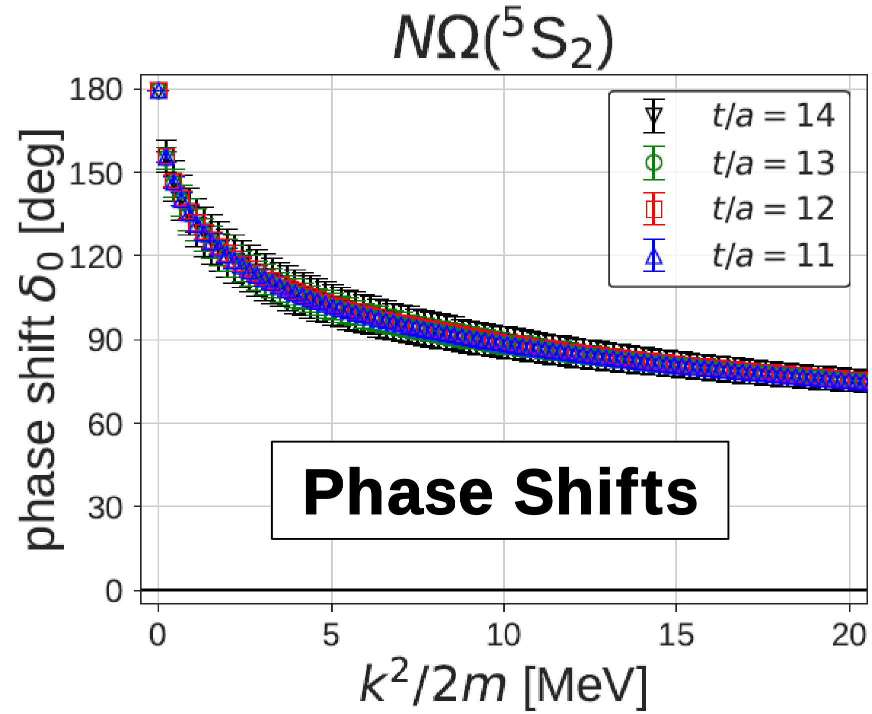
dineutron, $\Xi\Xi$ etc. (J=0)

H-dibaryon (J=0)

Deuteron (J=1)

NΩ system ($^5S_2$)

