Nucleon structure with lattice QCD

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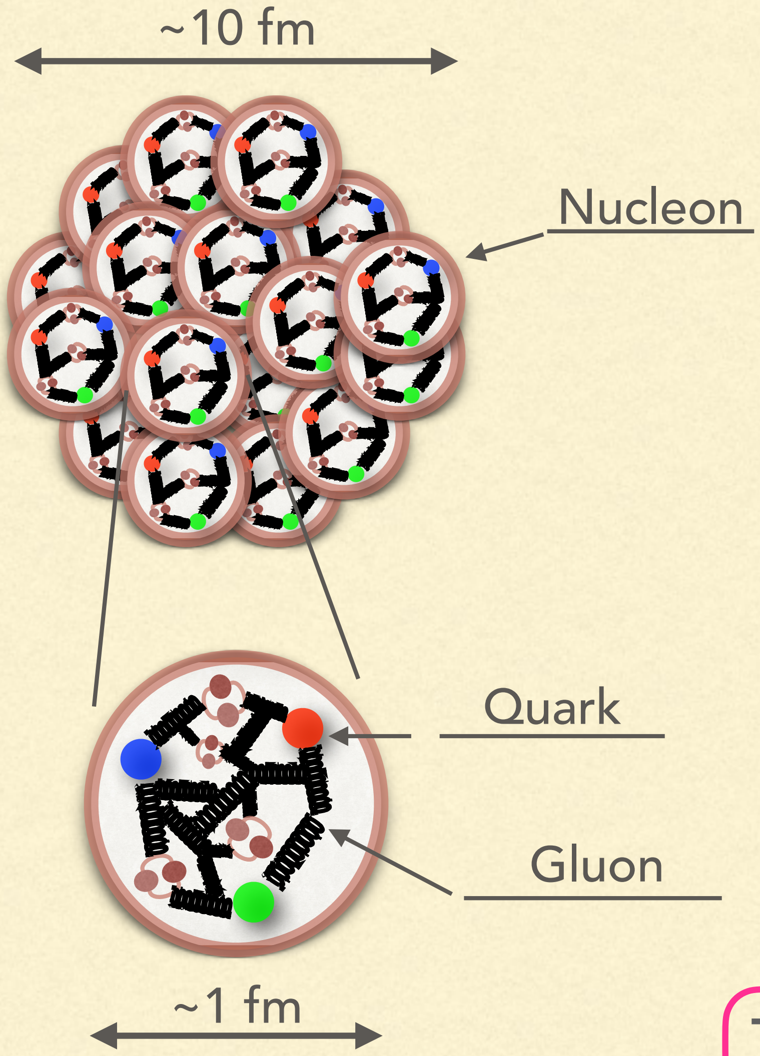
GPPU
GRADUATE PROGRAM ON PHYSICS FOR THE UNIVERSE

Introduction

- Many body problem with QCD
- Nucleon structure study
- Parton Distributions
- The conventional studies and our works

Nuclear physics & Nucleon structure



◉NULEAR PHYSICS

Quantum Many Body Prob.
Blocks -> Nucleon(point)
Int. -> Nuclear f. + Coulomb f.

However, Nucleon has structure

◉NUCLEON STRUCTURE

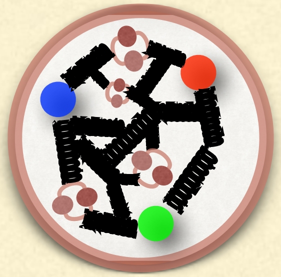
Quantum Many Body Prob.
Blocks -> Quarks & Gluons
Int. -> QCD

The STRUCTURE is NOT trivial itself

Nucleon has STRUCTURE

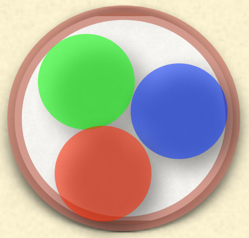
High Energy Nucleon

QUARK & GLUON pic.



? Spin crisis
? Origin of mass
? Momentum & helicity fractions

$\Lambda_{\mathrm{QCD}} \sim O(10^2)$ (MeV)



O Magnetic moment
O Mass gap
O Chiral SSB

Is the properties of Nucleon interpretable in terms of the dynamics of quark & gluon?

Perturbation dose NOT work

CONSTITUENT QUARK pic.

Low Energy Nucleon

Non perturbative analysis(*ab initio*) = Lattice QCD

* "Energy" corresponds to the resolution when we see nucleon($q \sim 100$(GeV) $\leftrightarrow$ resolution ~ 0.002 (fm)).

Parton Distributions

GTMD $W(x, \overrightarrow{k}_T, \Delta) \xrightarrow[\Delta^+ \to 0]{}$ Wigner $W(x, \overrightarrow{k}_T, \vec{r}_T)$

$\Delta = P_{\text{ini.}} - P_{\text{fin.}}$

Quantum phase-space distributions

Parton Distributions

Transverse Momentum Dependent Parton Distribution

Generalized Parton Distribution

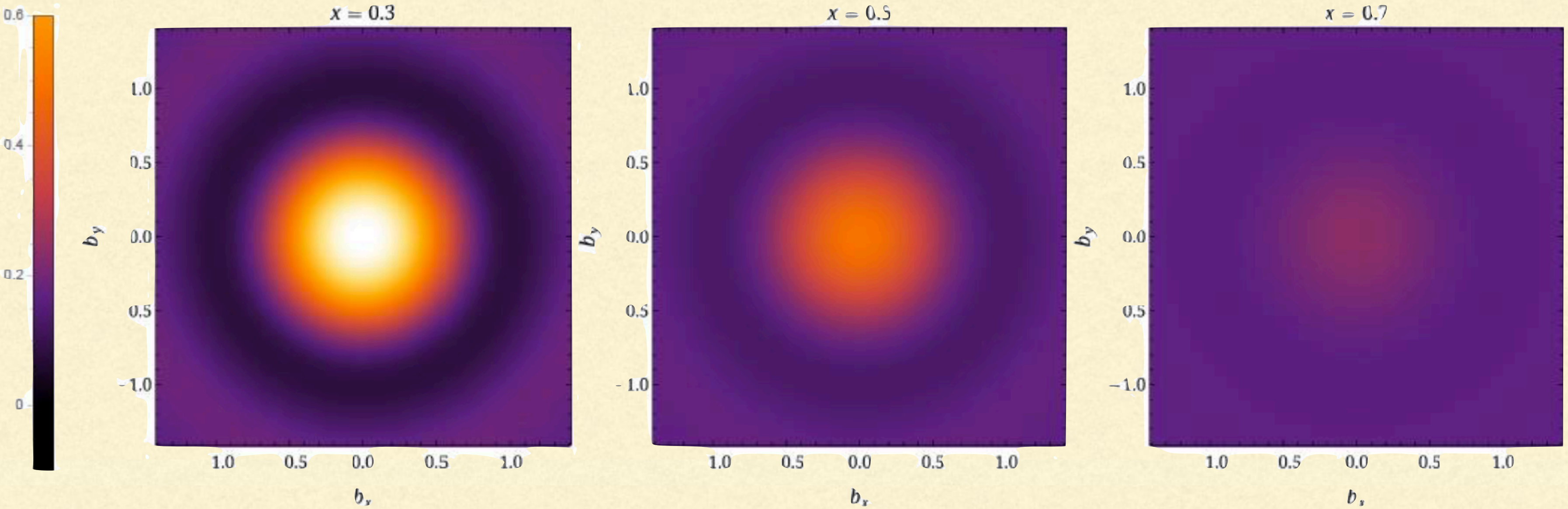
3D structure

$\Delta \to 0$ $\quad \int d\vec{r}_T \quad \int d^2 k_T$

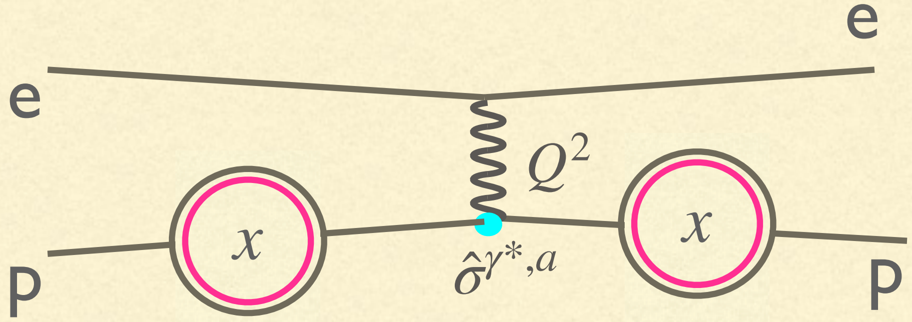
GTMD $W(x, \overrightarrow{k}_T, \Delta) \xrightarrow[\Delta^+ \to 0]{}$ Wigner $W(x, \overrightarrow{k}_T, \vec{r}_T)$

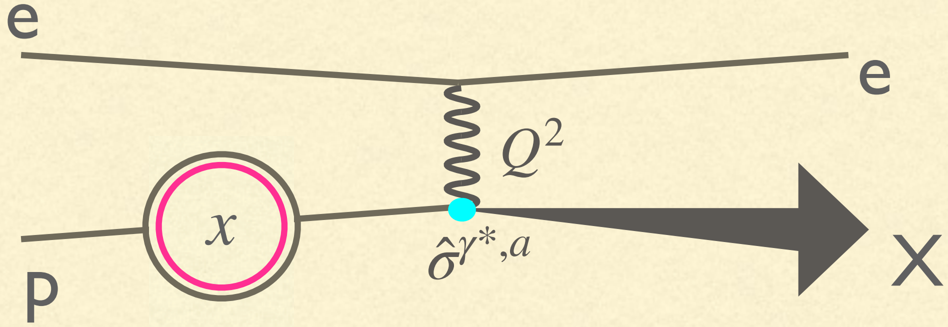
$\Delta = P_{\text{ini.}} - P_{\text{fin.}}$

e.g.) GPD and Nucleon tomography slide from H.-W. Lin @QCD Evolution Workshop 2021



Parton Distributions





GTMD $W(x, \overrightarrow{k}_T, \Delta)$ $\xrightarrow[\Delta^+ \to 0]{}$ Wigner $W(x, \overrightarrow{k}_T, \vec{r}_T)$

$\int dx \int d^2 k_T \downarrow$ $\Delta = P_{\mathrm{ini.}} - P_{\mathrm{fin.}}$ $\int d^2 k_T, \Delta \to 0 \downarrow$

1D structure ↓

Form Factor

- Elastic scattering

→ Nucleon's SPATIAL dis.

