Direct CP violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decay from the Standard Model

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PRD102,054509 (RBC & UKQCD Collaborations)

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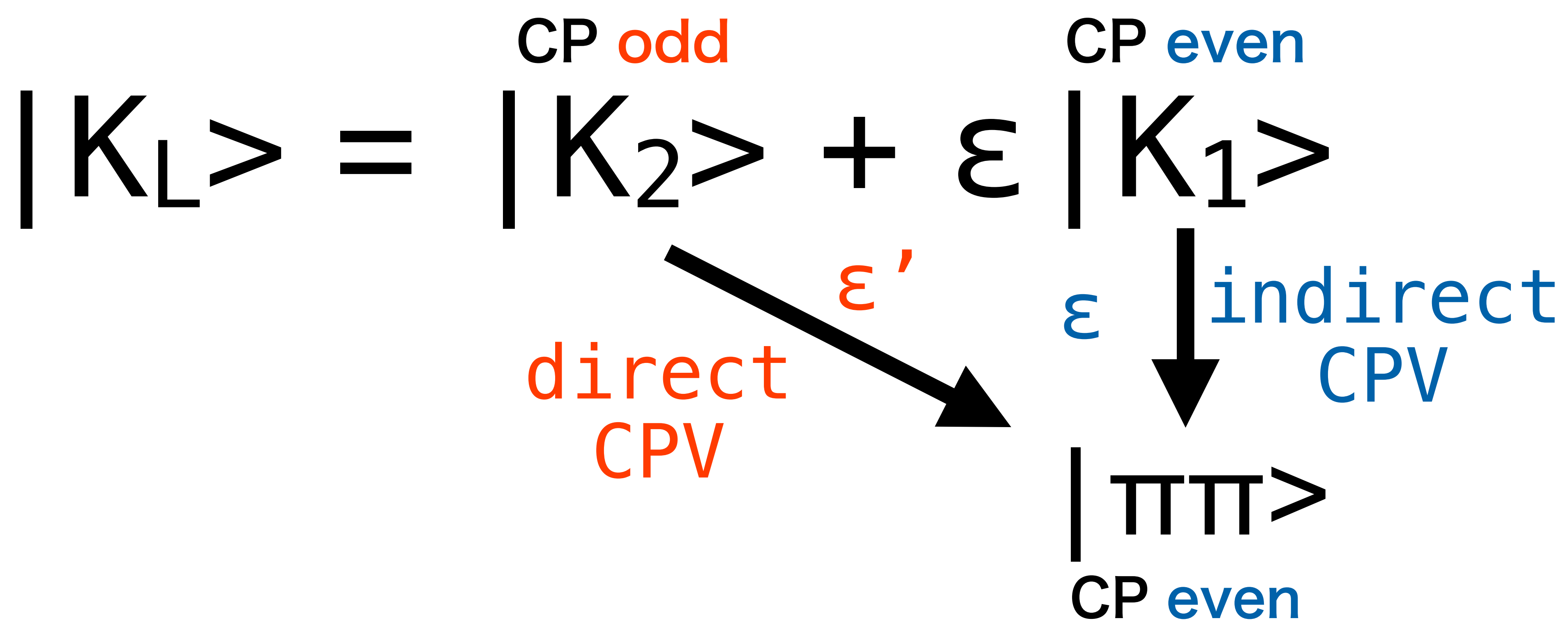
Contents

CP violation in K → ππ

- $K_L \to \pi\pi$ invalid in CP limit
- CP violated in reality



CP odd   CP even

$$|K_L\rangle = |K_2\rangle + \varepsilon|K_1\rangle$$

ε’

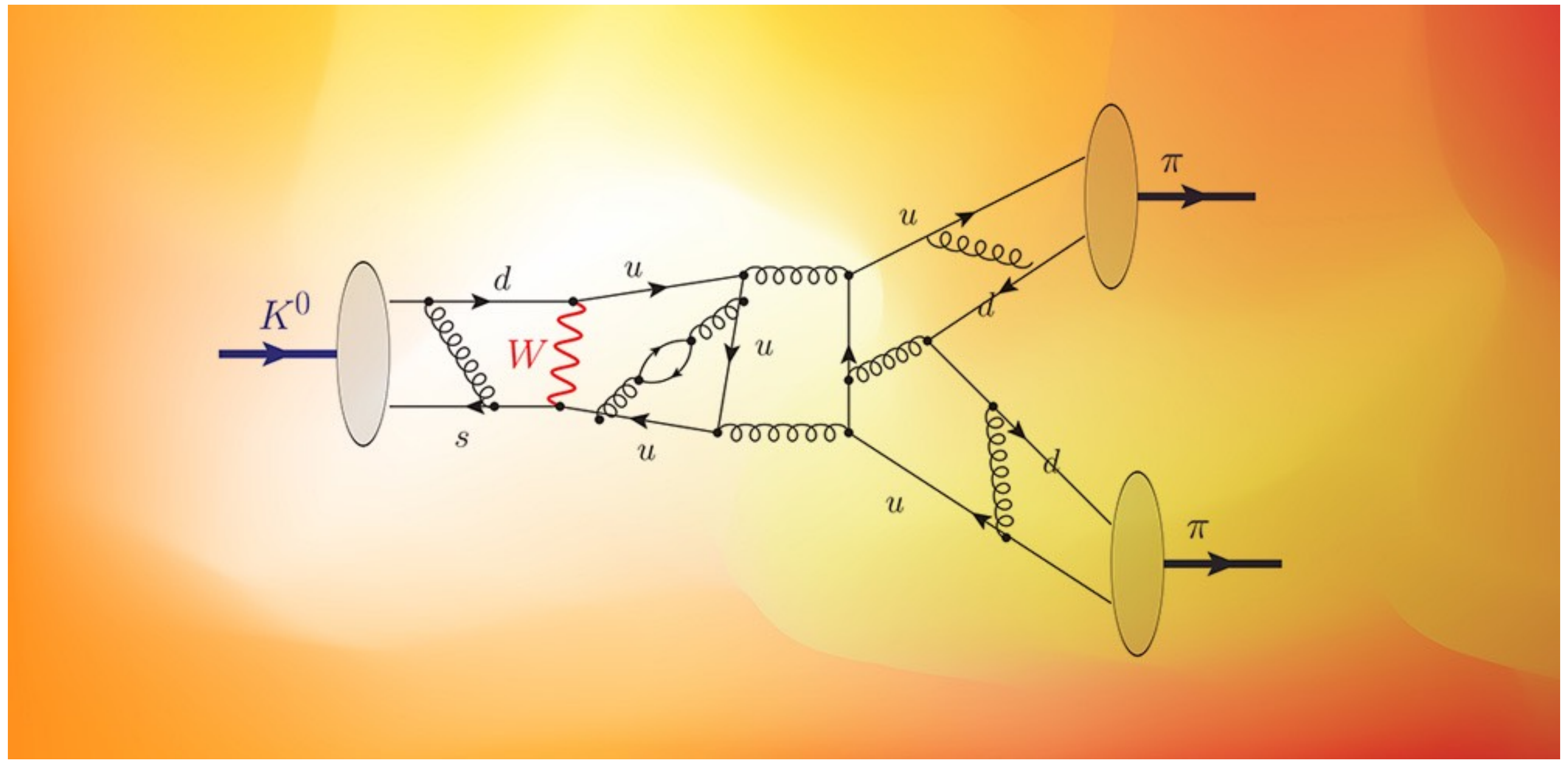
direct CPV

ε

indirect CPV

$|\pi\pi\rangle$

CP even



- 2 important measures ε’ & ε of CP violation
  - $\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\mathrm{exp}} = 16.6(2.3) \times 10^{-4}$
  - Can it be explained by the SM?
- Key to understanding matter/anti-matter asymmetry

I = 0 & I = 2 decay modes

$$\langle(\pi\pi)_{\mathrm{I}=0}| = \sqrt{1/3}\langle\pi^0\pi^0| + \sqrt{2/3}\langle\pi^+\pi^-|, \quad \langle(\pi\pi)^{\mathrm{I}_3=0}_{\mathrm{I}=2}| = -\sqrt{2/3}\langle\pi^0\pi^0| + \sqrt{1/3}\langle\pi^+\pi^-|$$

- Isospin-definite amplitudes

  $$\mathrm{A}_{\mathrm{I}} = \langle(\pi\pi)_{\mathrm{I}}|\mathrm{H}_{\mathrm{W}}|\mathrm{K}\rangle$$

- Convenient decomposition especially for isospin symmetric calculation
- A_2 precisely calculated (PRL108 (2012) 141601, PRD91 (2015) 074502)
  - 2 lattice spacings: 2.36 GeV, 1.73 GeV → continuum limit taken
  - Re A_2 = $1.50(4)_{stat}(14)_{sys} \times 10^{-8}$ GeV, Im A_2 = $-6.99(20)_{stat}(84)_{sys} \times 10^{-13}$ GeV

    cf: $(\mathrm{Re}\,A_2)_{exp} = 1.479(4) \times 10^{-8}$ GeV
- ε’: needs both of A_0 & A_2