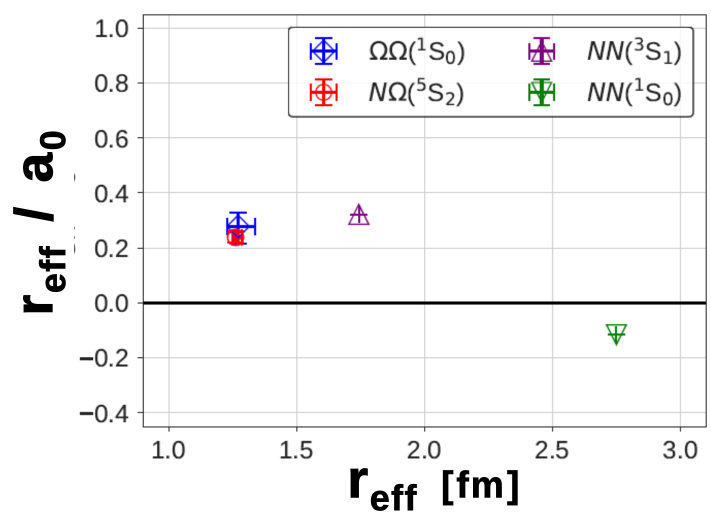




(Quasi) Bound state

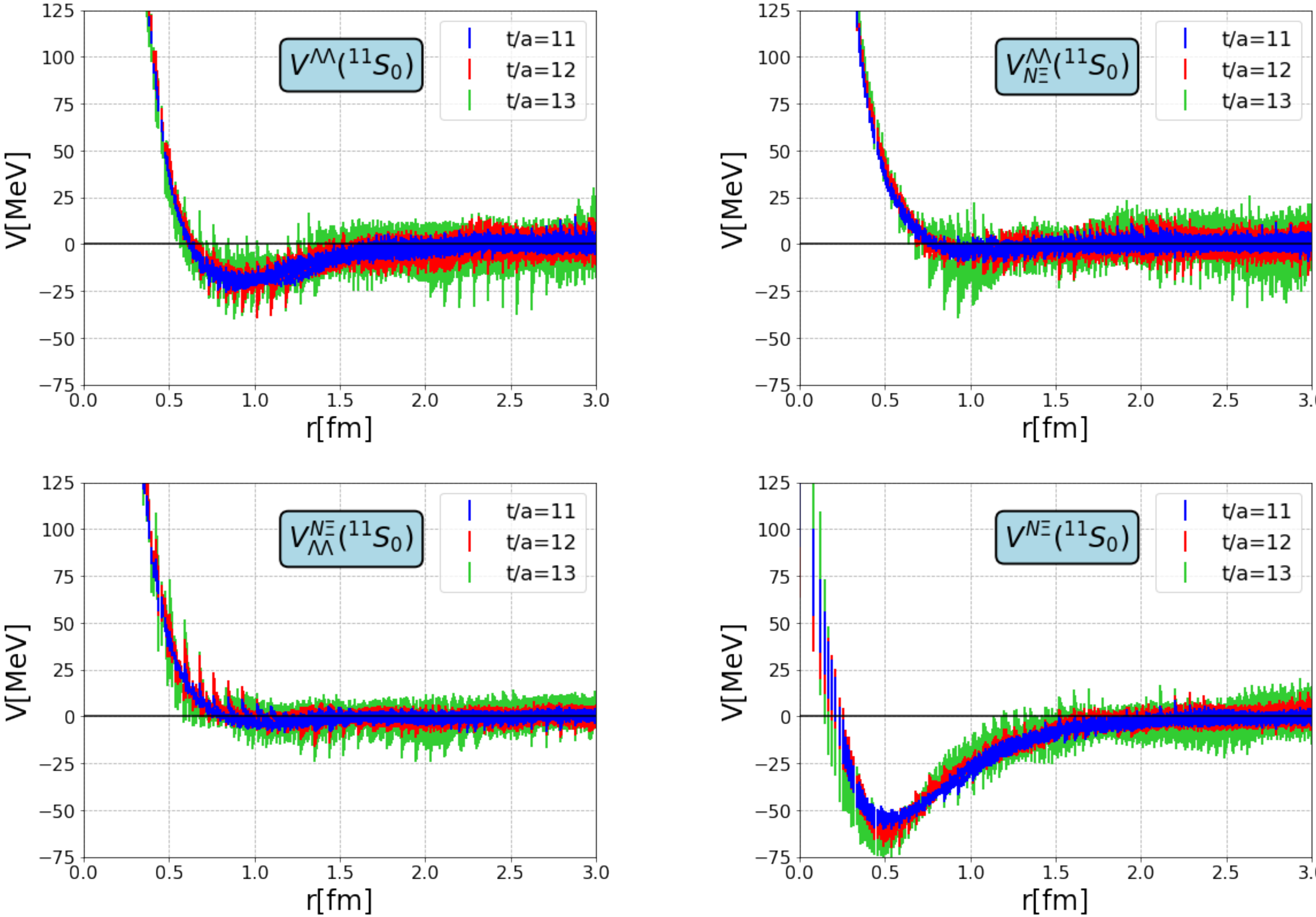
[~ Unitary limit]

$$B_{N\Omega} = 1.54(0.30)(^{+0.04}_{-0.10})\ \text{MeV}$$
$$B_{p\Omega^-} = 2.46(0.34)(^{+0.04}_{-0.11})\ \text{MeV}$$