Proton radius puzzle
Nucleon transversity
Quark EDM e.t.c.

Parton Distribution Function

- Deep inelastic scattering

→ Partons' MOMENTUM/HEL-CITY dis. inside nucleon

Proton spin crisis, SSA,
Gluon saturation e.t.c.

Parton Distributions

Transverse Momentum Dependent Parton Distribution

$\Delta \to 0$

$\int d\vec{r}_T$

Generalized Parton Distribution

$\int d^2k_T$

3D structure

GTMD $W(x, \overrightarrow{k}_T, \Delta)$ $\xrightarrow[\Delta^+ \to 0]{}$ Wigner $W(x, \overrightarrow{k}_T, \vec{r}_T)$

$\int dx \int d^2k_T$

$\Delta = P_{\text{ini.}} - P_{\text{fin.}}$

ongoing projects

$\int d^2k_T, \Delta \to 0$

1D structure

Form Factor

- Elastic scattering

→ Nucleon's SPATIAL dis.

Proton radius puzzle
Nucleon transversity
Quark EDM e.t.c.

Parton Distribution Function

- Deep inelastic scattering

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Parton Distributions

Transverse Momentum Dependent Parton Distribution

Generalized Parton Distribution

3D structure

$\Delta \to 0$

$\int d\vec{r}_T$

$\int d^2 k_T$

GTMD $W(x, \overrightarrow{k}_T, \Delta)$ $\xrightarrow[\Delta^+ \to 0]{}$ Wigner $W(x, \overrightarrow{k}_T, \vec{r}_T)$

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1D structure

Form Factor

- Elastic scattering

$\to$ Nucleon's SPATIAL dis.

Proton radius puzzle

Nucleon transversity

Quark EDM e.t.c.

Our works

Paper

- K.-I. Ishikawa et al., Phys. Rev. D**98** (2018) 074510 [1807.03974].
- E.Shintani et al., Phys. Rev. D**99** (2019) 014510 [1811.07292]; (Erratum; Phys. Rev. D**102** (2020) 019902.)
- N.Tsukamoto et al., PoS Lattice2019 (2020) 132 [1912.00654].
- K.-I. Ishikawa et al., arXiv:2107.07085 (2021). e.t.c.

Talk

- R.T. et al., "Nucleon axial, tensor and scalar charges using lattice QCD with physical quark masses", JPS2021年秋季大会 e.t.c.

Parton Distributions

Transverse Momentum Dependent Parton Distribution

Generalized Parton Distribution

3D structure

$\Delta \to 0$

$\int d\vec{r}_T$

$\int d^2k_T$

GTMD $W(x, \overrightarrow{k}_T, \Delta)$ $\xrightarrow[\Delta^+ \to 0]{}$ Wigner $W(x, \overrightarrow{k}_T, \vec{r}_T)$

$\int dx \int d^2k_T$

$\Delta = P_{\text{ini.}} - P_{\text{fin.}}$

$\int d^2k_T, \Delta \to 0$

1D structure

Form Factor

- Elastic scattering

→ Nucleon's SPATIAL dis.

Proton radius puzzle

Nucleon transversity

Quark EDM e.t.c.

Parton Distribution Function

- Deep inelastic scattering

→ Partons' MOMENTUM/HEL-CITY dis. inside nucleon

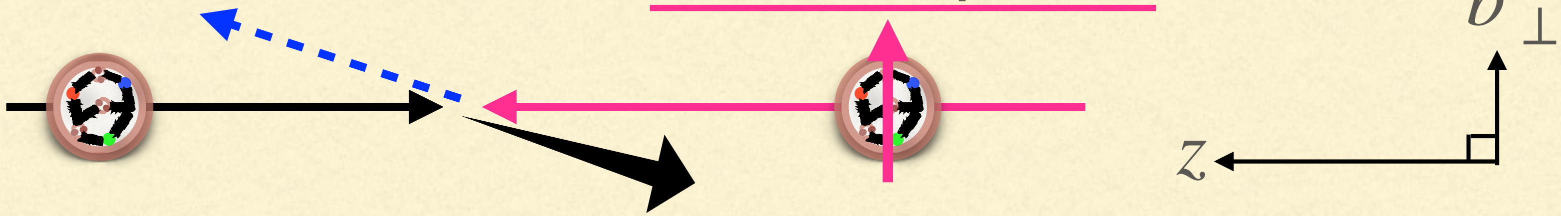
Proton spin crisis, SSA, Pickup

Gluon saturation e.t.c.

Single Spin Asymmetry(SSA)

e.g.) $p + \underline{p\uparrow} \rightarrow \underline{\pi_0} + X$ process (Polarized pp collision)

Neutral pion (detect)   Polarized proton $\vec{b}_\perp$



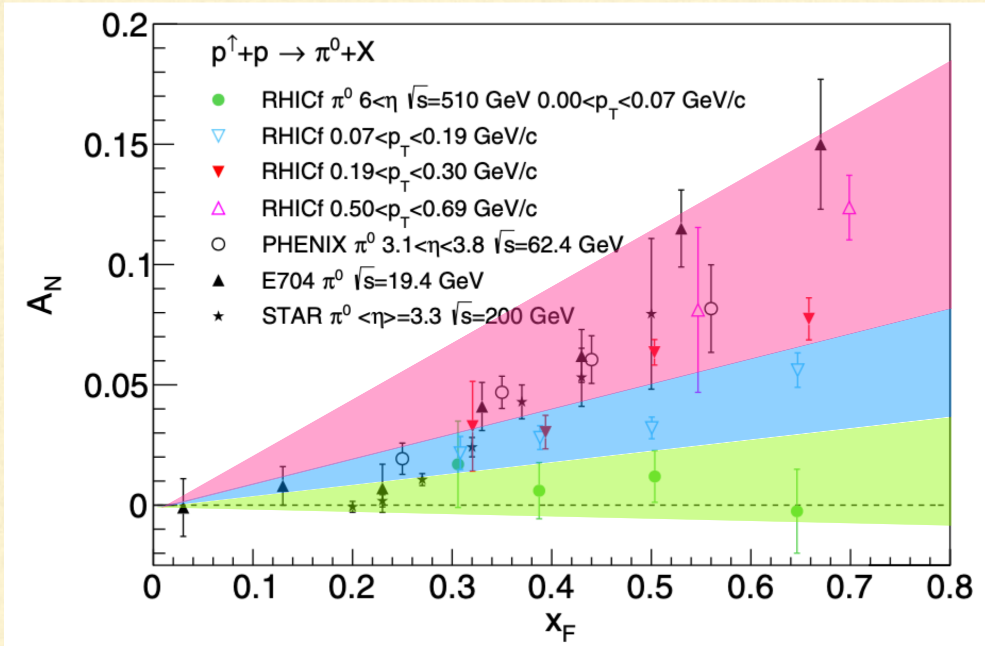
The π_0's production cross section depends on proton's spin.

$$\text{Asymmetry} : A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$$\lesssim 0.3$$

$\leftrightarrow$ < 0.0001 in naive pQCD

Problem: LARGE asymmetry



Mechanisms of SSA -roughly

The mechanisms are classified by their kinematical scale.

① $\Lambda_{\rm QCD} \leq P_\perp \ll \sqrt{Q^2} \to P_\perp \sim$ Partons' $k_\perp \sim \Lambda_{\rm QCD} \leq M_N$

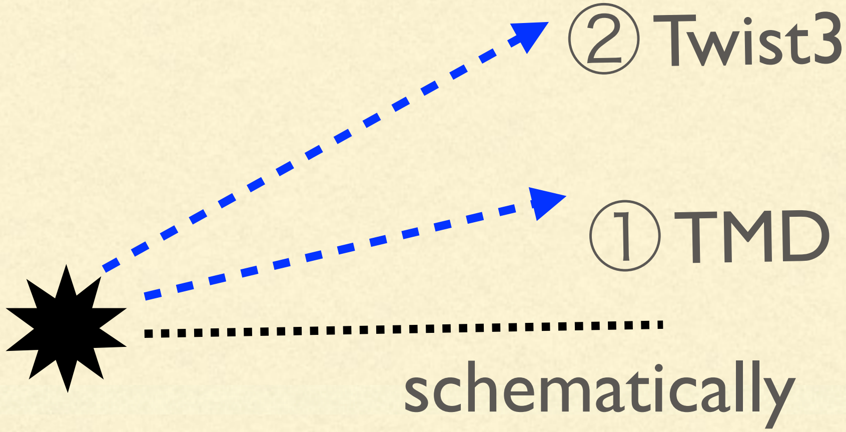
- Transverse Momentum Dependent PDFs (TMD PDFs)
  : Non perturbative generation of SSA (No α_s/Q suppression)

$$\sigma \sim \sum_{ab} f_{a,T}^{\perp}(x, k_\perp, \mu_F) \otimes \hat{\sigma}^{\gamma^*, a\to b}(x, Q, \mu_F, \mu_D) \otimes D_{h/b}(z, k'_\perp, \mu_D)$$

② $\Lambda_{\rm QCD} \ll P_\perp \sim \sqrt{Q^2} \to$ Factorize with $P_\perp$

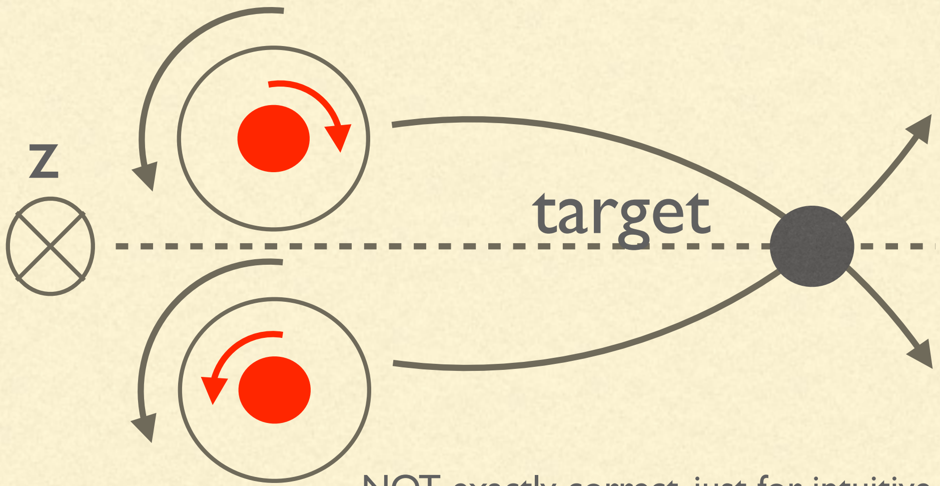
- Twist3 matrix effects
  : Hadron spin flip through gluon
  ($O(\Lambda_{\rm QCD}/Q,\ M_N/Q)$ effects)

Both mechanisms WORK!