The ΔI = 1/2 rule

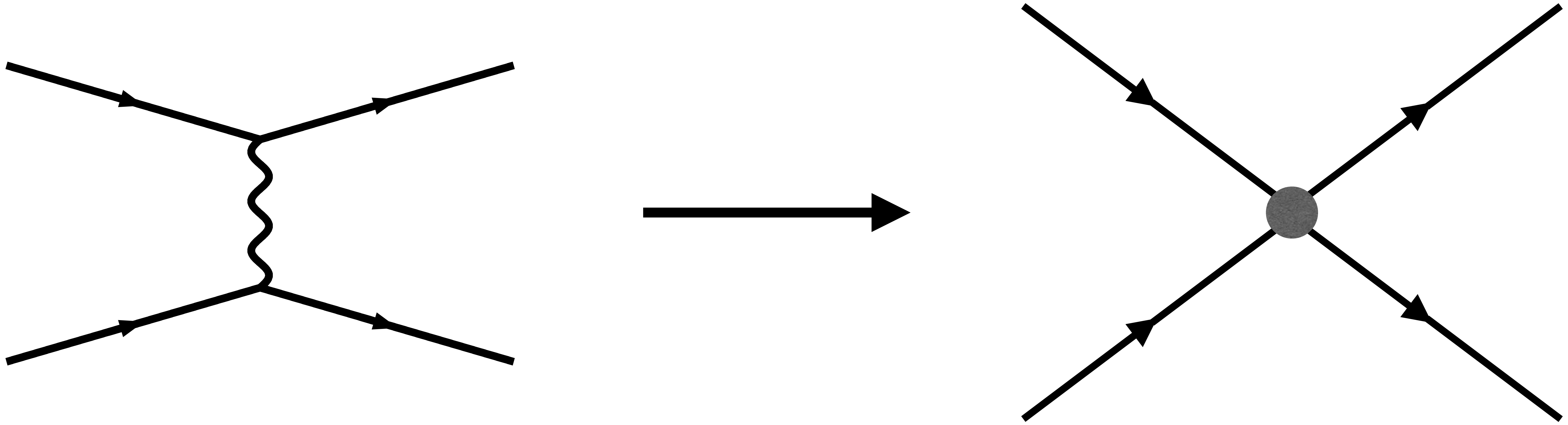
- Experimental fact

$$\frac{\mathrm{Re}\,\mathrm{A}_0}{\mathrm{Re}\,\mathrm{A}_2} = 22.45(6)$$: large suppression of ΔI = 3/2 (A_2) mode

- Estimation at LO pQCD: Re A_0 = 2 Re A_2
- Extra factor x10 coming from full QCD or BSM?

Approach to weak processes

- Large scale separation
  - Weak interactions: m_W = 80 GeV, m_Z = 91 GeV
  - QCD scale: $\Lambda_{QCD} \approx$ 300 MeV
- Low-energy effective theory
  - contributions from heavy particles: effective interactions



4-Fermi interactions

Lattice calculation of MEs

- Effective Hamiltonian

Wilson coefficients
- Information of t, W, Z, …
- Calculated by pQCD

$$\mathrm{H_W} = \sum_{\mathrm{i}} \underline{\mathrm{w_i^R}(\mu)}\,\underline{\mathrm{O_i^R}(\mu)}$$

Effective operators (e.g. 4-Fermi)
- Composed of light particles
- MEs calculated by lattice QCD

- Lattice calculation of $\mathrm{M_i^{lat}} = \langle \mathrm{out}|\mathrm{O_i^{lat}}|\mathrm{in}\rangle$
  - can involve all contributions from QCD
- Renormalization
  - Non-perturbatively $\mathrm{O_i^{lat}} \to \mathrm{O_i^R}(\mu), \quad \mathrm{M_i^{lat}} \to \mathrm{M_i^R}(\mu)$
  - Perturbatively $\mathrm{w_i^{\overline{MS}}}(\mu) \to \mathrm{w_i^R}(\mu)$

ΔS = 1 effective operators

- $(\bar{s}q)_{V-A}(\bar{q}'q'')_{V\pm A} = \bar{s}\gamma_\mu(1-\gamma_5)q' \cdot \bar{q}'\gamma_\mu(1\pm\gamma_5)q''$
- α, β: color indices

$$\left.\begin{aligned}
Q_1 &= (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}}\,(\bar{u}_\beta d_\alpha)_{\mathrm{V-A}}\,, \\
Q_2 &= (\bar{s}u)_{\mathrm{V-A}}\,(\bar{u}d)_{\mathrm{V-A}}\,,
\end{aligned}\right\}$$

Current-current operators

- $Q_1^c = (\bar{s}_\alpha c_\beta)_{\mathrm{V-A}}(\bar{c}_\beta d_\alpha)_{\mathrm{V-A}}$ & $Q_2^c = (\bar{s}c)_{\mathrm{V-A}}(\bar{c}d)_{\mathrm{V-A}}$

  enter when $n_f \geq 4$

$$\left.\begin{aligned}
Q_3 &= (\bar{s}d)_{\mathrm{V-A}}\sum_q(\bar{q}q)_{\mathrm{V-A}}\,, \\
Q_4 &= (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}}\sum_q(\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}\,, \\
Q_5 &= (\bar{s}d)_{\mathrm{V-A}}\sum_q(\bar{q}q)_{\mathrm{V+A}}\,, \\
Q_6 &= (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}}\sum_q(\bar{q}_\beta q_\alpha)_{\mathrm{V+A}}\,,
\end{aligned}\right\}$$

QCD penguin operators

- sum over q runs for all active quarks

$$\left.\begin{aligned}
Q_7 &= \frac{3}{2}\,(\bar{s}d)_{\mathrm{V-A}}\sum_q e_q\,(\bar{q}q)_{\mathrm{V+A}}\,, \\
Q_8 &= \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{\mathrm{V-A}}\sum_q e_q\,(\bar{q}_\beta q_\alpha)_{\mathrm{V+A}}\,, \\
Q_9 &= \frac{3}{2}\,(\bar{s}d)_{\mathrm{V-A}}\sum_q e_q\,(\bar{q}q)_{\mathrm{V-A}}\,, \\
Q_{10} &= \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{\mathrm{V-A}}\sum_q e_q\,(\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}\,,
\end{aligned}\right\}$$

EW penguin operators

ε’ calculation

- Isospin-limit formula

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{ \frac{i\omega e^{i\overset{\textbf{ππ phase shifts}}{\boxed{\delta_2-\delta_0}}}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\} \qquad (\omega = \mathrm{Re}\,A_2/\mathrm{Re}\,A_0)$$

$$A_I = \overset{\textbf{Lellouch-Lüscher finite volume correction}}{\boxed{F}}\frac{G_F}{2}V_{us}^{*}V_{ud}\sum_{i,j}\underbrace{[z_i(\mu)+\tau y_i(\mu)]}_{\substack{\textbf{Wilson coefs.}\\ \textbf{pQCD}}}\underbrace{\overset{\textbf{Renormalization matrix}}{Z_{ij}(\mu)}}_{\substack{\textbf{LQCD}\\ \textbf{(+pQCD)}}}\underbrace{\langle(\pi\pi)_I|Q_j^{lat}|K\rangle}_{\textbf{LQCD}}$$

- F & δ$_I$ extracted from ππ scattering study
    - 2pt function $\langle O_{\pi\pi}(\vec{p},t)O_{\pi\pi}(\vec{p},0)^{\dagger}\rangle$
    - Lüscher’s method [Commun.Math.Phys. 219 (2001) 31]

(RBC/UKQCD is preparing a companion paper on this calculation)

First result for ε’ in 2015

Z. Bai et al, (RBC/UKQCD) PRL115(2015) 21, 212001

- Simulation parameters
  - 32^3 x 64 (2+1 Möbius domain-wall fermions)
  - near physical pion & kaon: m_π = 143.1(2.0) MeV, m_K = 490.6(2.2) MeV
  - lattice cutoff: 1.3784(68) GeV
  - 216 configurations
- Re A_0 & Im A_0: large statistical & systematic errors

disconnected diagrams Truncation of pQCD (small renormalization scale)

- ε’

2.2σ

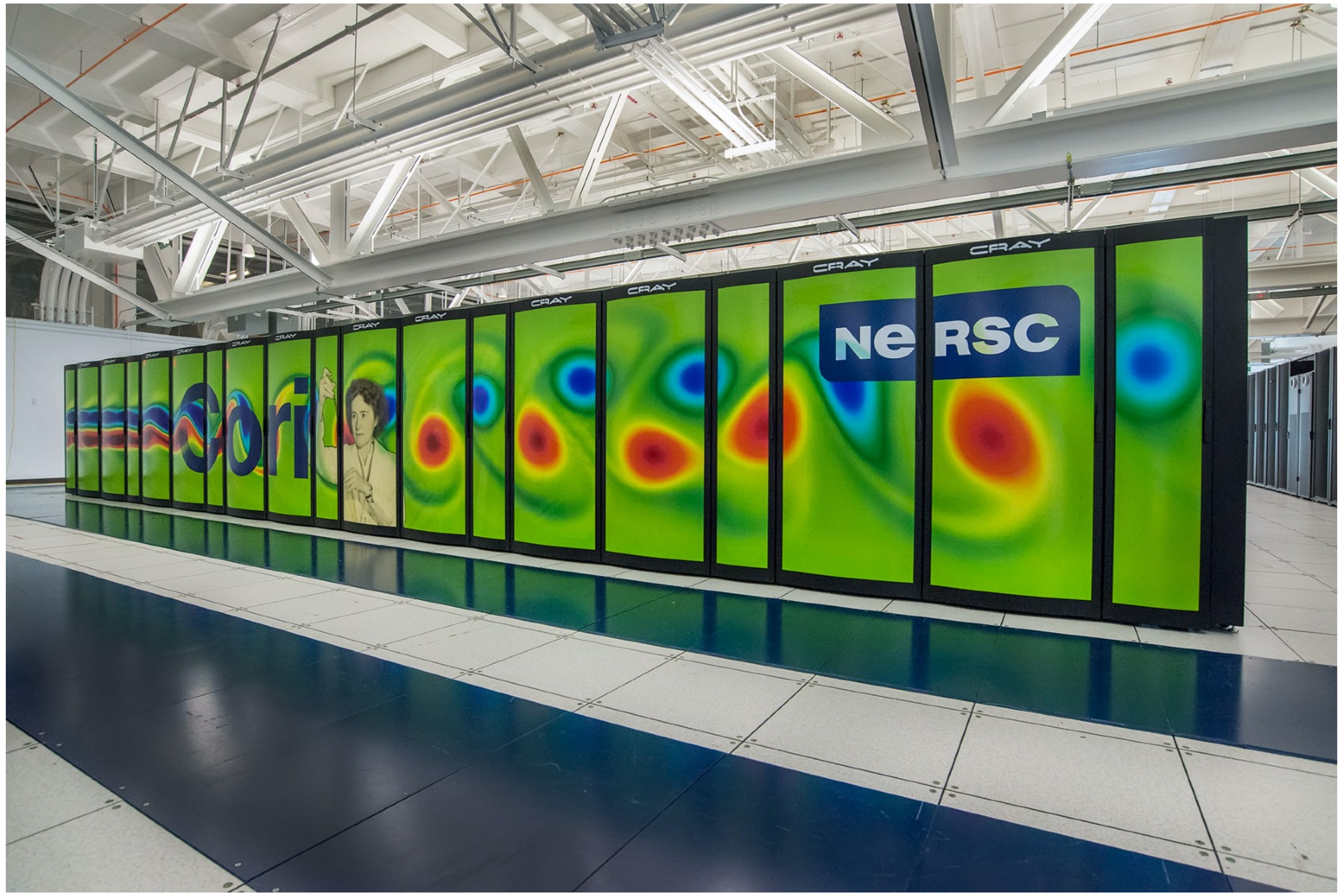
$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{SM}} = 1.38(5.15)_{\mathrm{stat}}(4.59)_{\mathrm{sys}} \times 10^{-4} \quad \Longleftrightarrow \quad \mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{exp}} = 16.6(2.3) \times 10^{-4}$$