T. Iritani et al. (HAL Coll.), PLB792(2019)284

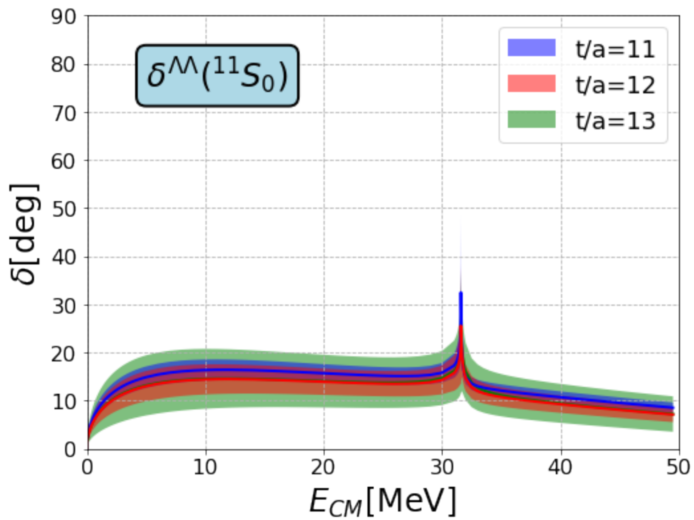
ΛΛ, NΞ (effective) 2x2 coupled channel analysis

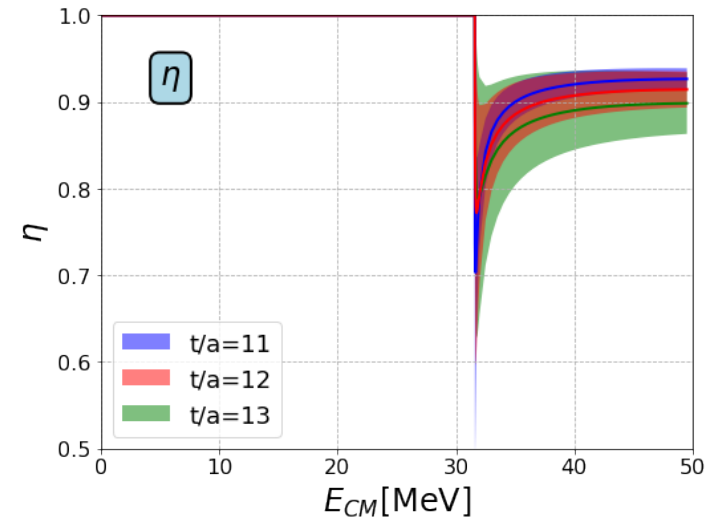


NΞ ($^{11}S_0$) channel is attractive
NΞ–ΛΛ coupling is small

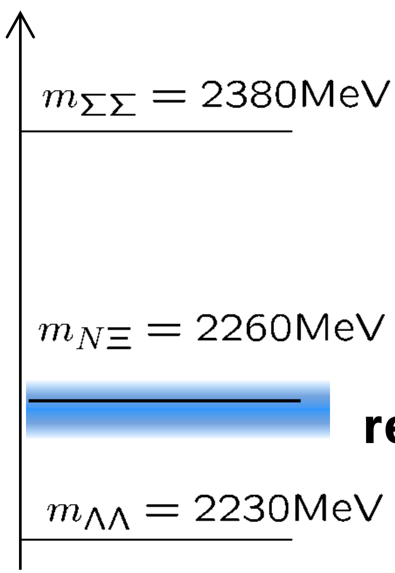
K. Sasaki et al., (HAL Coll.), NPA998(2020)12137

ΛΛ, NΞ 2x2 coupled channel analysis



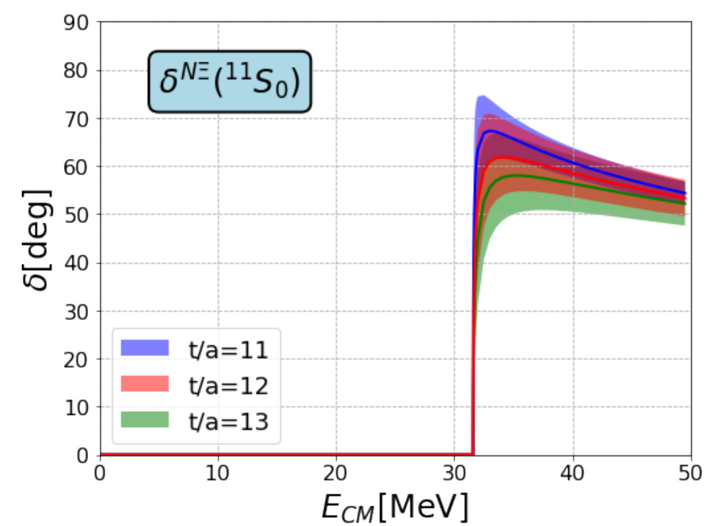


a_0 = -0.81(23)(+0.00/-0.13) [fm]
r_{eff} = 5.47(78)(+0.09/-0.55) [fm]



J-PARC E42 (Ahn)

NΞ ~ unitary limit
remnant of "H-dibaryon"



(N.B. NΞ = 1rep 50%, 27rep 30% in SU(3))

How/Where can we examine these exotic states / interactions ?