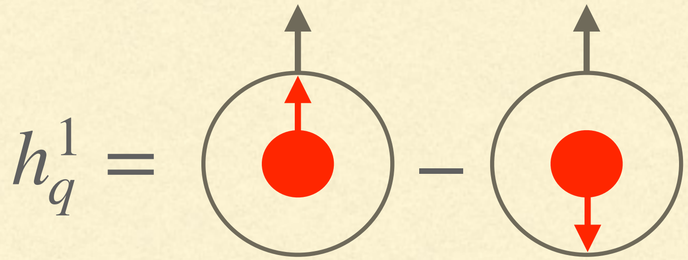


SSA and lattice contributions

TMD PDF is essential but difficult to obtain with experiments.



One of TMD-Collins func. h_q^1:



Use Lattice to constrain Exp.

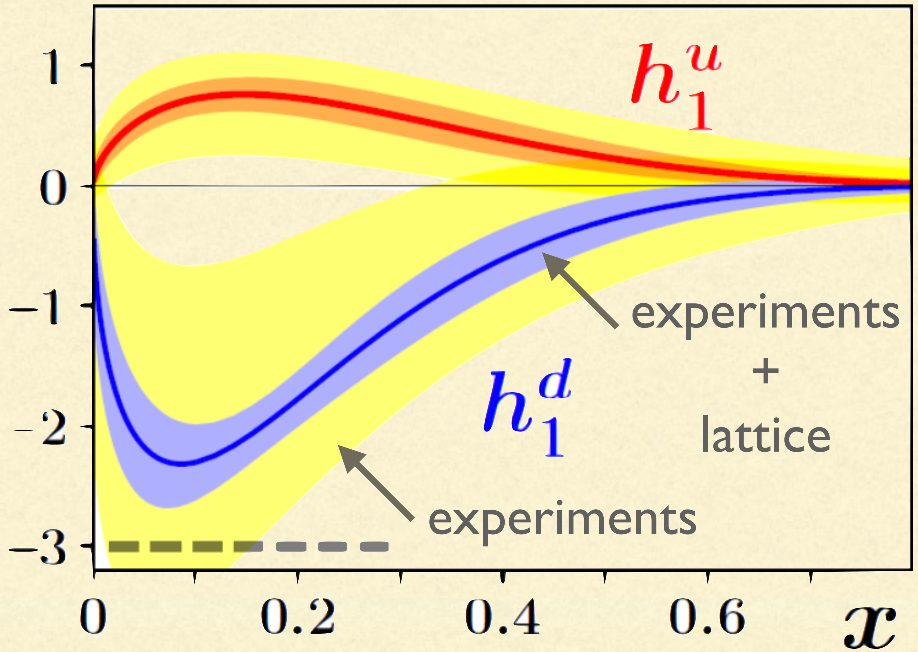
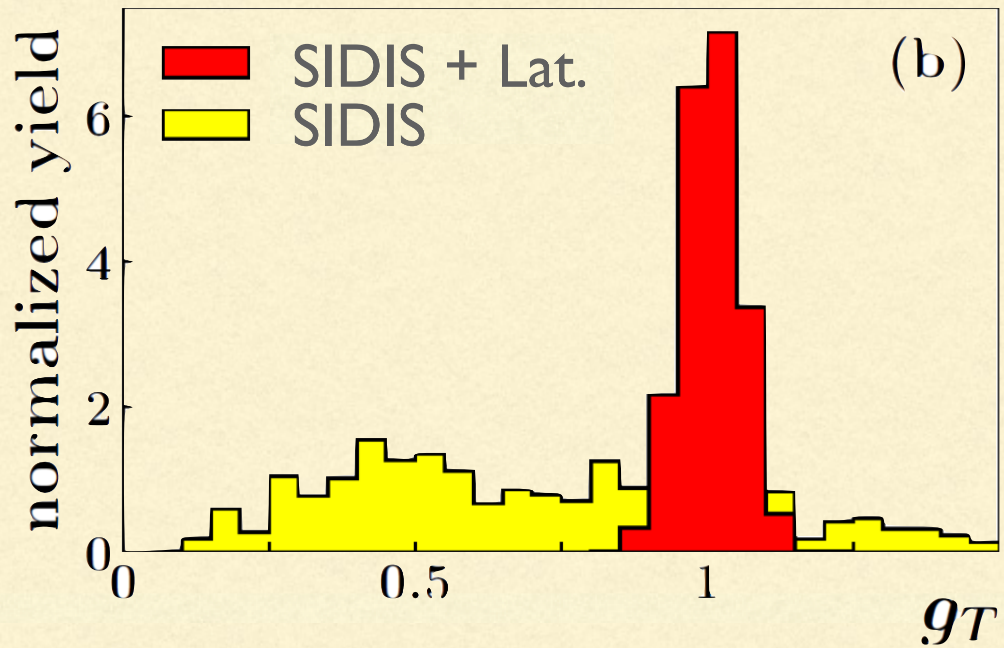