This work

- Same gauge ensemble but…
  - 216 → 741 configurations (864 → 5,864 MD time)
  - Multiple ππ operators → Excited-state contaminations well managed
  - Renormalization scale nonperturbatively lifted up by step scaling
→ significant reduction of systematic error

Computational resources

- Main resource
  - Cori @ NERSC (National Energy Research Scientific Computing Center)
  - 430 M NERSC hours (~core hours)
- Supplemental
  - BlueGene/Q (BNL), Hokusai (RIKEN), Mira (Argonne), KEKSC 1540 (KEK), DiRAC (Edinburgh), Blue Waters (Illinois)



Contents

- ☑ Introduction
- K → ππ matrix elements
  - Extracting on-shell kinematics
  - ππ scattering phase shift & ππ puzzle
  - K → ππ
- Operator renormalization
- On going projects

What's needed for K → ππ MEs

- Euclidean correlation function (0-momentum case)

$$\underline{\int d^3x_{\pi\pi} d^3x_K} \langle O_{\pi\pi}(t_{\pi\pi},\vec{x}_{\pi\pi}) H_W(t,\vec{0}) O_K(t_K,\vec{x}_K)^\dagger \rangle$$

zero-momentum projection ($e^{i\vec{p}\cdot\vec{x}} = 1$)

$$= \sum_{\underline{m,n}} \langle 0|O_{\pi\pi}|\pi\pi, m\rangle \frac{1}{2E_{\pi\pi,m}} \langle \pi\pi, m|H_W|K, n\rangle \frac{1}{2E_{K,n}} \langle K, n|O_K^\dagger|0\rangle e^{-m_{\pi\pi,m}(t_{\pi\pi}-t)} e^{-m_{K,n}(t-t_K)}$$

all possible zero-(total)momentum states that have the same quantum numbers as $O_{\pi\pi}/O_K$

If the lightest state is interesting…
look at large $t_{\pi\pi}$ - t & t - t_K:

$$\to \langle 0|O_{\pi\pi}|\pi\pi, 0\rangle \frac{1}{2E_{\pi\pi,0}} \langle \pi\pi, 0|H_W|K, 0\rangle \frac{1}{2E_{K,0}} \langle K, 0|O_K^\dagger|0\rangle e^{-m_{\pi\pi,0}(t_{\pi\pi}-t)} e^{-m_{K,0}(t-t_K)}$$

What’s needed for K → ππ MEs

- Euclidean correlation function (0-momentum case)

$$\underline{\int d^3x_{\pi\pi} d^3x_K} \langle O_{\pi\pi}(t_{\pi\pi},\vec{x}_{\pi\pi}) H_W(t,\vec{0}) O_K(t_K,\vec{x}_K)^\dagger \rangle$$

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$$= \sum_{\underline{m,n}} \langle 0|O_{\pi\pi}|\pi\pi, m\rangle \frac{1}{2E_{\pi\pi,m}} \langle \pi\pi, m|H_W|K, n\rangle \frac{1}{2E_{K,n}} \langle K, n|O_K^\dagger|0\rangle e^{-m_{\pi\pi,m}(t_{\pi\pi}-t)} e^{-m_{K,n}(t-t_K)}$$

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$$\rightarrow \langle 0|O_{\pi\pi}|\pi\pi, 0\rangle \frac{1}{2E_{\pi\pi,0}} \underline{\langle \pi\pi, 0|H_W|K, 0\rangle} \frac{1}{2E_{K,0}} \langle K, 0|O_K^\dagger|0\rangle e^{-m_{\pi\pi,0}(t_{\pi\pi}-t)} e^{-m_{K,0}(t-t_K)}$$

Is this what we wanted?

Heavier ππ state needed

- Kaon lightest state is its physical ground state
- The lightest ππ state off-shell with 2 stationary pions, $E_{\pi\pi,0} \approx 270$ MeV
  - need to extract $| E_{\pi\pi} = m_K \approx 500 \text{ MeV} \rangle$
- Possible approaches
  - 💡 Finite box → individual pion momenta and spectrum not continuous (2nd lightest can be of interest)
  - Analyze correlation functions considering multiple states
  - Manipulate boundary condition so that the lightest state vanishes (employed)

I = 2 calculation

(PRL108 (2012) 141601, PRD91 (2015) 074502)

- Impose anti-periodic boundary conditions (APBC) on d quark in n directions:
  - $d(x + L\hat{e}_{x_1,\ldots,x_n}) = -d(x)$
  - Charged pions: anti-periodic on those boundaries:

    $\pi^{\pm}(x + L\hat{e}_{x_1,\ldots,x_n}) = -\pi^{\pm}(x)$
  - $\widetilde{\pi}^{\pm}(\vec{p}, t)|_{p_i=0} = 0, \quad i = 1, \ldots, n$ → lightest state energy: $E^2_{\pi^\pm} = m^2_\pi + n^2(\pi/L)^2$

    L (& n) should be tuned so $E_{\pi^+\pi^+} = m_K$
- Isospin rotation (Wigner-Eckart theorem):

$$\langle(\pi\pi)^{I_3=1}_{I=2}|H^{\Delta I_3=1/2}_{\Delta I=3/2}|K^+\rangle = \frac{3}{2}\underbrace{\langle(\pi\pi)^{I_3=2}_{I=2}}_{\langle\pi^+\pi^+|}|H^{\Delta I_3=3/2}_{\Delta I=3/2}|K^+\rangle$$

  - A_2 can be calculated at on-shell kinematics with APBC d quark

G-parity boundary conditions for I = 0

- A_0 must be calculated with $\pi^0\pi^0$ final state → APBC useless
- G-parity boundary conditions:

$$f(x + L\hat{e}_{x_1,\dots,x_n}) = \hat{G}f(x) = \underbrace{\hat{C}}_{\textbf{Charge conjugation}}\underbrace{e^{-i\pi\hat{I}_y}}_{\textbf{180° isospin rotation}}f(x) \qquad (f:\ \text{isospin representation})$$

  - $\hat{G}\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} -C\bar{d}^T \\ C\bar{u}^T \end{pmatrix}, \quad \hat{G}\begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix} = \begin{pmatrix} -u^T C^{-1} \\ d^T C^{-1} \end{pmatrix}$
  - All pions: G-parity odd → can extract on-shell kinematics
- Dirac matrix mixes u & d → Computationally expensive
- Several other challenges confront

ππ scattering essential

T. Wang, C. Kelly, et al (RBC/UKQCD) 2103.15131

- Relations among ππ phase shift δ_0, energy $E_{\pi\pi}$ and "momenta" k
  - ex: 2-boson system in 1+1-dim w/ periodic BC in spatial direction