I already experimentally(?) observe one at Haneda/Tokyo Airport !

Di-Omega State !

Baryon-Baryon correlation in HIC

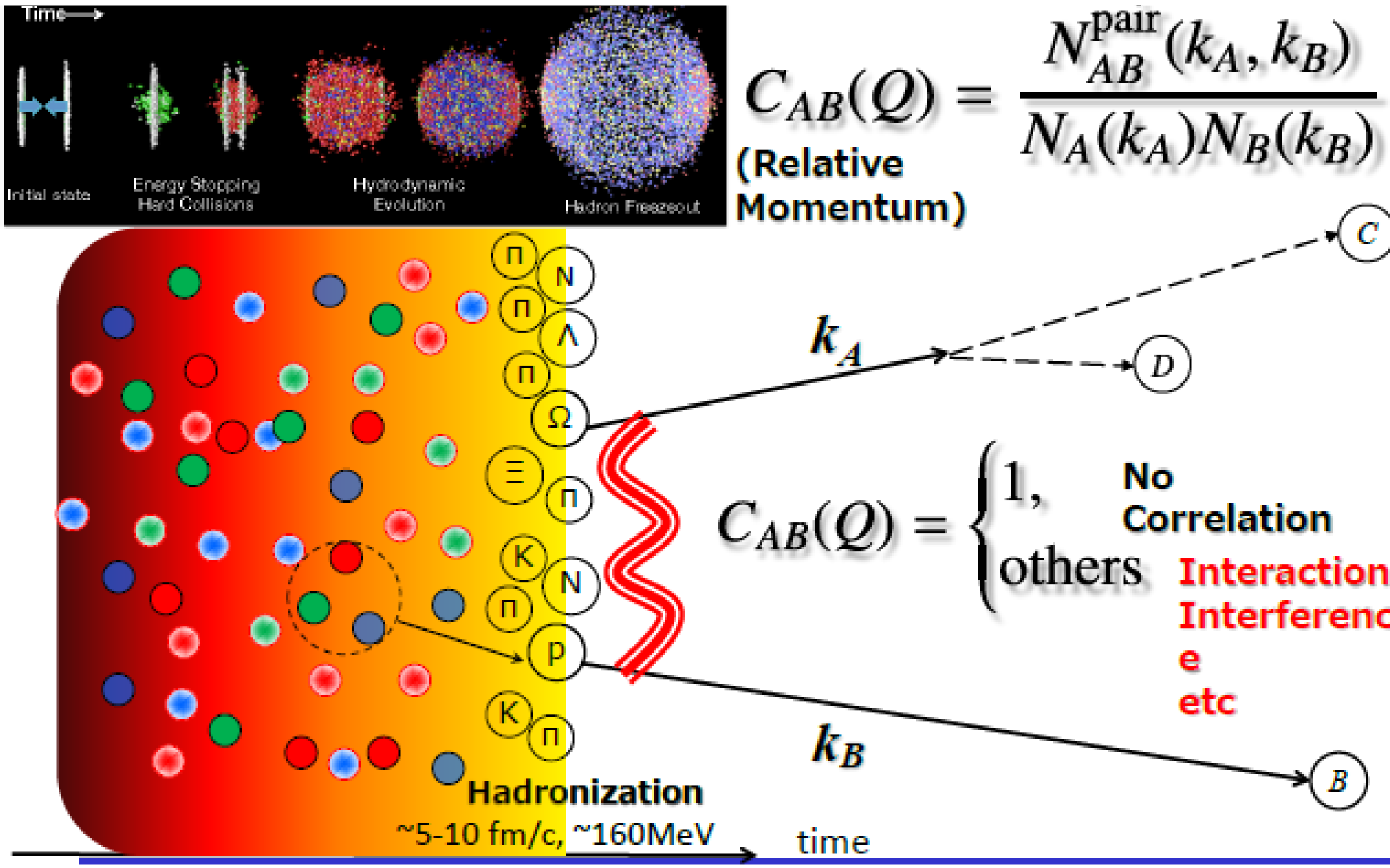


Fig. from K. Morita

BB-correlation
←→ BB- (final state) interaction

Ratio of correlation between small/large source size is useful to mask Coulomb effect