fig. from H.-W. Lin @QCD Evolution Workshop 2021



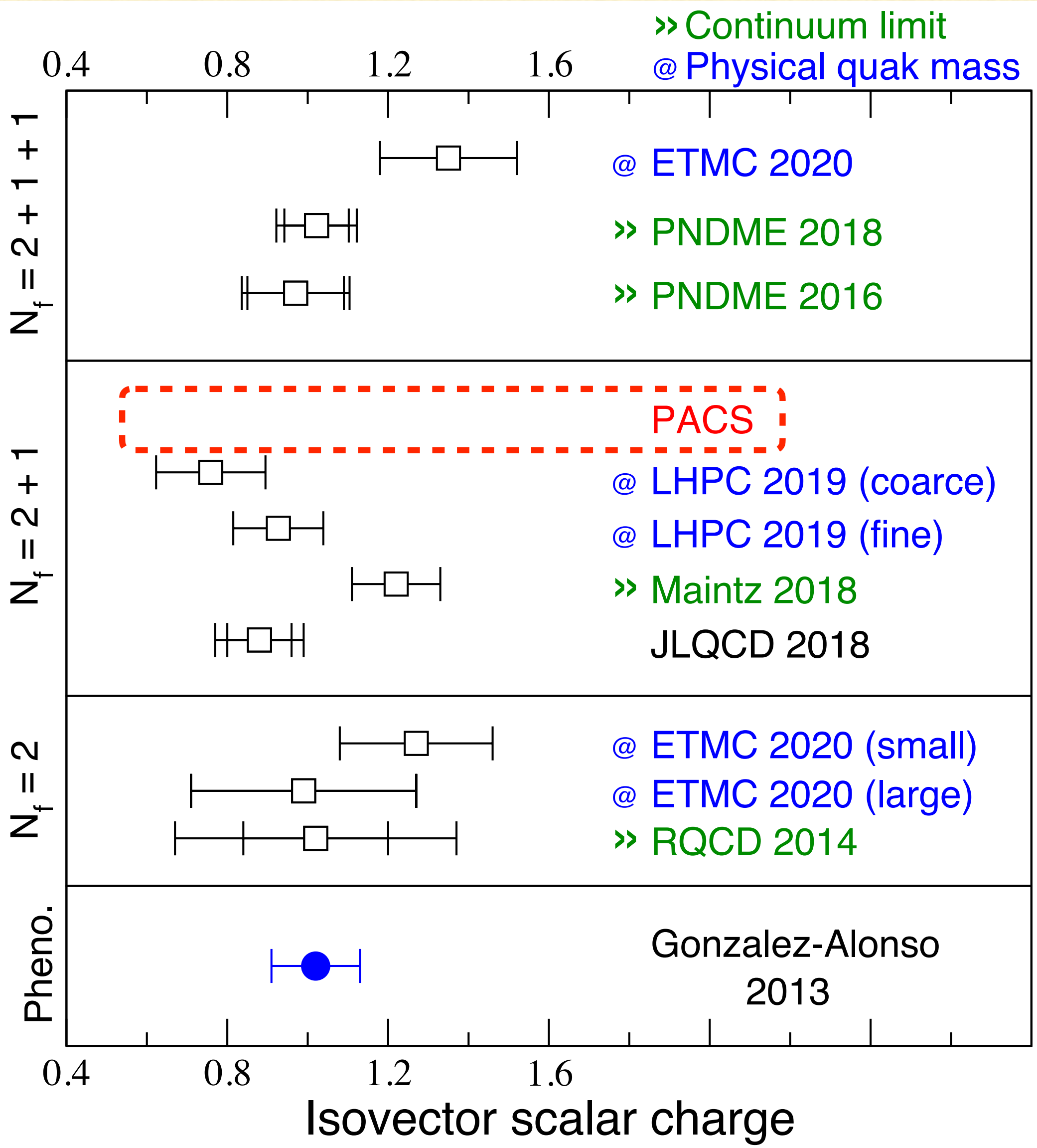
Nucleon structure with lattice QCD

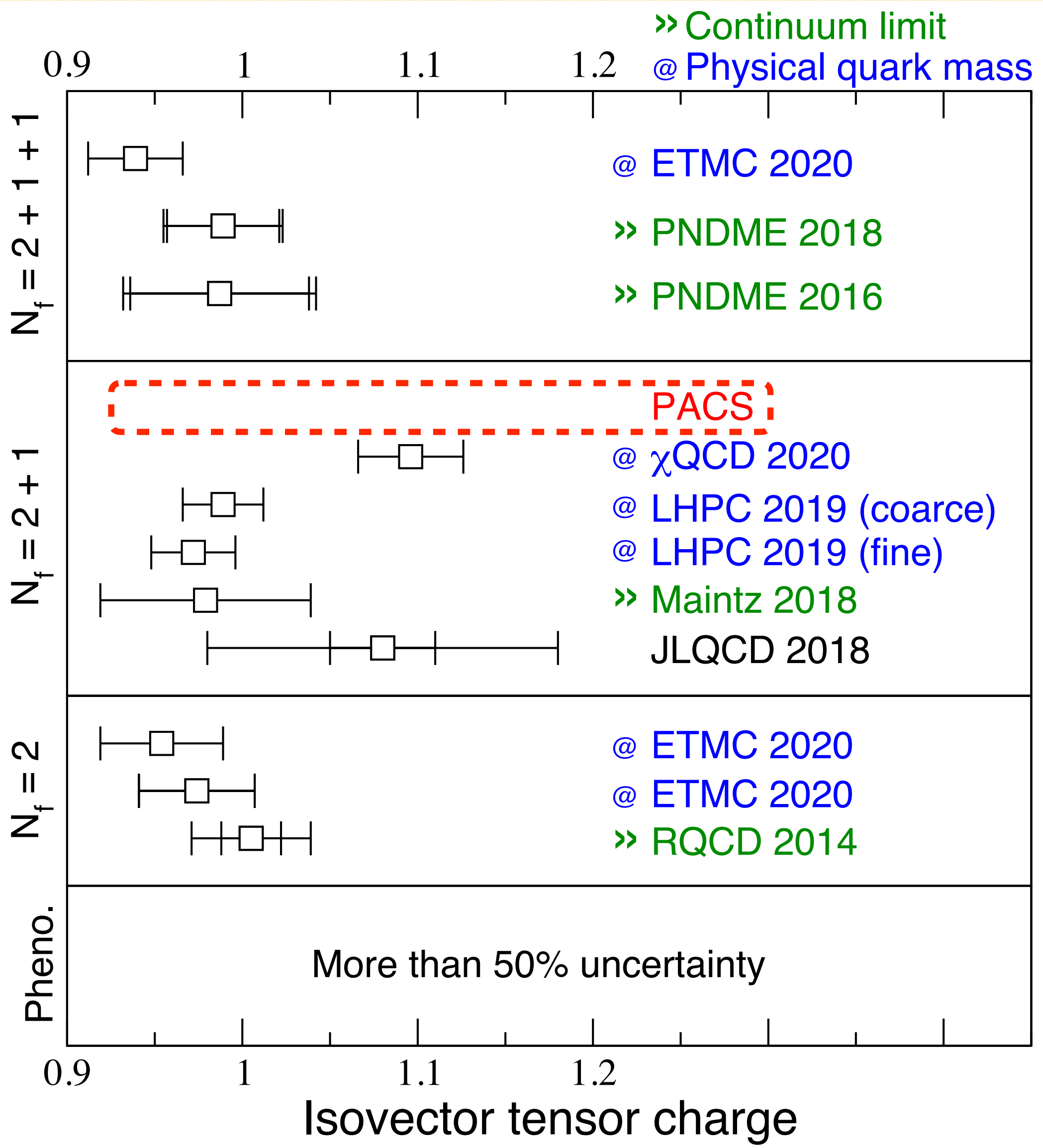
After 2011, lattice can approach Parton Distributions directly.
However, can lattice overcome experimental precision/accuracy?

→ ✓BENCHMARK calculations, indirect one, are also needed

	Matrix elements	Feature	Experiments/Remark
✓	g_A	Nucleon axial charge	$g_A^{exp.} = 1.2756(13)$
	g_S	Direct Dark Matter detection $\langle N \vert \psi 1 \bar{\psi} \vert N \rangle$	Both isoscalar and isovector are needed for practical use
	g_T	0th moment of Colins func. $\langle N \vert \psi \sigma_{\mu\nu} \bar{\psi} \vert N \rangle$	
✓	$\langle x \rangle_{u-d}$	1st moment of unp. PDF	$\langle x \rangle_{u-d}^{\mathrm{PDF4LHC}} = 0.155(5)$
✓	$\langle x \rangle_{\Delta u - \Delta d}$	1st moment of pol. PDF	$\langle x \rangle_{\Delta u - \Delta d}^{\mathrm{BENCHMARK}} = 0.199(16)$

Conventional studies -isovector





High-precision & High-accuracy = Purpose of PACS(this work)

Lattice QCD & Assessment of error

- Lattice QCD
- Major systematic uncertainties
- Methods for assessing the uncertainties

Calculation strategy

Our targets :

- Non-perturbative information of nucleon
  → Calculate them in Lattice QCD
- Depend on the renormalization
  → Need the renormalization constants additionally

Therefore:

(Renormalized value)
= (Bare matrix element) × (Renormalization constant)

→ Evaluate both the bare matrix elements and the renorm -alization constants with high accuracy in Lattice QCD

High accuracy in Lattice QCD(*ab initio* cal.)?

$\overline{\mathrm{MS}}$ (Modified Minimal subtraction) scheme at 2 GeV for the subtraction point is widely used now.

Lattice QCD and its accuracy

Path integration of QCD = High-dimensional integrals

$$\langle O \rangle = \frac{1}{Z} \int \mathscr{D}[U] \mathscr{D}[\overline{\psi}] \mathscr{D}[\psi] \, O[U, \overline{\psi}, \psi] \, \mathrm{e}^{-J_{\mathrm{QCD}}[U, \overline{\psi}, \psi]}$$

→ Estimate stochastically = Monte Carlo integration

(Importance sampling)

High accuracy in Lattice QCD means

1. Statistically improved

→ [1] All-mode-averaging

2. Fewer systematic uncertainties

→ Eliminate [2] some by Set-ups, but NOT enough

Assess the residual systematic uncertainties

[1] T. Blum et al., Phys.Rev. D **88,** 094503(2013)、[2] One stem from finite spatial volume and heavy pion mass

Residual systematic uncertainties

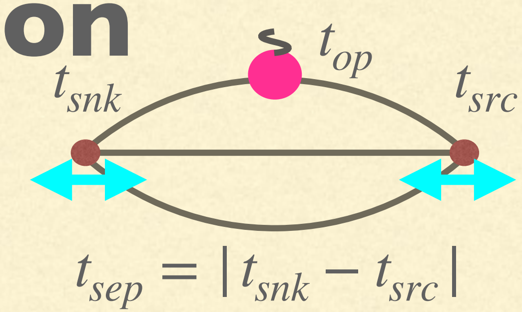
① : (Bare matrix element) × ② : (Renormalization constant)

Both have systematic uncertainties, and we mainly focus on

Parts	Systematic uncertainty	Origin
①	Excited state contamination	• Nucleon's excited states e.g. $\langle N(t)N(0)^\dagger\rangle = \sum_i a_i \, \mathrm{e}^{-E_i t}$
②	Perturbative truncation Non-perturbative effects ⟵ Fitting functions/range	• Chiral S.S.B • Gluon condensation e.g. $Z_O^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) \supset \frac{m_{val}^2}{p^2}, \frac{\langle q\bar{q}\rangle^2}{p^6}, \frac{\langle A_\mu\rangle^2}{p^2}$

Problem : How can we assess systematic uncertainties?

① Excited state contamination



Nucleon matrix elements obtained from the ratio of 3pt. function to 2pt. function

$$\frac{\langle N(t_{snk})O(t_{op})N(t_{src})^\dagger\rangle}{\langle N(t_{snk})N(t_{src})^\dagger\rangle} = \frac{\sum_{ij}\langle 0|N(0)|i\rangle\langle i|O(0)|j\rangle\langle j|N^\dagger|0\rangle e^{-E_i t_{sep}}}{\sum_i |\langle 0|N(0)|i\rangle|^2 e^{-E_i t_{sep}}}$$

$$\rightarrow \quad \langle N|O(0)|N\rangle + Ae^{-(E_1 - M_N)t_{sep}} + \cdots$$

Actually, ~~$t_{sep} \gg t_{op} \gg 0$~~

All excited states appearing in the ratio depend on t_{sep}

→ Calculate the ratio for several t_{sep} and gaze t_{sep} independence

= confirm no excited states contamination

→ Average after the ground state saturation

② Non-perturbative effect

$$Z^{\overline{\rm MS}}(2\ {\rm GeV}) = \underbrace{\frac{Z^{\overline{\rm MS}}(2\ {\rm GeV})}{Z^{\overline{\rm MS}}(\mu)} \cdot \frac{Z^{\overline{\rm MS}}(\mu)}{Z^{\rm Lattice}(\mu)}}_{\text{Perturbative matching}} \times \underbrace{Z^{\rm Lattice}(\mu)}_{\text{Non-perturbative}}$$

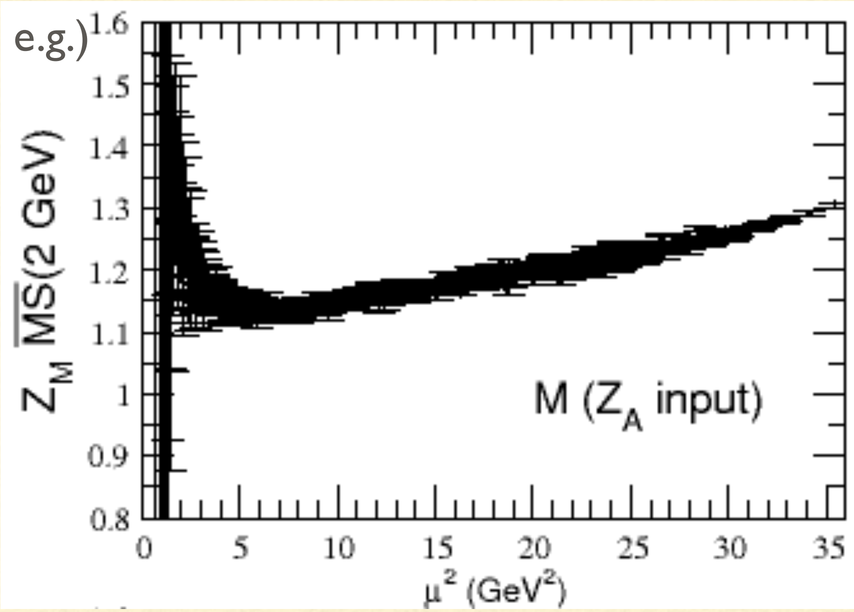
→ Ideally, $Z^{\overline{\rm MS}}(2\ {\rm GeV})$ is independent of matching scale: μ

However, the dependence appear

↓

Remove such scale dependence by FIT and extract the scale-free renormalization constant

e.g.)