Relation b/w plane wave at x=L & x=0: $e^{ikL+2i\delta(k)} = 1$

$$\rightarrow \underline{k_n L + 2\delta(k_n) = 2n\pi}$$

"Lüscher formula" for this toy case

$$(\text{cf: } k_n = 2n\pi/L \text{ in non-interacting case})$$

Dispersion relation: $E_n = 2\sqrt{m^2 + k_n^2}$

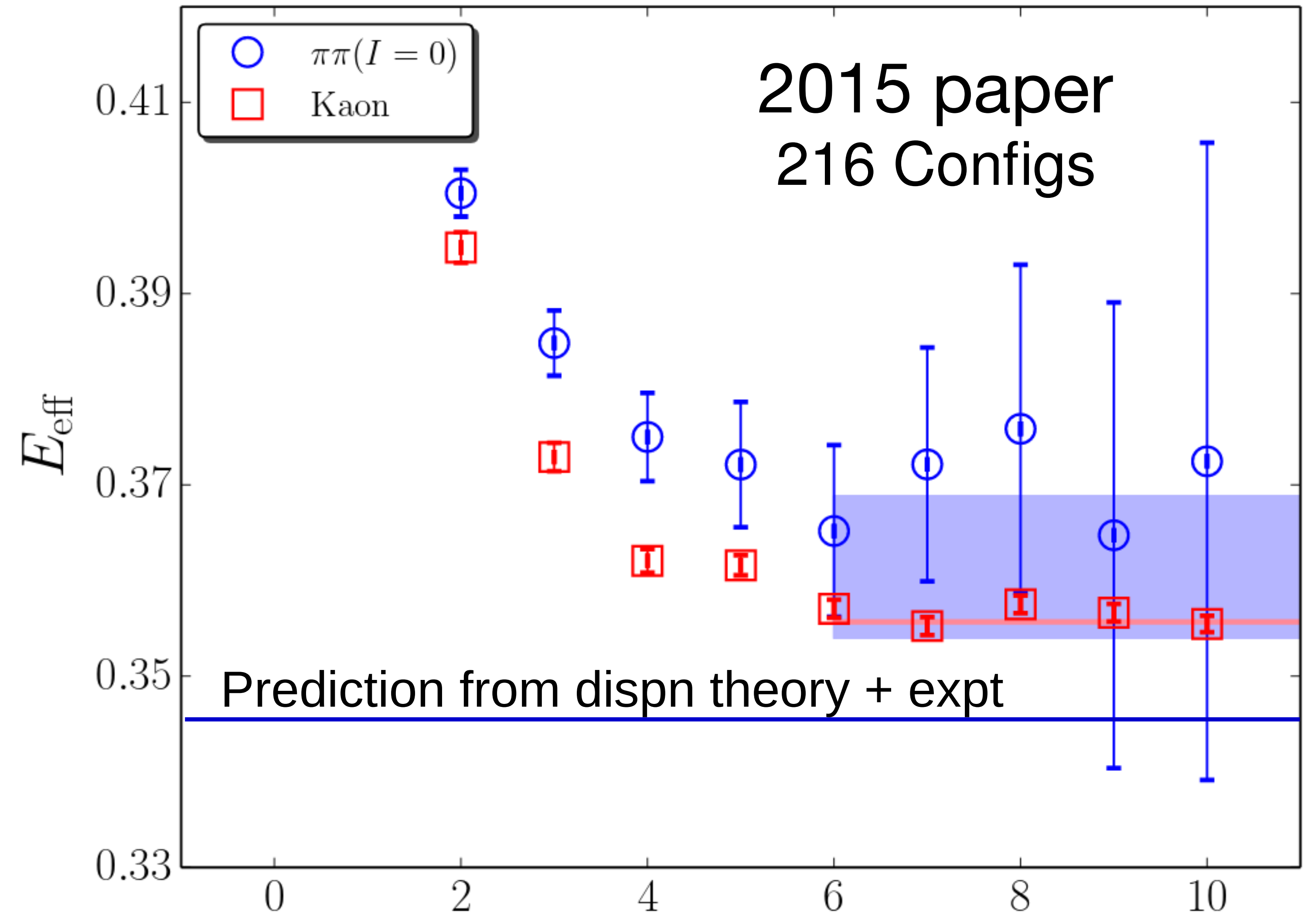
- Different relation for 3+1-dim & G-parity BC but same steps:
  - Extract a few energy levels E_n from ππ 2pt function (next slide) then corresponding k_n
  - Then $\delta(k_n)$ can be obtained from Lüscher formula
  - Derivative $\delta'(k_{\pi(\text{on-shell})})$ also needed for Lellouch-Lüscher finite-volume factor F

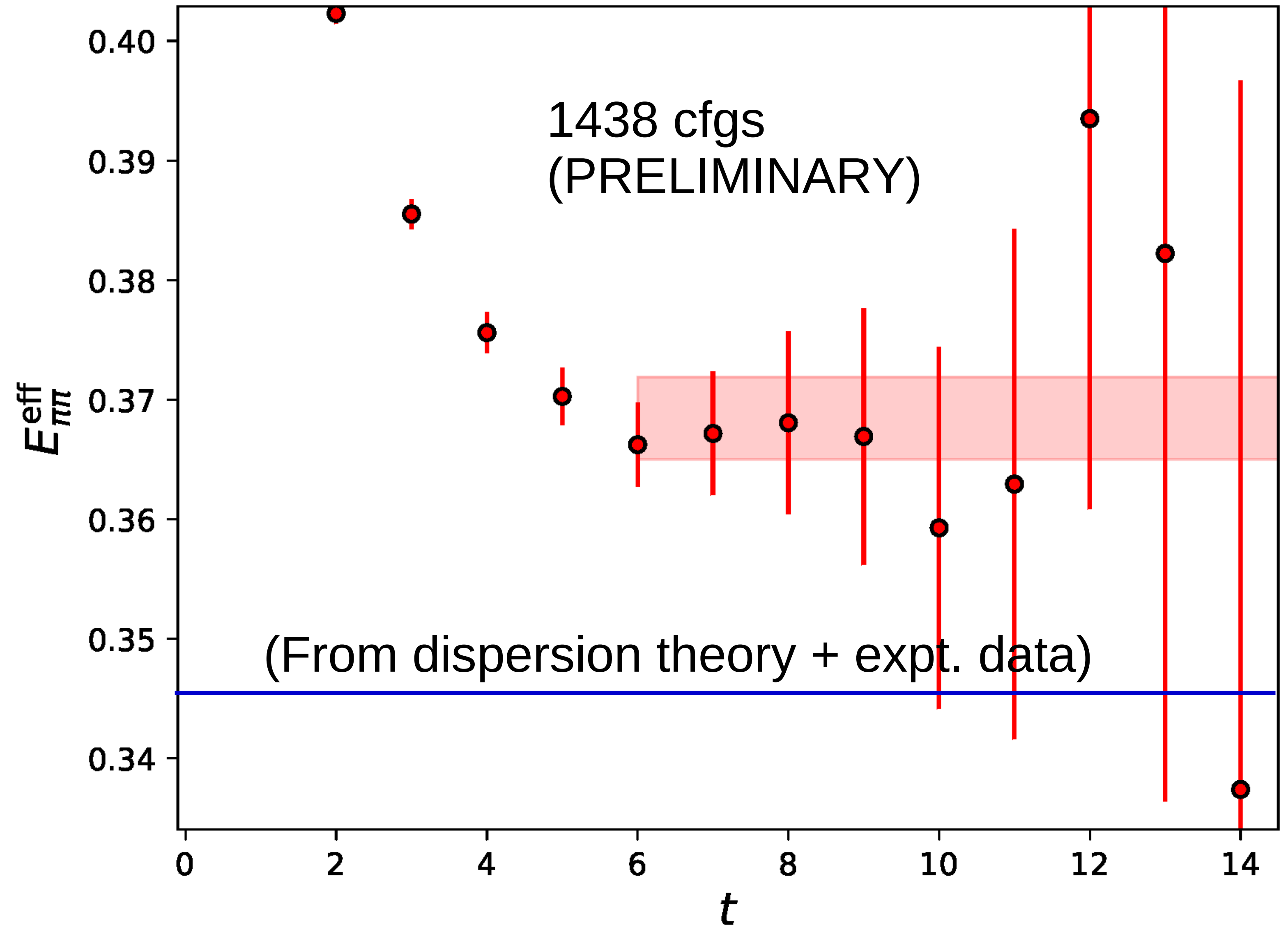
The “ππ puzzle”

- Large discrepancy b/w lattice & pheno+exp
  - $\delta_0^{2015} = 23.8(4.9)(2.2)^\circ, \quad \delta_0^{2020'} = 19.1(2.5)(1.2)^\circ$
  - $\delta_0^{\text{ph+exp}} = 36^\circ$
- Lattice analysis was fairly stable, unchanged by…
  - testing single- & 2-state fits

    $$G(t) = z_0 e^{-E_0 t} \;\&\; G(t) = z_0 e^{-E_0 t} + z_1 e^{-E_1 t}$$

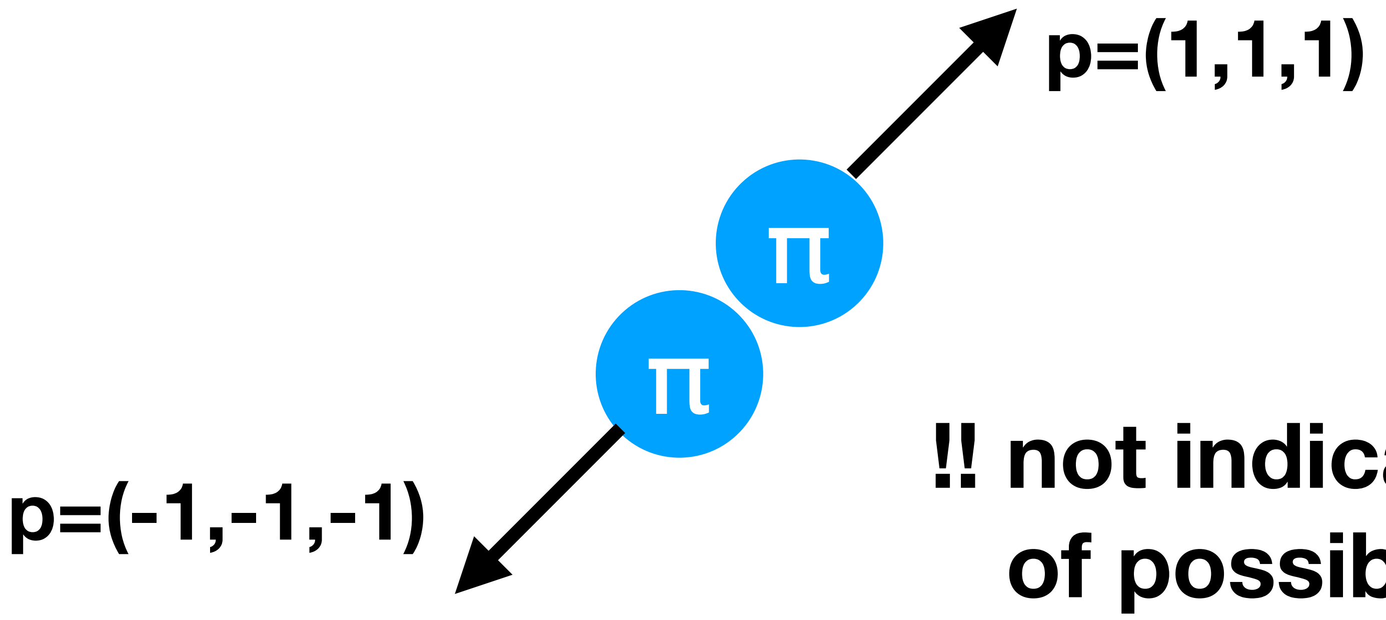
  - stable w/ varying fit range in the plateau region
- How it can be explained?
  - A big reason why we needed to retry I=0 calculation
  - our conclusion: excited states still significant





Resolving the ππ puzzle

- Introduce multiple ππ operators
  - In 2015
    $$O_{\pi\pi} = \pi\pi(1,1,1)$$
  - Additions in 2020
    $$\pi\pi(3,1,1), \qquad \sigma = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$



!! not indicating pion momenta of possible states

- 2pt functions

$$G_{ij}(t) = \langle O_i(t) O_j(0)^\dagger \rangle = \sum_n A_{i,n} A^\dagger_{j,n} e^{-E_n t}$$