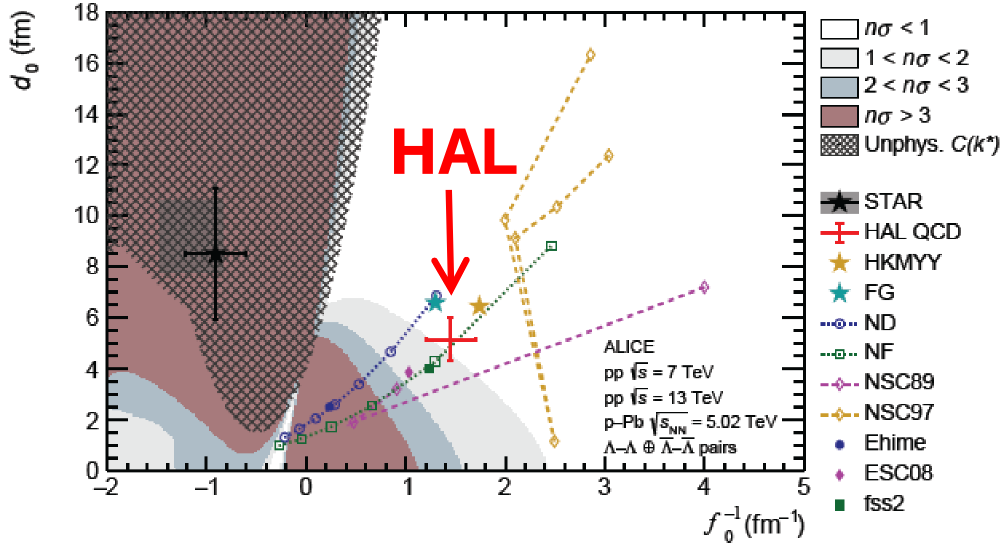
K. Morita et al., PRC94(2016)031901
K. Morita et al., PRC102(2020)015201

Femtoscopy @ nucleus collisions

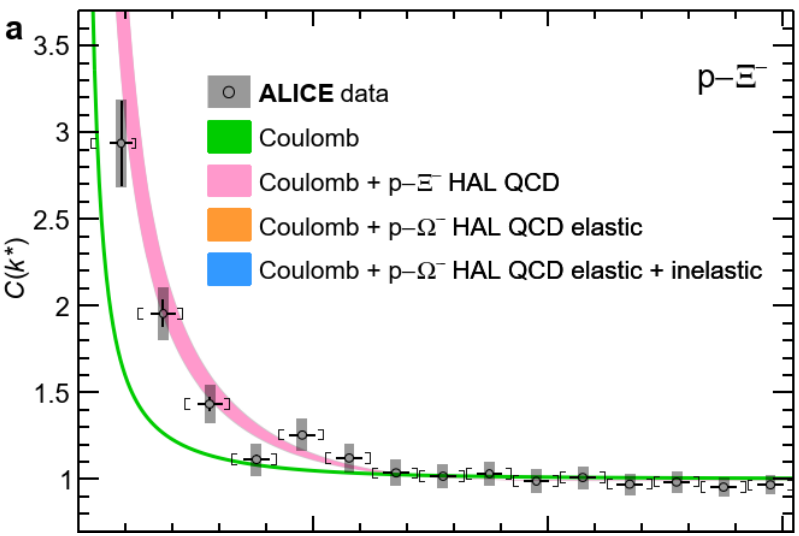
ΩΩ & charmed forces studied in LHC RUN3 (2022-)