Skip how we can calculate the renormalization constants on lattice for saving time here.

FIT and systematic error

Matching scale dependence stems from :

- IR : Non-perturbative effect
- UV : Discretization error

$$\rightarrow Z_O^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = \frac{c_{-1}}{(a\mu)^2} + c_0 + \sum_i c_i (a\mu)^{2i}$$

→ Extract scale-free renormalization constants by FIT

FIT TYPE	IR	* UV	* FIT range
IR-pole ansatz	Pole term	Polynomial	$1\ (\mathrm{GeV}) < \mu$
IR-truncated ansatz	Truncation	Polynomial	$2\ (\mathrm{GeV}) \leq \mu$

Discrepancies between FIT types ~ Evaluation of FIT ansatz
→ appropriate the discrepancies to the systematic error

* Each order of polynomial and FIT range is suit for the most stable FIT. And also supported by RMS error analysis.

Numerical results

- Nucleon matrix elements
- Renormalization constants
- Renormalized quark momentum/helicity fraction

Simulation details -PACS configuration

	128^4 lattice	64^4 lattice
Lattice size	128^4 [1]	64^4 [2]
Lattice spacing	~ 0.084 fm	
Pion mass	135 MeV	139 MeV[3]
Spatial vol.	$\sim (10.8\ \mathrm{fm})^3$	$\sim (5.4\ \mathrm{fm})^3$

Eliminate 2 systematic uncertainties

~~Finite size effect~~

~~Chiral extrapolation~~

Highest precision of g_A [1]

$$g_A^{128^4} = 1.273(24)_{\mathrm{sta.}}(5)_{\mathrm{sys.}}(9)_{\mathrm{ren.}}$$

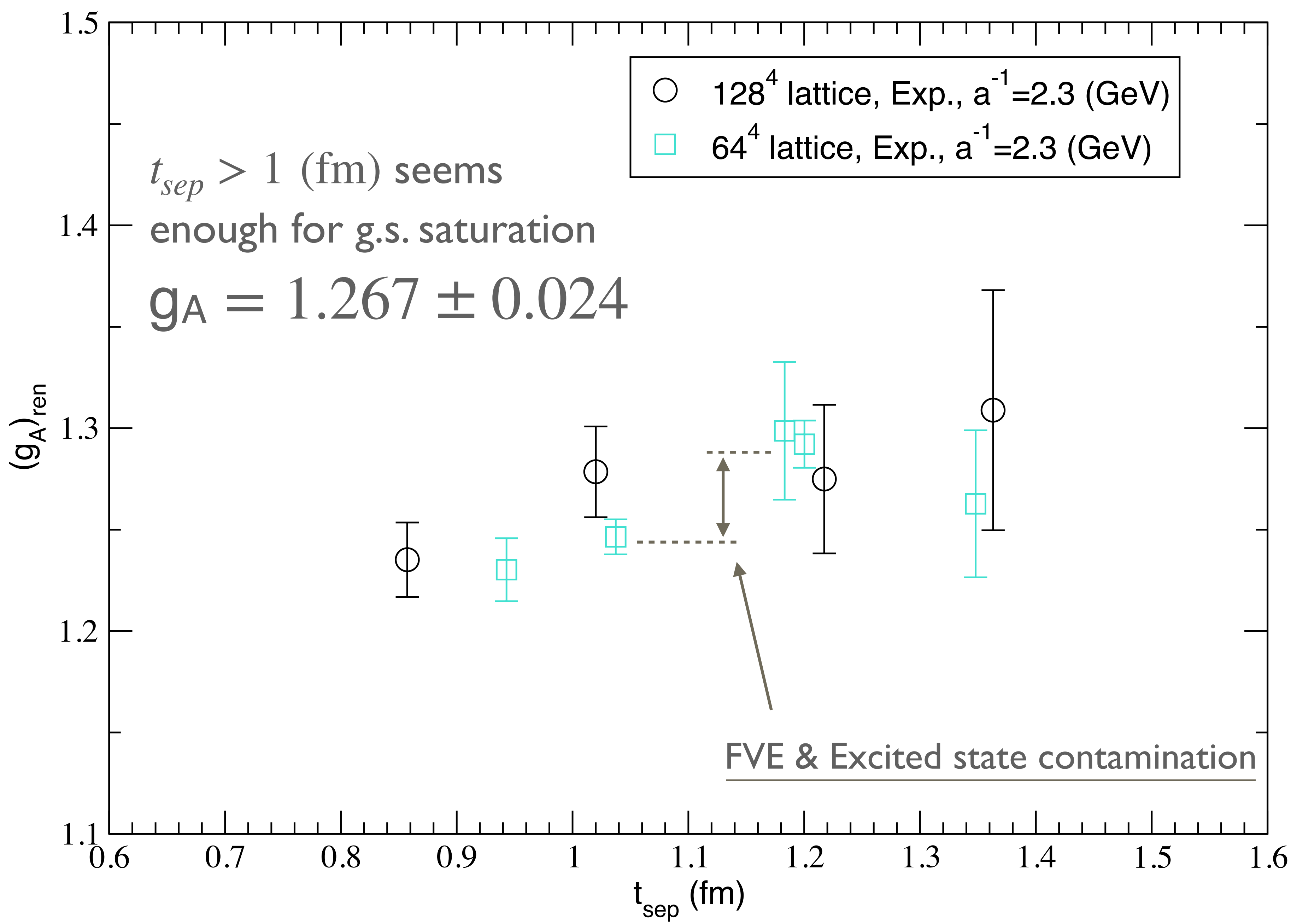
$$g_A^{\mathrm{exp.}} = 1.2756(13)$$

[1] E. Shintani et al., Phys. Rev. D **99**, 014510(2019) [2] K.-I. Ishikawa et al., Phys. Rev. D **99**, 014504(2019)

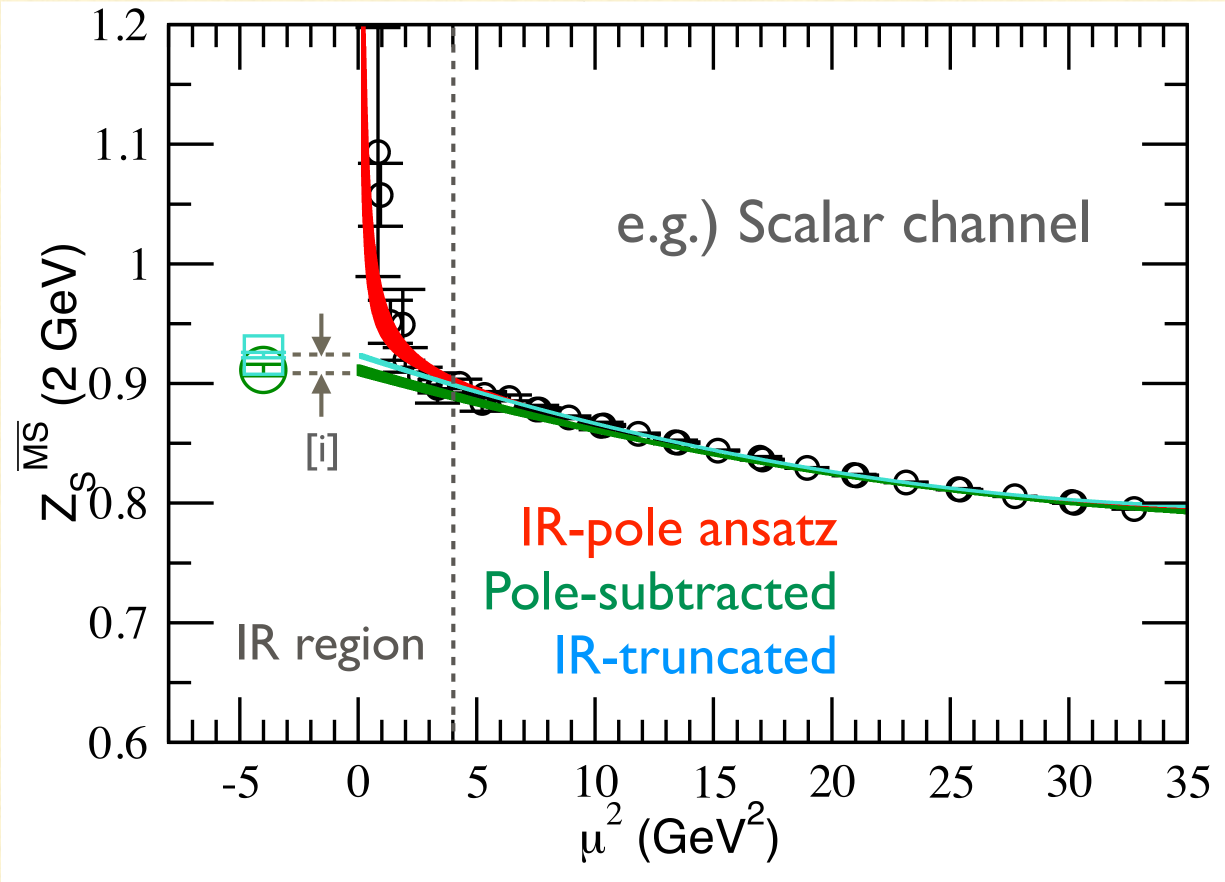
The stout-smeared $O(a)$ improved Wilson fermions and Iwasaki gauge action.