  - possible to isolate a few lightest states
  - better way to investigate/manage excited-state contamination

Effect of multi operators on ππ

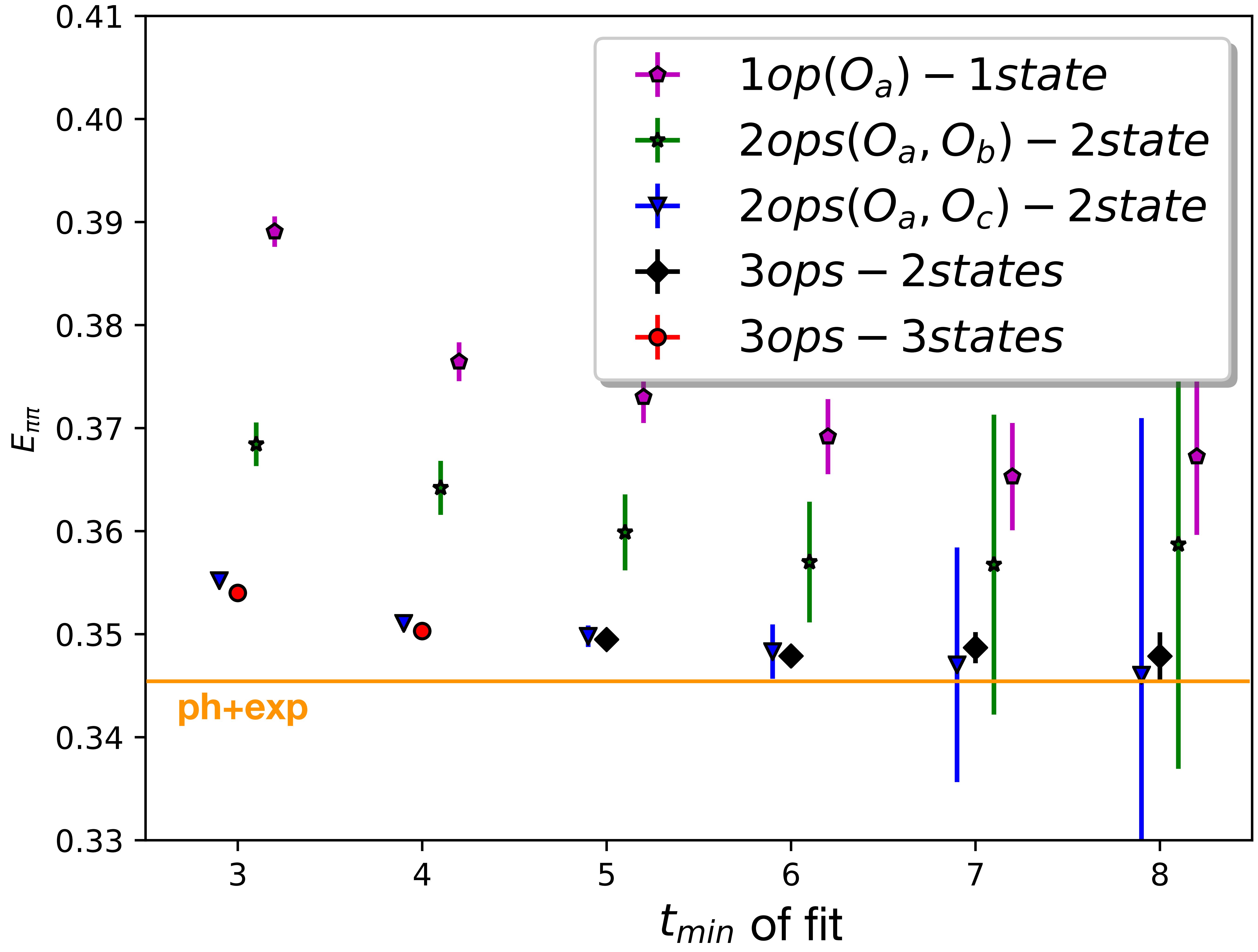
Result compatible with ph+exp:

$$\delta_0^{(1,1,1)} = 19.1(2.5)(1.2)^\circ$$

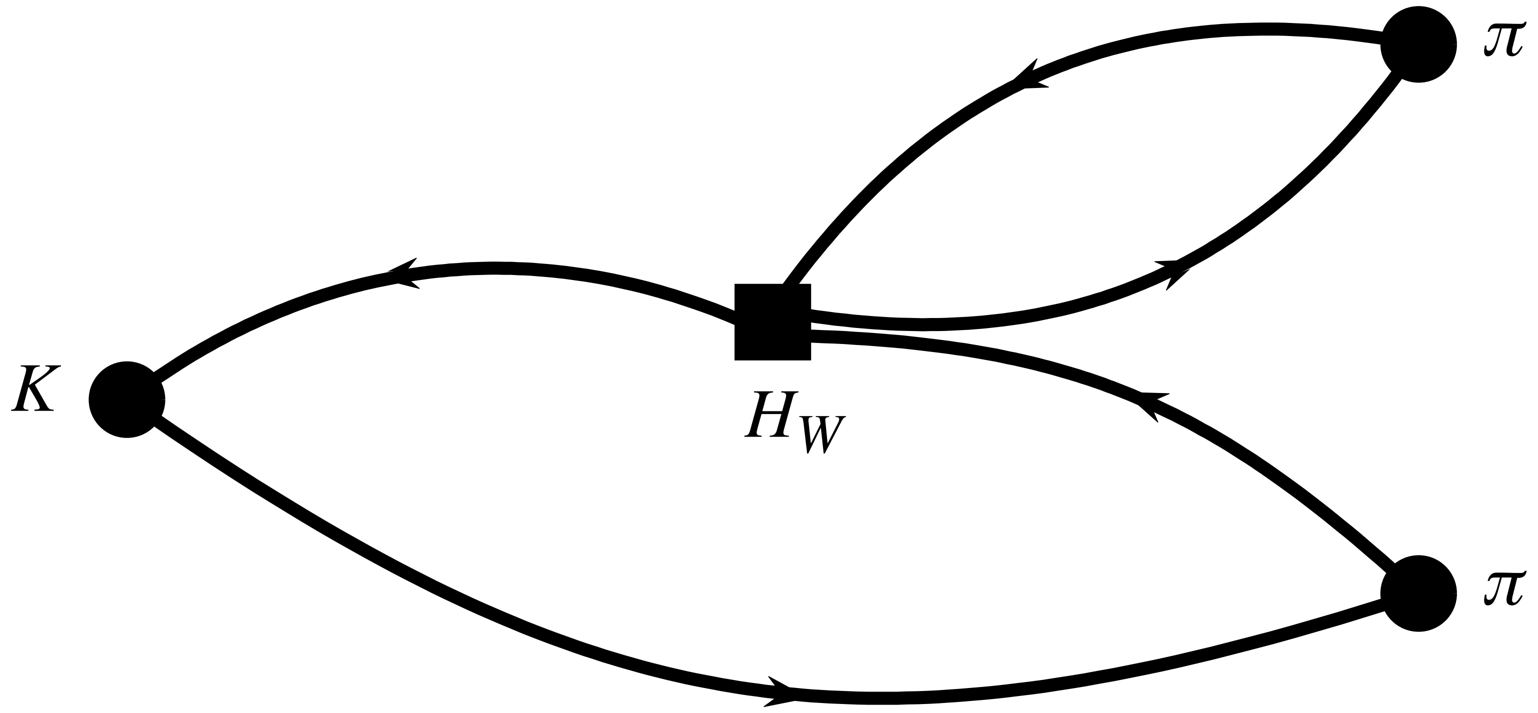
↓

$$\delta_0^{2021}(471\ \mathrm{MeV}) = 32.3(10)(14)^\circ$$

fit result for lightest (on-shell) ππ energy in lattice units



$_{I=0}\langle\pi\pi|\, Q_i \,|K\rangle$ 3pt functions



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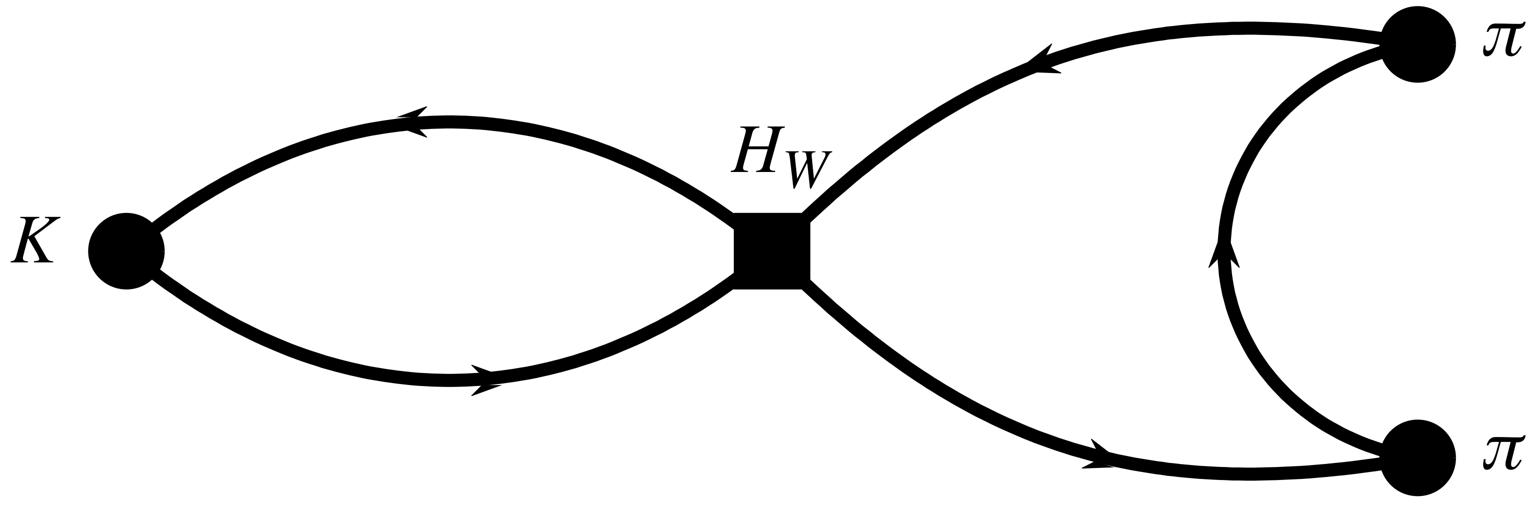