ΛΛ

ALICE Coll., PLB797(2019)134822

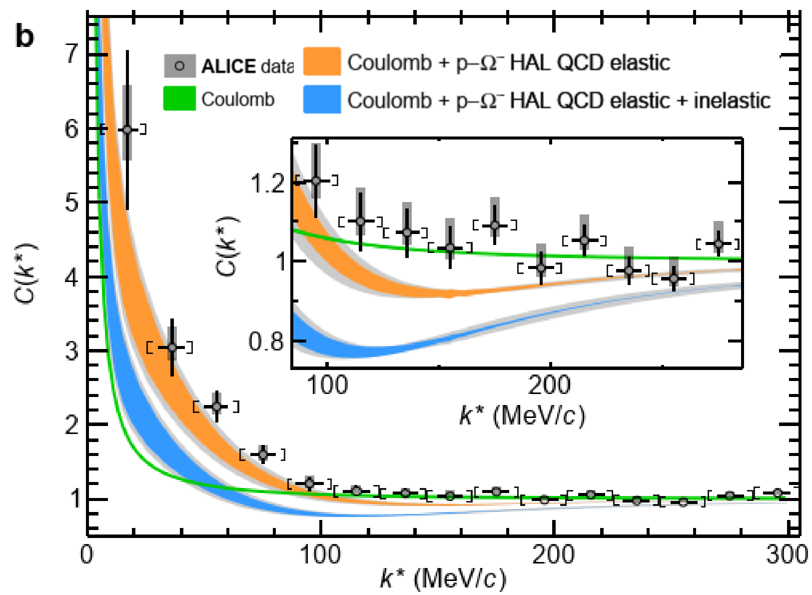


pΞ⁻



pΩ

ALICE Coll., Nature 588 (2020) 232



2019 2020 2021 2022 2030s

From "near the physical point"
To "on the physical point"

(uncertainty from quark mass dep
~ B.E. of dibaryon, hypernuclei)

P-wave

LS-forces

3-body forces

Config generation

Dibaryons

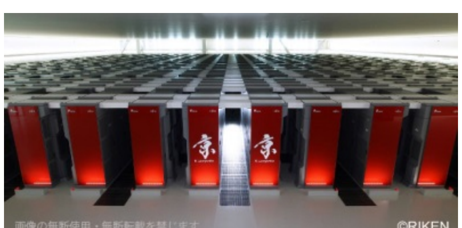
Hyperon forces

Charmed forces

Exotics, Resonances

K comp ←→ ~ 8yr
(K: 5.5yr + HOKUSAI: 2.5yr)

Fugaku ←→ ~ 2yr
(S/D-waves)



11 PFlops

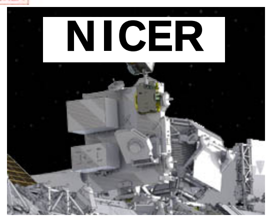
440 PFlops

2019 2020 2021 2022 2030s

YN, YY, (YNN)
Exotic hyper-nuclei

J-PARC ExHEF (2028(?)-)

HIAF (2024-)



GW in NS merger
NS radius

➔ EoS

3NF (I=1/2, 3/2)
r-process in NS merger

FRIB (2022-)

FAIR (2025(?)-)

Baryon correlation

Exotic hadrons

LHC RUN3 (2022-24)

Exotic hadrons

K-computer

→

Fugaku

→

Summary

- The 1st LQCD for Baryon Interactions near the phys. point
  - m(pi) ~= 146 MeV, L ~= 8fm, 1/a ~= 2.3GeV
  - Nuclear/Hyperon forces and Charmed forces in P=(+) channel
- Di-baryon families predicted near unitarity!
  - The most charming dibaryon $\Omega_{ccc}\Omega_{ccc}$ and most strange dibaryon $\Omega\Omega$
  - (Quasi-) dibaryon $N\Omega$ and remnant of H-dibaryon as $N\Xi$ virtual state
- New method to quantify sys err in derivative expansion
  - Clear evidence that HAL potential can be reliably extracted regardless correlator is dominated by G.S. or excited states
- Fugaku (2021-)
  - Baryon forces on the physical point
  - Dibaryons, Hypernuclei, charmed systems, bottomed systems
  - Future: LS-forces, P=(-) channel, 3-baryon forces, etc., & EoS
  - Resonances & Exotics