[3] Finite volume-size effect

Nucleon axial charge g_A



Renormalization and systematic error



Systematic uncertainty of

[i] Perturbative truncation

[ii] Non-perturbative effects

[iii] Fitting function/range

→ Three major systematic uncertainties are quoted

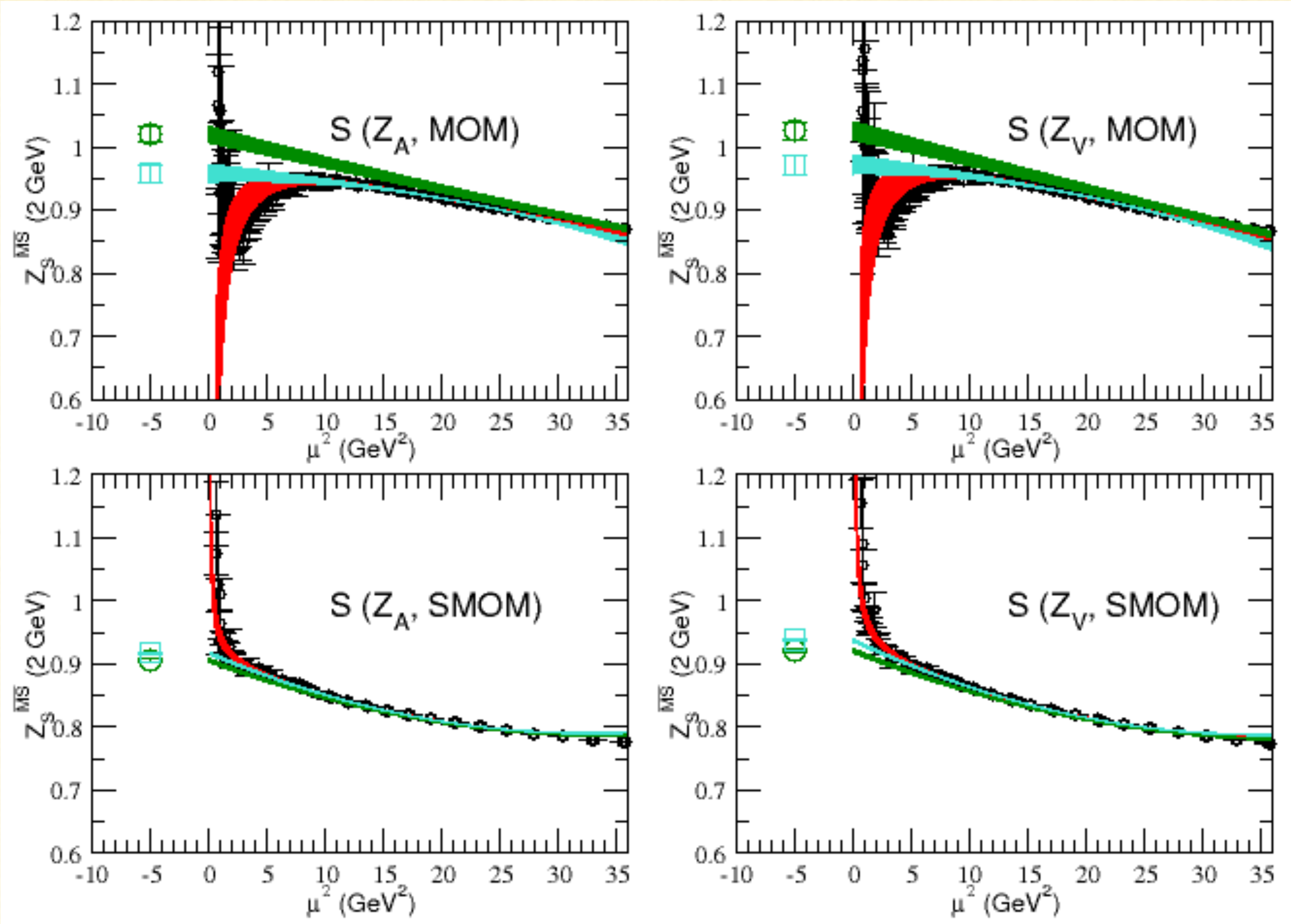
High-precision & High-accuracy

Contribution to error	Sta.	[i]	[ii]	[iii]	Total
$Z_S = 0.910(3)_{\rm sta}(5)_{\rm [i]}(7)_{\rm [ii]}(8)_{\rm [iii]}$	0.34%	0.58%	0.80%	0.83%	1.3%
$Z_T = 1.030(2)_{\rm sta}(23)_{\rm [i]}(5)_{\rm [ii]}(3)_{\rm [iii]}$	0.18%	2.2%	0.50%	0.26%	2.3%