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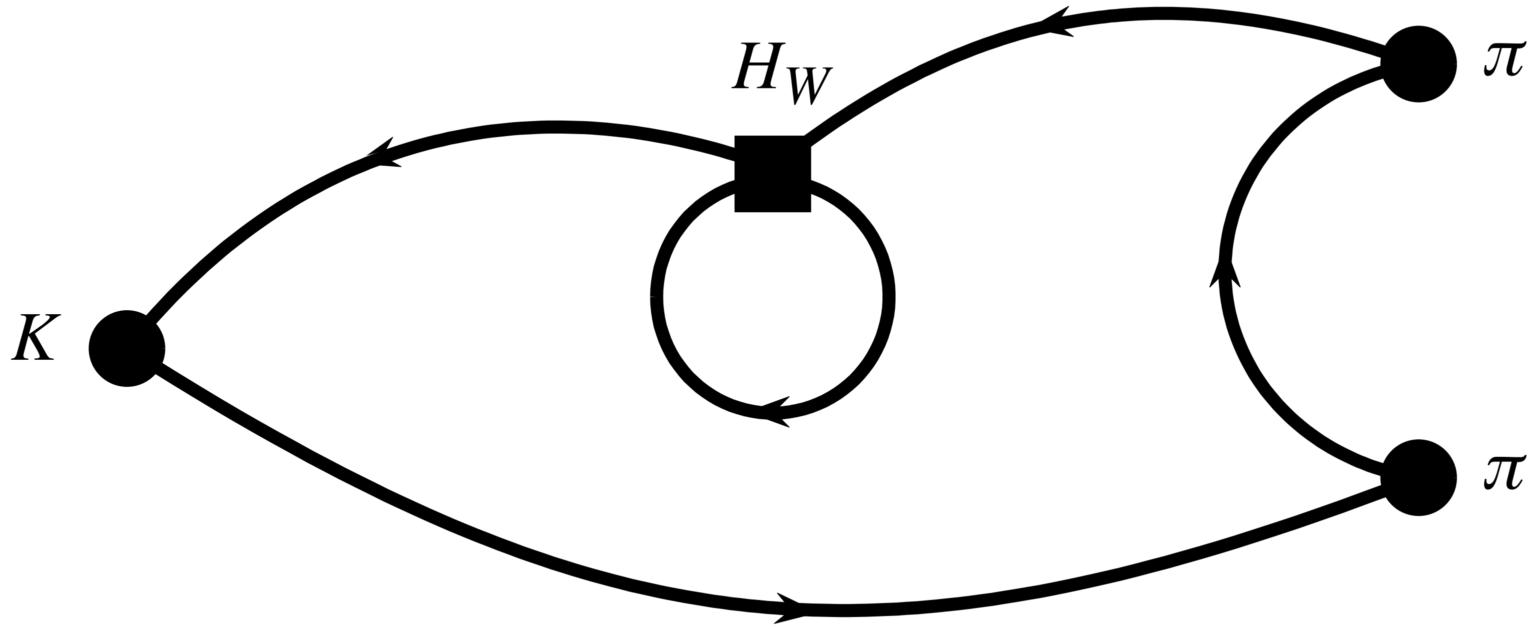

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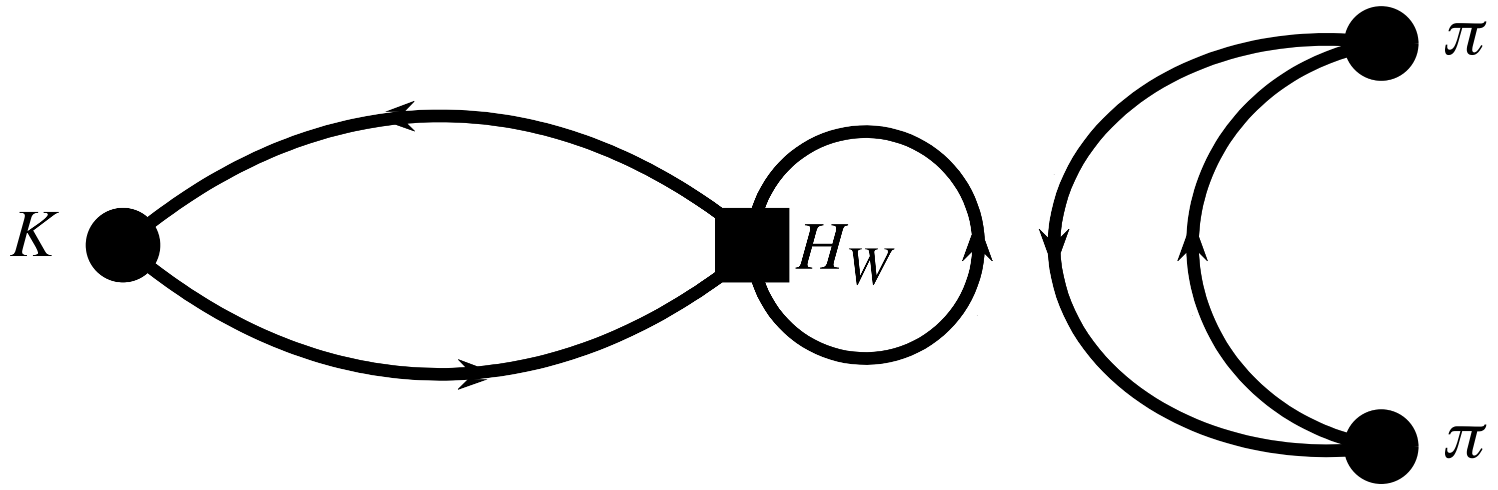


type 4

A2A propagators, V & W vectors

$$D^{-1}_{A2A} = \sum_{l=1}^{N_l} |\phi_l\rangle \frac{1}{\lambda} \langle\phi_l| + \frac{1}{N_h} \sum_{h=1}^{N_h} \underbrace{\left(D^{-1} - \sum_{l=1}^{N_l} |\phi_l\rangle \frac{1}{\lambda} \langle\phi_l| \right)}_{D^{-1}_{defl}} |\eta_h\rangle\langle\eta_h|$$

$$= \sum_{i=1}^{N_l+N_h} |V_i\rangle\langle W_i|$$

- V & W vectors

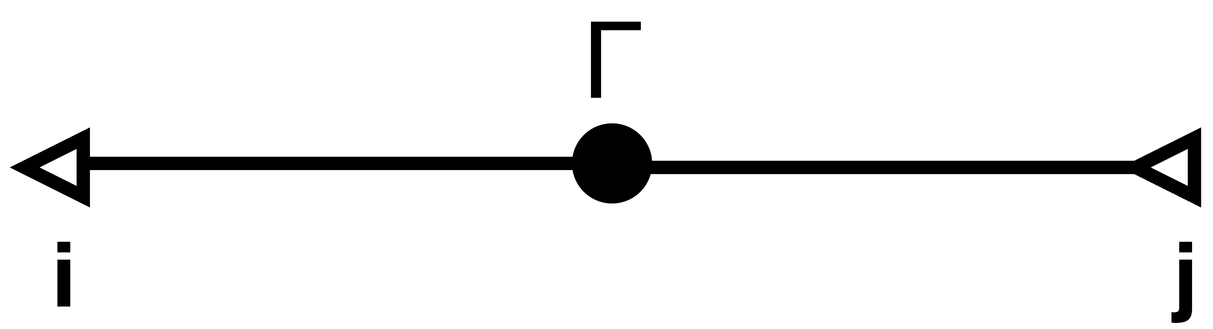
$$1 \leq i \leq N_l \;\; \Rightarrow \;\; \underline{|V_i\rangle = \frac{1}{\lambda}|\phi_i\rangle, \quad |W_i\rangle = |\phi_i\rangle}$$

$$N_l + 1 \leq i (= N_l + h) \leq N_l + N_h \;\; \Rightarrow \;\; \underline{|V_i\rangle = \frac{1}{N_h} D^{-1}_{defl} |\eta_h\rangle, \quad |W_i\rangle = |\eta_h\rangle}$$

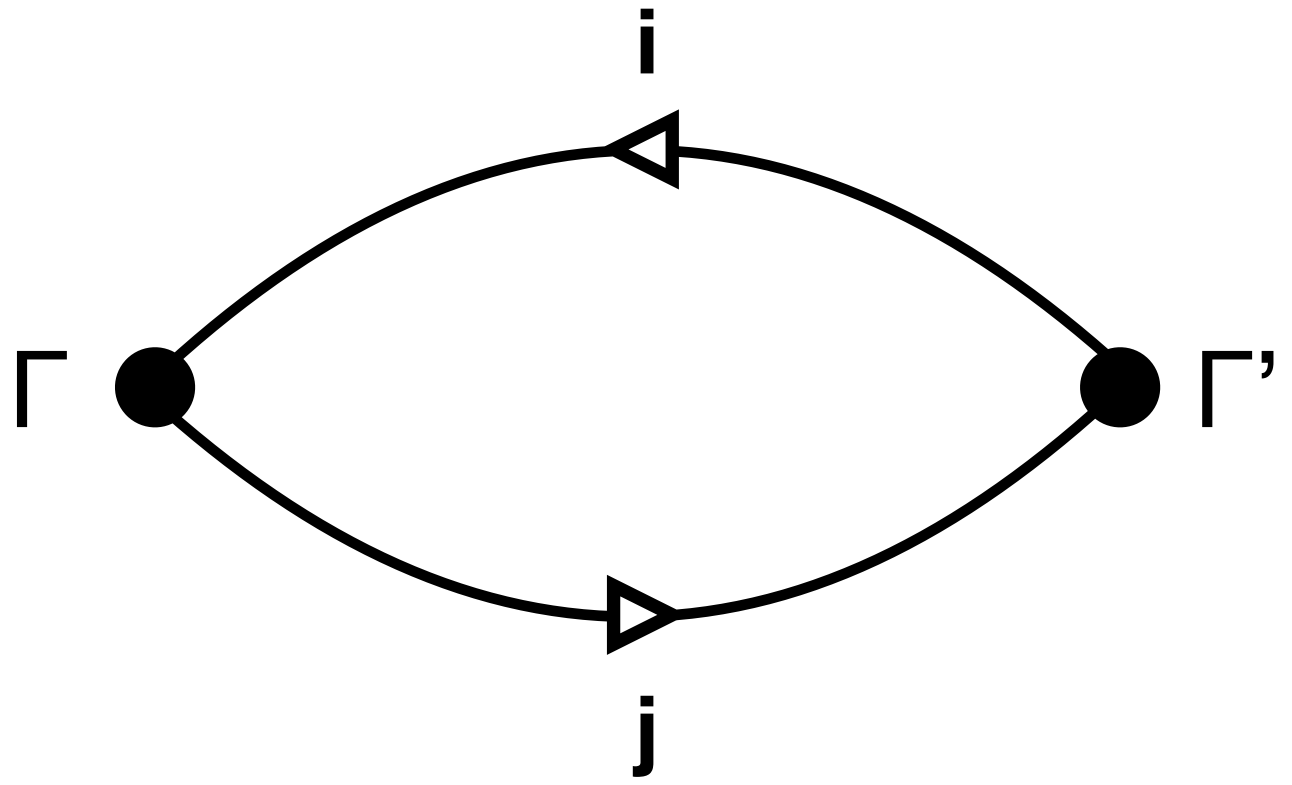
Meson fields

- Spin & color contractions leaving mode indices i, j
- Easily summed over time slice → savable data size
- Multiplied with any other meson fields to construct correlation functions

meson field



$$\Pi_{\Gamma,ij}(t) = \langle W_i|\Gamma|V_j\rangle_t$$



$$\Pi_{\Gamma,ij}(t)\Pi_{\Gamma',ji}(t')$$

A2A parameters & index

- 900 low modes from Lanczos algorithm for light quarks (not for strange)
- Random noise vectors
  - spin-color and time dilution

$$\eta_{h;s,c}(\vec{x},t) = \xi(\vec{x})\delta_{h,s+N_s(c+N_c t)}$$

  - N_s x N_c x N_t = 768 noise vectors
- # of V & W vectors: 1,668 for light quark, 768 for strange quark
- Pion fields: 1,668 x 1,668 matrix; Kaon field: 1,668 x 768 matrix

Effective MEs

- Tried with 3 ππ sink operators

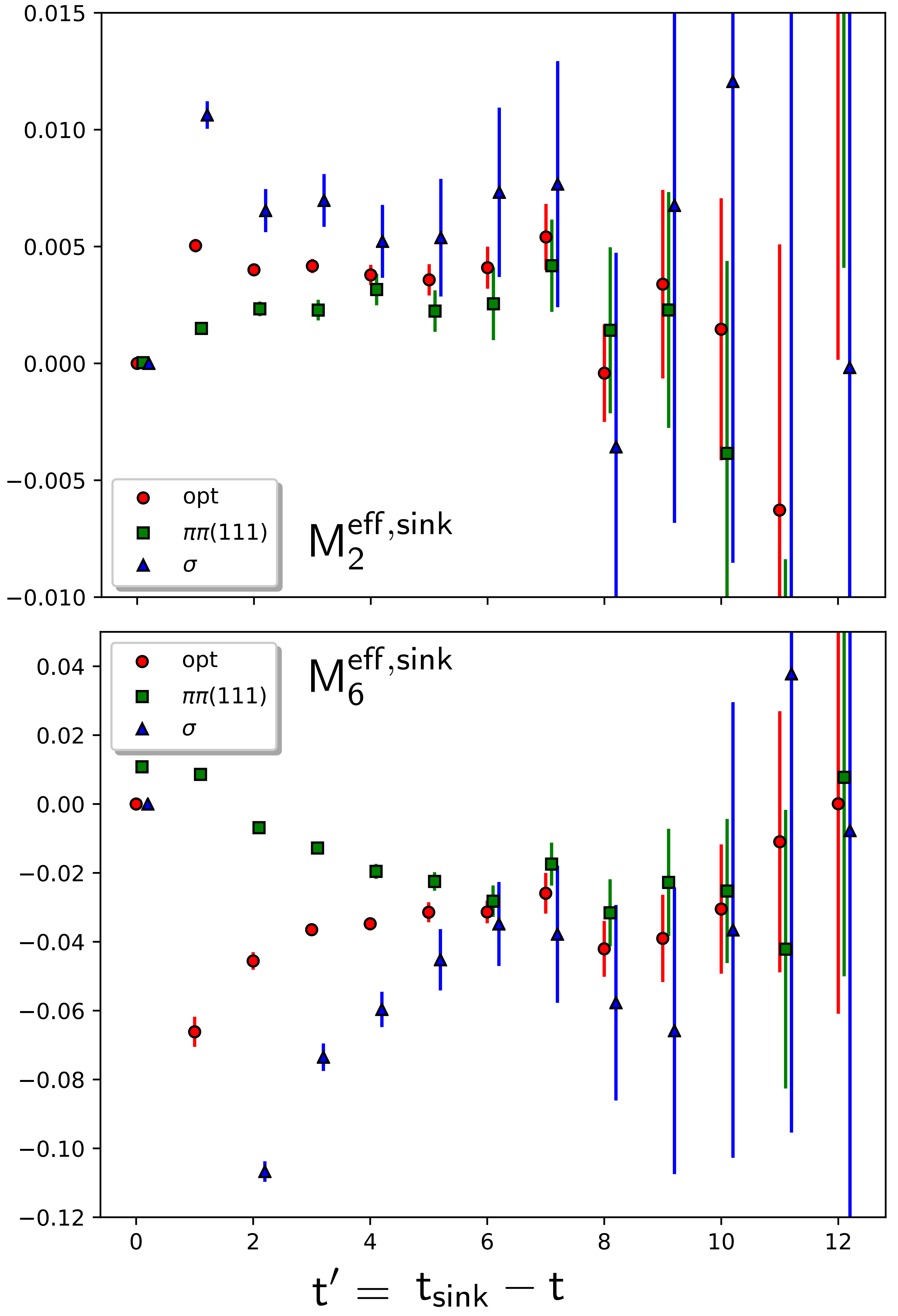
$$O_{\text{sink}} = O_{\pi\pi(1,1,1)},\ \ O_\sigma,\ \ O_{\text{opt}}$$

- Optimal combination of ππ(1,1,1) & σ

$$O_{\text{opt}} = r_1 O_{\pi\pi(1,1,1)} + r_2 O_\sigma$$

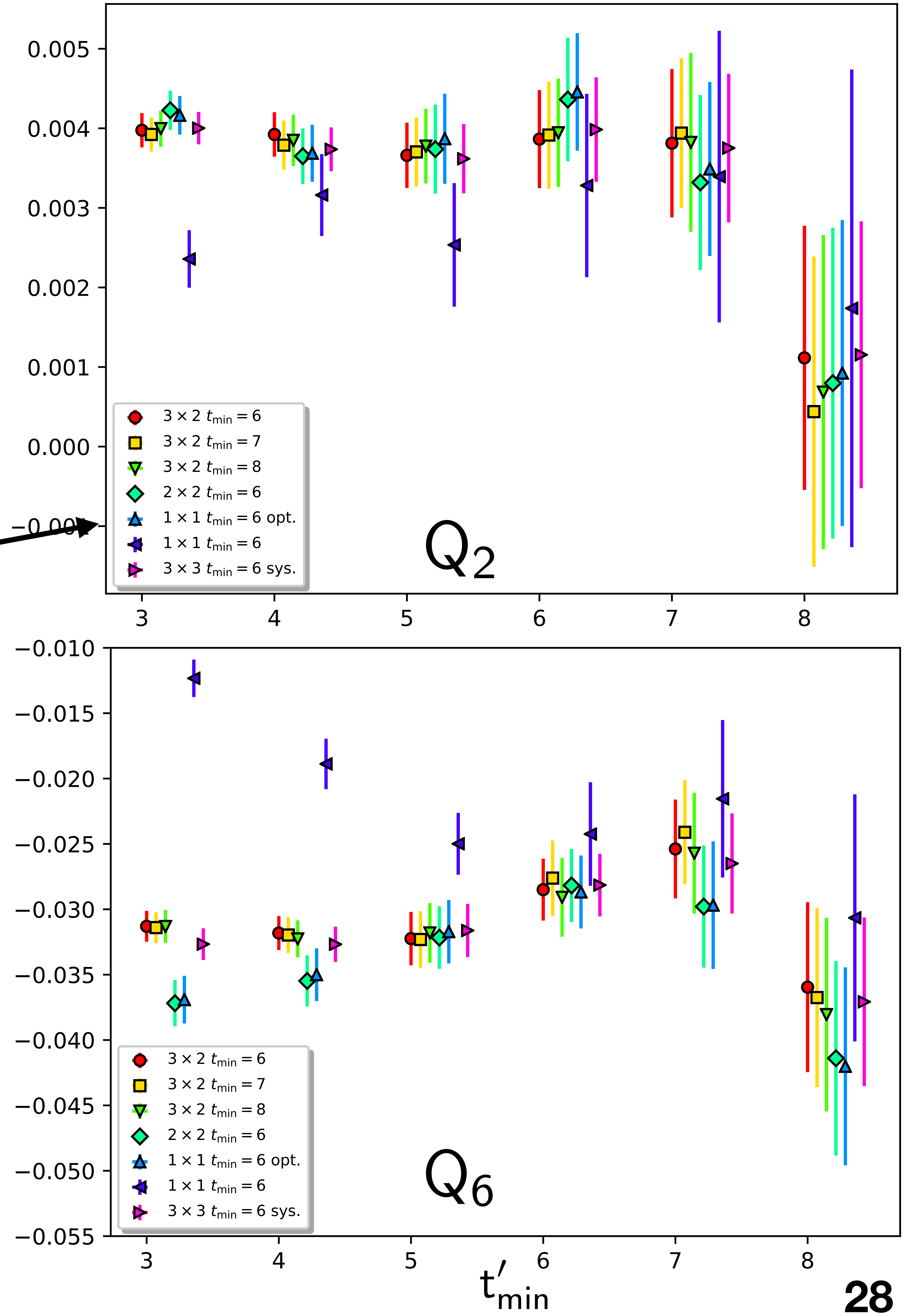
  - r_1 & r_2 determined from ππ 2pt functions
  - Orthogonal to 1st excited state