RI/MOM and RI/SMOM

Scalar operator = suffer from chiral symmetry breaking strongly

= Z_S depends on how we treat IR strongly



Extract constant with

- Pole + quadratic using IR data
- Quadratic truncating IR data

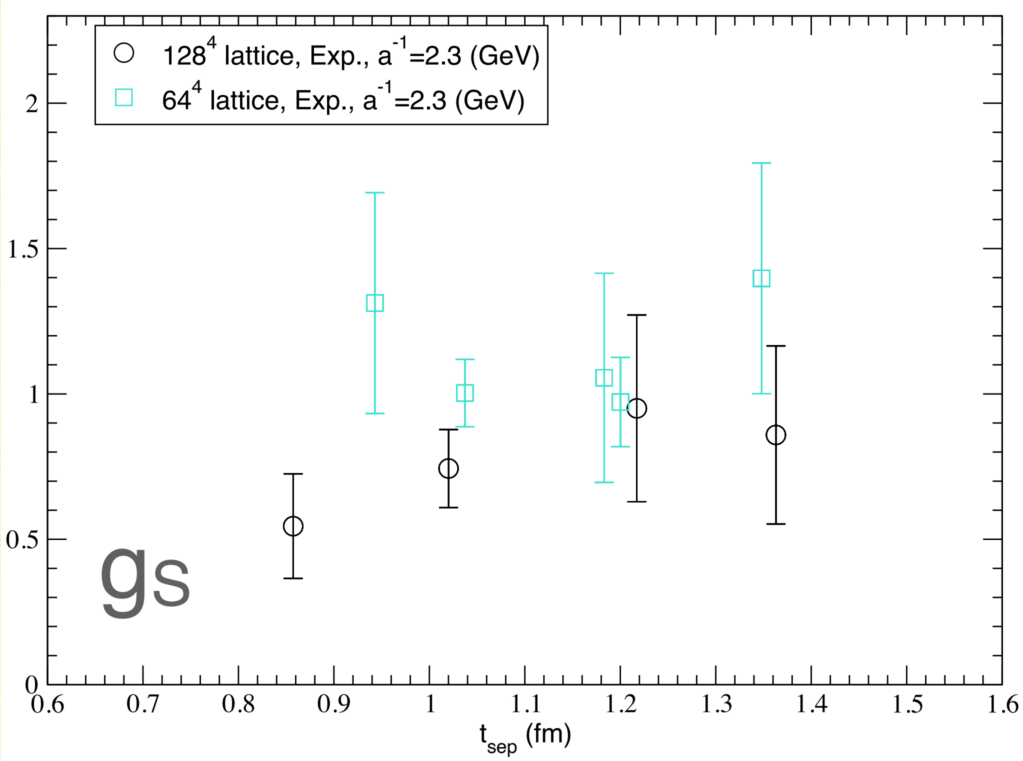
The discrepancy are

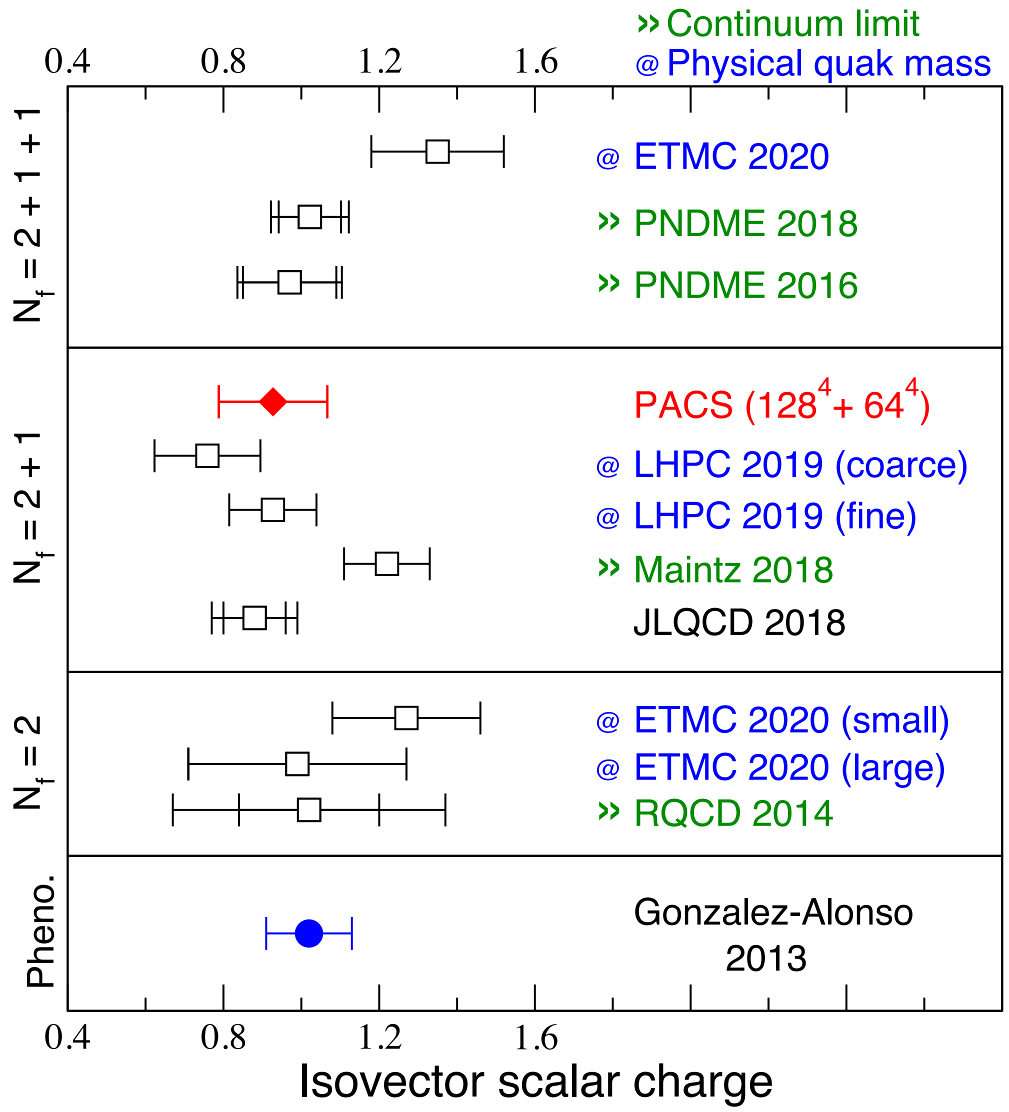
~6 % for MOM
~2 % for SMOM

Sys. err. is under control with improved scheme

C. Sturm, Y. Aoki, N. H. Christ, T. Izubuchi, C. T. C. Sachrajda and A, Soni, Phys. Rev. D **80,** 014501(2009).

Renormalized scalar couplings



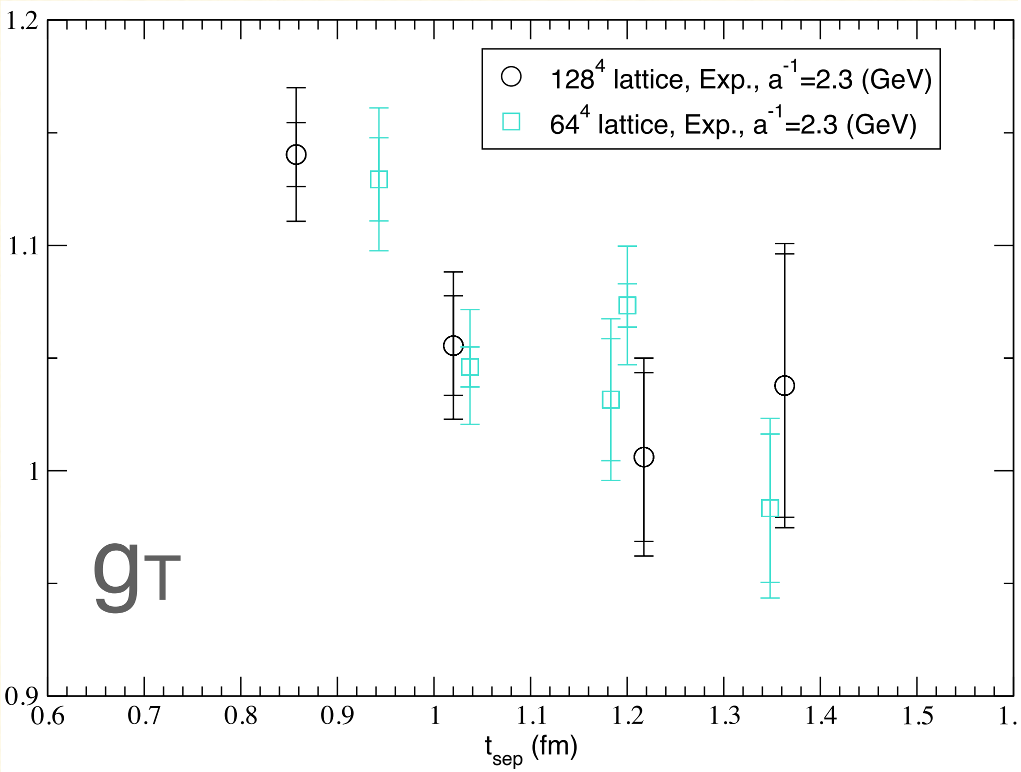


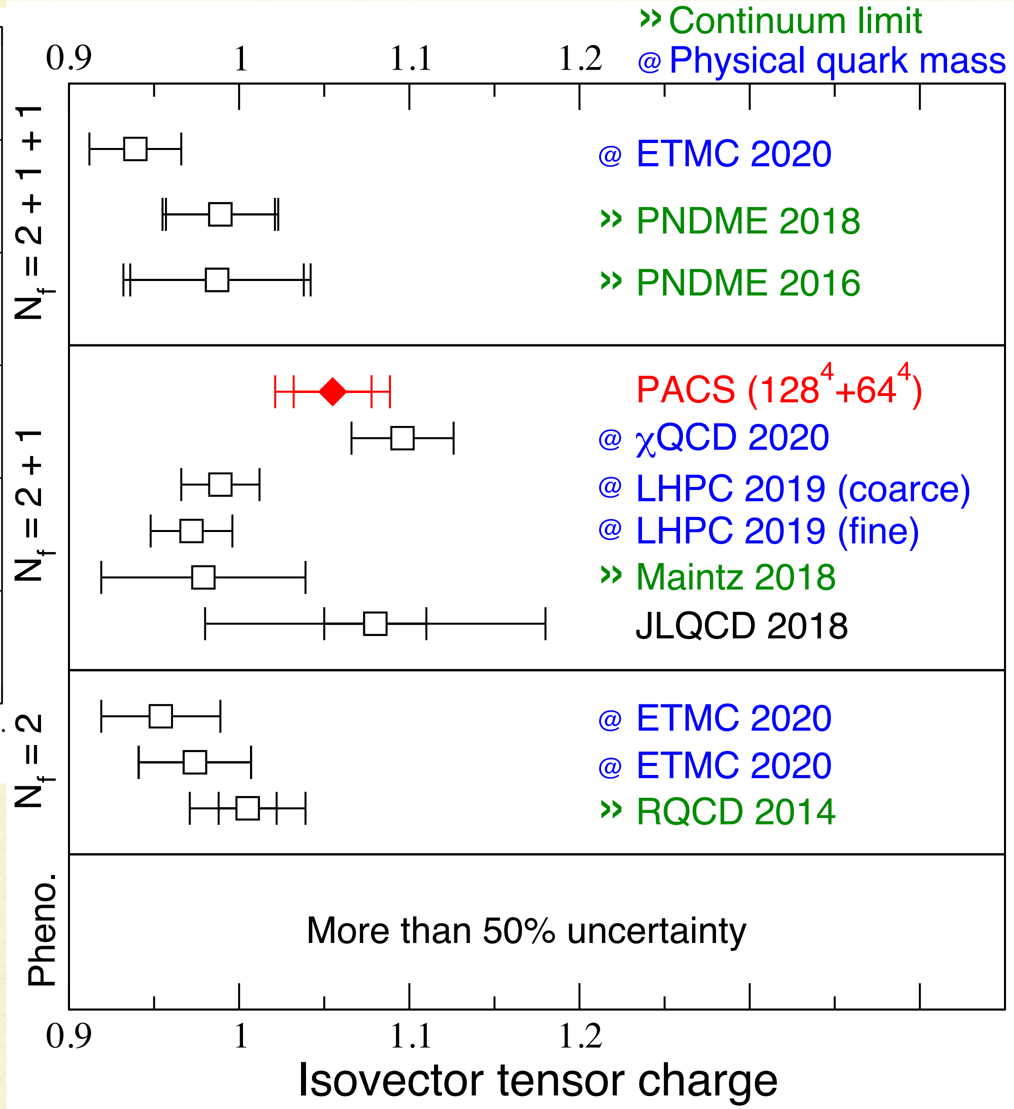
$t_{sep} > 1$ (fm) seems enough for g.s. saturation

$$g_S = 0.927(139)_{sta.}(11)_{sys.}$$

[FLAG2019] Aoki. S et al., Eur. Phys. J. C. 80, 113 (2020).
[PNDME2018] R. Guputa et al., Phys. Rev. **D98** (2018) 034503.
[PNDME2016] T. Bhattacharya et al., Phys. Rev. **D94** (2016) 054508.
[ETMC2020] C. Alexandrou et al., Phys. Rev. **D102** (2020) 054517.
[LHPC2019] N. Hasan et al., Phys. Rev. **D99** (2019) 114505.
[Mainz2018] K. Ottnad et al., in Proceedings, Lattice2018.
[JLQCD2018] N. Yamanaka et al., Phys. Rev. **D98** (2018) 054516.
[RQCD2014] G. S. Bali et al., Phys. Rev. **D91** (2015) 054501.
[Pheno,] M. Gonzalez-Alonso et al., Phys. **Lett** 112 (2014) 04501.

Renormalized tensor couplings





$t_{sep} > 1$ (fm) seems enough for g.s. saturation

$$g_T = 1.055(23)_{sta.}(25)_{sys.}$$

[FLAG2019] Aoki. S et al., Eur. Phys. J. C. 80, 113 (2020).
[χQCD2020] D. Horkel et al., arXiv:2002.06699v1 (2020).
[PNDME2018] R. Guputa et al., Phys. Rev. **D98** (2018) 034503.
[PNDME2016] T. Bhattacharya et al., Phys. Rev. **D94** (2016) 054508.
[ETMC2020] C. Alexandrou et al., Phys. Rev. **D102** (2020) 054517.
[LHPC2019] N. Hasan et al., Phys. Rev. **D99** (2019) 114505.
[Mainz2018] K. Ottnad et al., in Proceedings, Lattice2018.
[JLQCD2018] N. Yamanaka et al., Phys. Rev. **D98** (2018) 054516.
[RQCD2014] G. S. Bali et al., Phys. Rev. **D91** (2015) 054501.

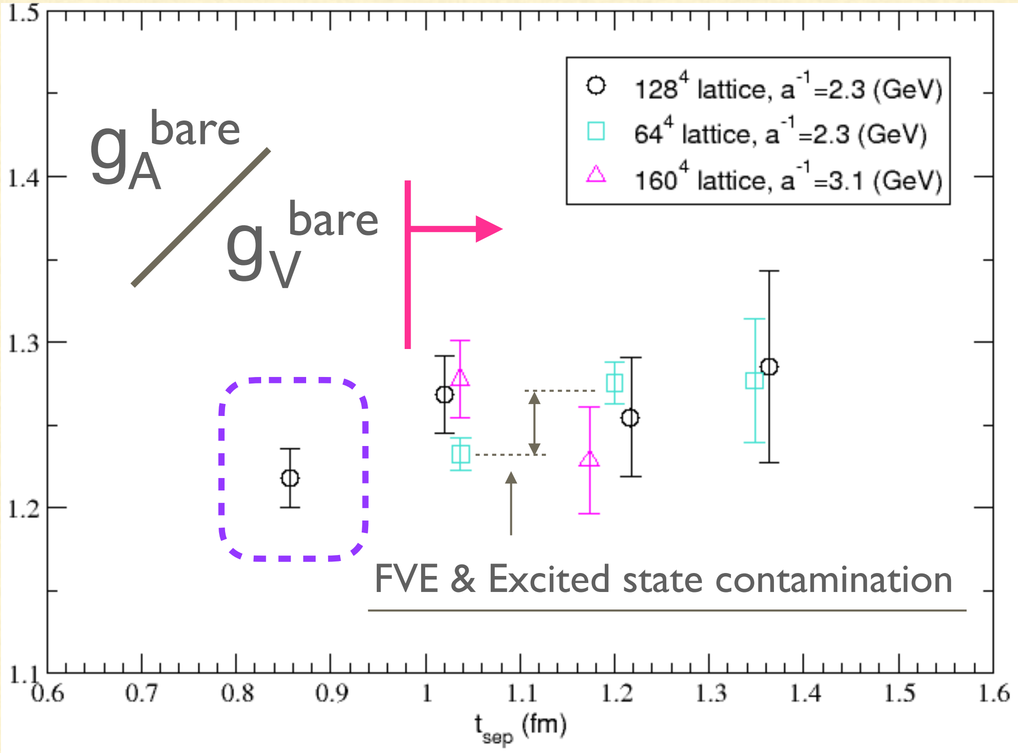
Simulation details -PACS10 configuration[1][2]

	P.R.D **99**, 014510(2019)	Running
Lattice size	128^4 [1]	160^4 [2]
Spacial volume	$\sim (10.8\ \mathrm{fm})^3$	$\sim (10.3\ \mathrm{fm})^3$
Pion mass	135 MeV	135 MeV
Nucleon mass	~ 0.942 GeV	~ 0.939 GeV
$\lvert t_{sink} - t_{src} \rvert / a$	10, 12, 14, 16	16,19
Lattice spacing	~ 0.084 fm	~ 0.064 fm

[1] E. Shintani et al., Phys. Rev. D **99**, 014510(2019)
[2] E. Shintani and Y.Kuramashi, Phys.Rev. D **100**, 034517(2019)

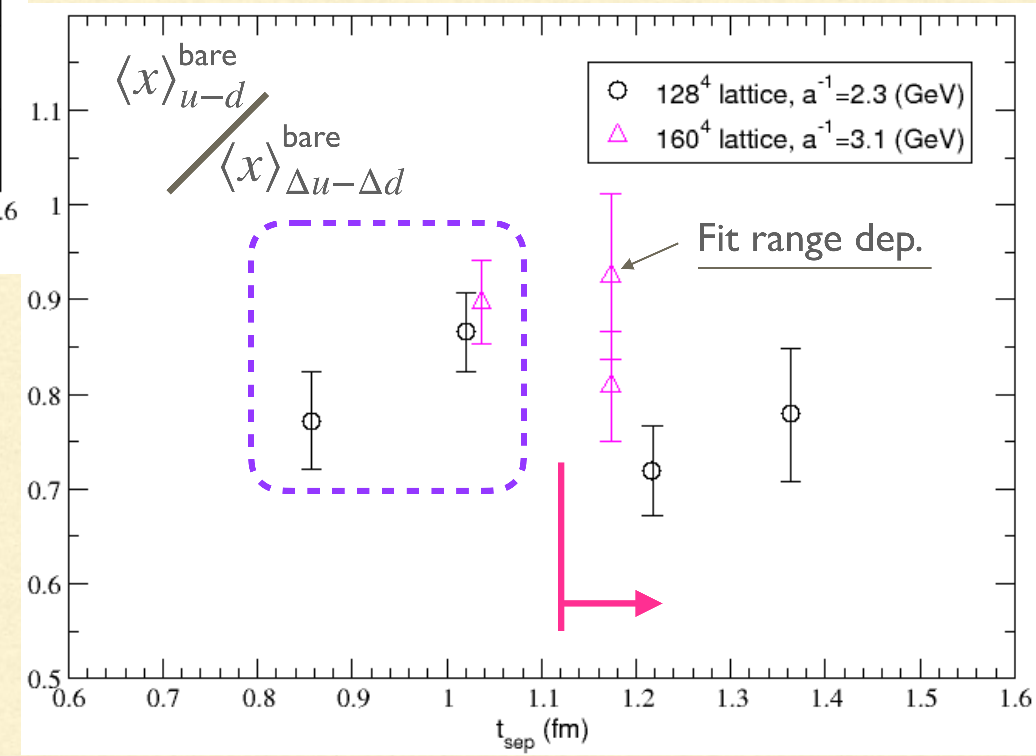
The stout-smeared $O(a)$ improved Wilson fermions and Iwasaki gauge action.

Excited state contamination



$t_{sep} > 1$ (fm) for g_A / g_V
$t_{sep} > 1.2$ (fm) for $\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u - \Delta d}$
seems enough for g.s. saturation

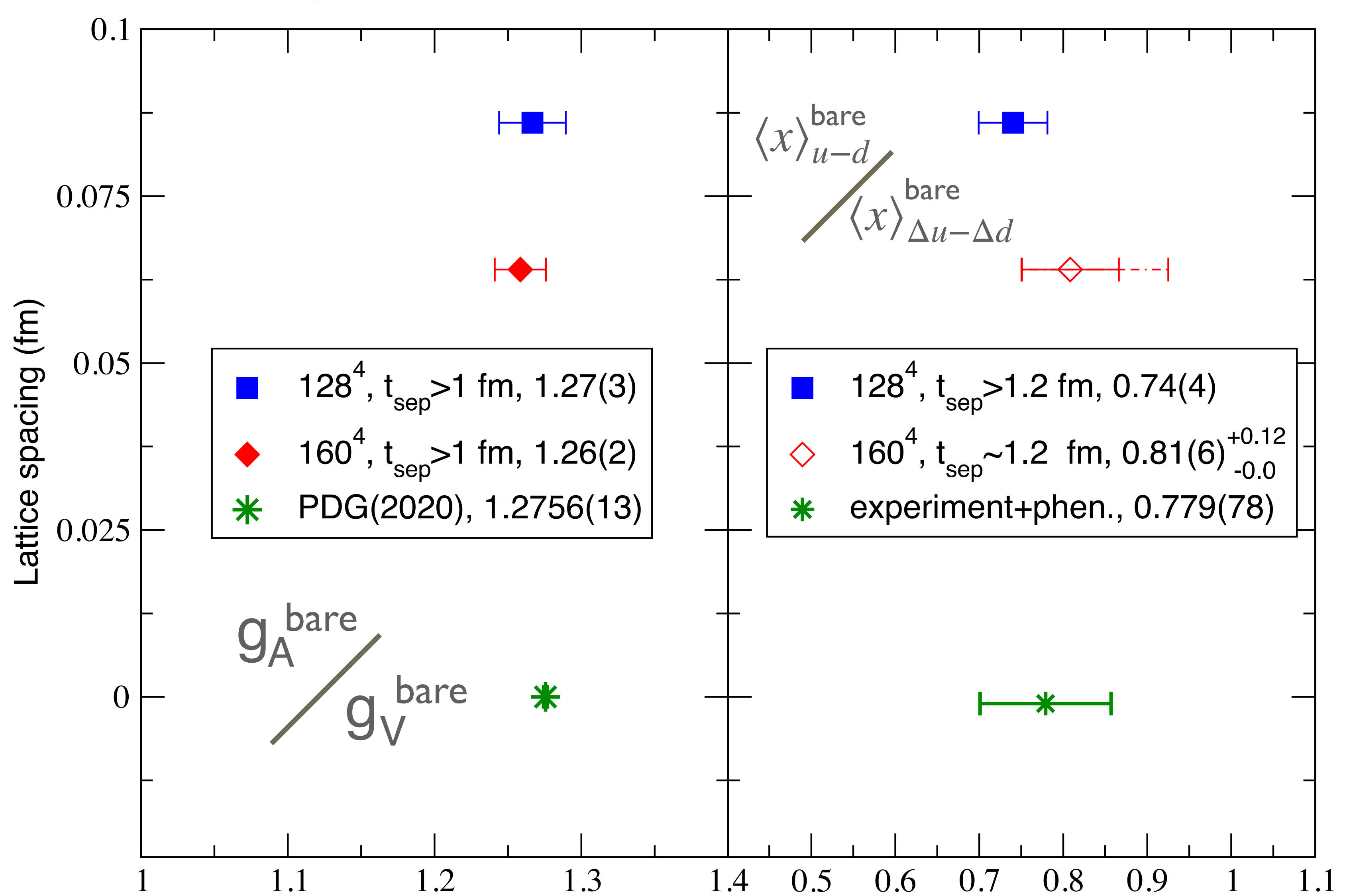
Excited state contamination?
Not enough statistics?



Preliminary

- Approach the continuum limit

→ g_A / g_V and $\langle x \rangle_{u-d}$ / $\langle x \rangle_{\Delta u - \Delta d}$ are consistent with experiments.



Summary and perspectives

- Conclusion of this talk
- Future works

Summary and Perspectives

High-precision and high-accuracy determination:

*$g_S = 0.927(139)_{sta.}(11)_{sys.}$ and $g_T = 1.055(23)_{sta.}(25)_{sys.}$

→ Lattice QCD is able to predict quantities associated with quantum many body correlation.

NEXT!

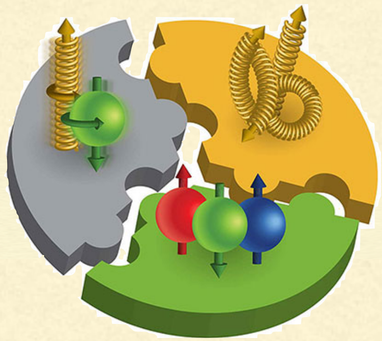
Approach continuum limit with high-precision and high-accuracy

① **Particle physics (Intensity frontier)**

Beyond S.M. = Experiment - S.M. of particle physics

eg)
- Quark EDM
- Muon g-2
- Proton decay

Nucleon structure



eg)
- Proton radius/spin problem
- Short range correlation in nuclei
- SSA

② **Nuclear physics (Many body problem)**

* We can also use these for searching the physics beyond the Standard Model (Intensity frontier).