- Including ππ(3,1,1) → unstable
  - 2-state fit with 2 ππ operators



Fit results

- Various fits
  - t'_{min}: min of (t_{sink} - t) [3-8]
  - t_{min}: min of (t-t_K)   [6-8]
  - (# of operators) x (# of states considered)



- In 2015, effects of excited states were significantly underestimated

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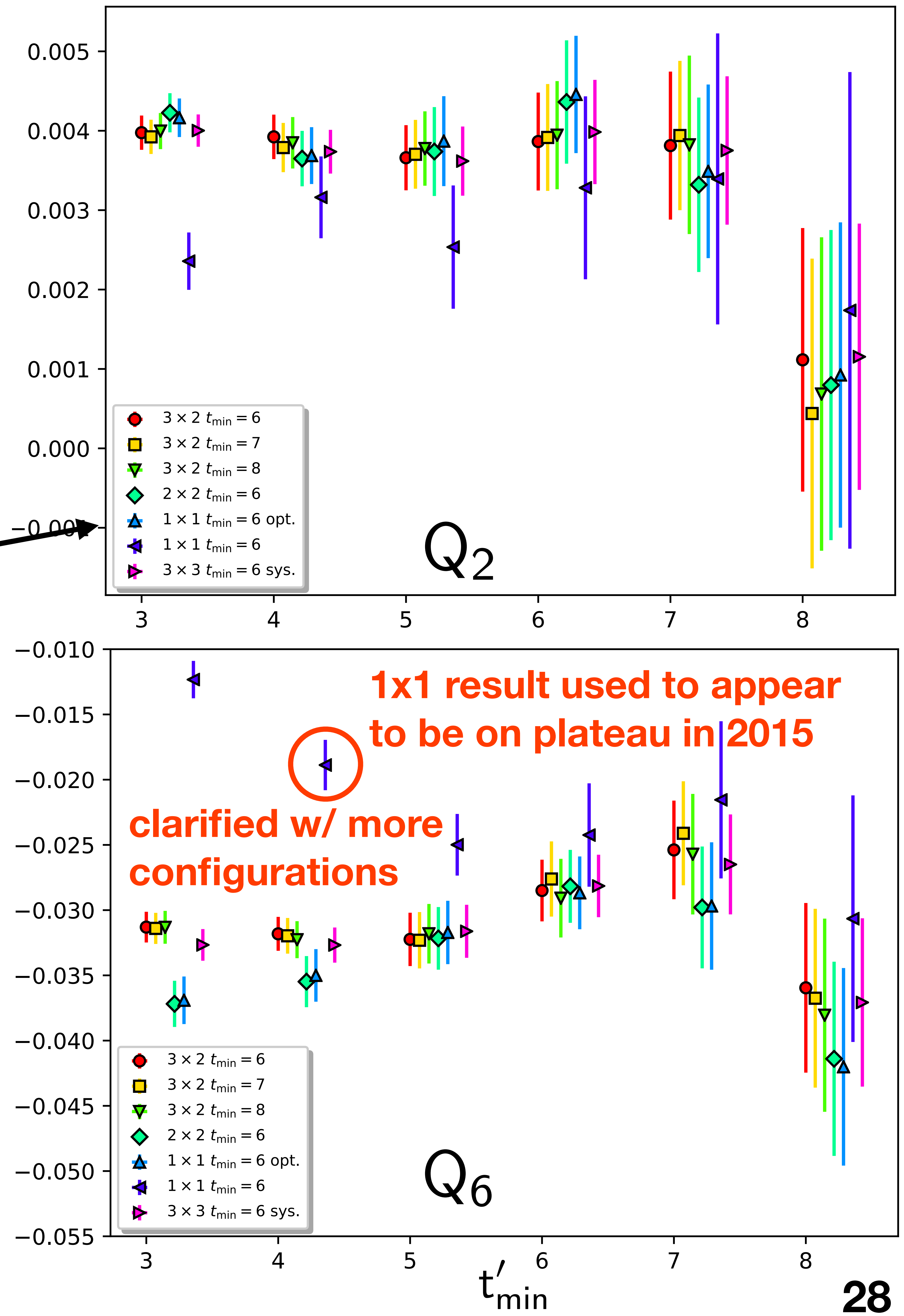
    - (# of operators) x (# of states considered)
- In 2015, effects of excited states were significantly underestimated



1x1 result used to appear to be on plateau in 2015

clarified w/ more configurations

ε’ calculation

- Isospin-limit formula

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i\overset{\text{✓ ππ phase shifts}}{\boxed{\delta_2-\delta_0}}}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2}-\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\} \qquad (\omega = \mathrm{Re}\,A_2/\mathrm{Re}\,A_0)$$

$$A_I = \overset{\text{✓ Lellouch-Lüscher finite volume correction}}{\boxed{F}}\frac{G_F}{2}V_{us}^*V_{ud}\sum_{i,j}\underbrace{[z_i(\mu)+\tau y_i(\mu)]}_{\text{Wilson coefs. pQCD}}\overset{\text{Renormalization matrix}}{\underbrace{Z_{ij}(\mu)}_{\text{LQCD (+pQCD)}}}\underbrace{\langle(\pi\pi)_I|Q_j^{\mathrm{lat}}|K\rangle}_{\text{LQCD ✓}}$$

- G-parity BCs are used to extract on-shell kinematics
- Significant ππ excited states are treated better than in 2015

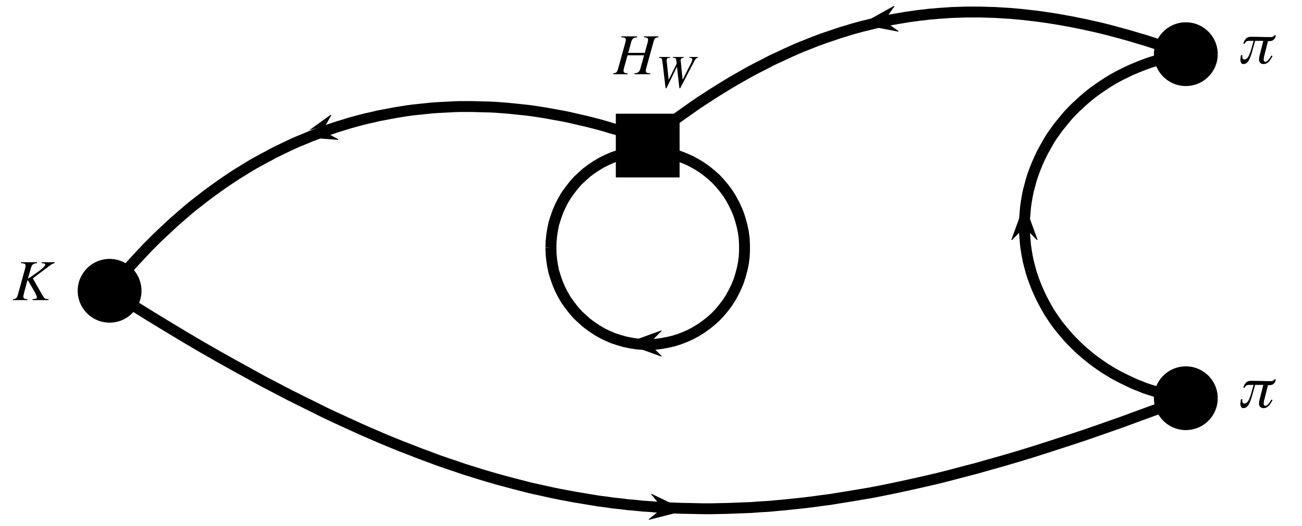
Remaining topic: Renormalization

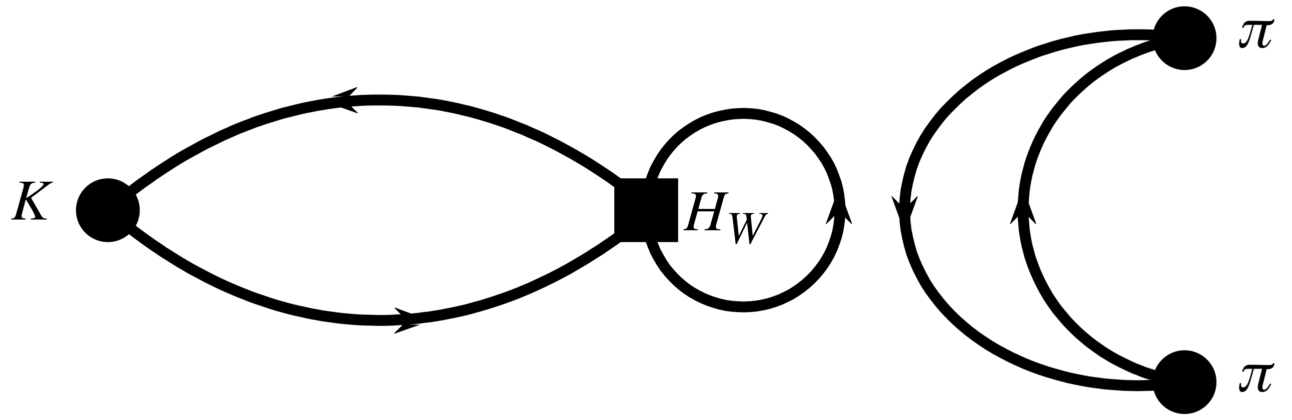
Contents

- ☑ Introduction
- ☑ K → ππ matrix elements
- Operator renormalization
  - RI/SMOM scheme & window problem
  - Step scaling
  - Our final result
- On going projects

Power divergence

- Quadratic divergence (~ a^{-2}) appears in MEs from

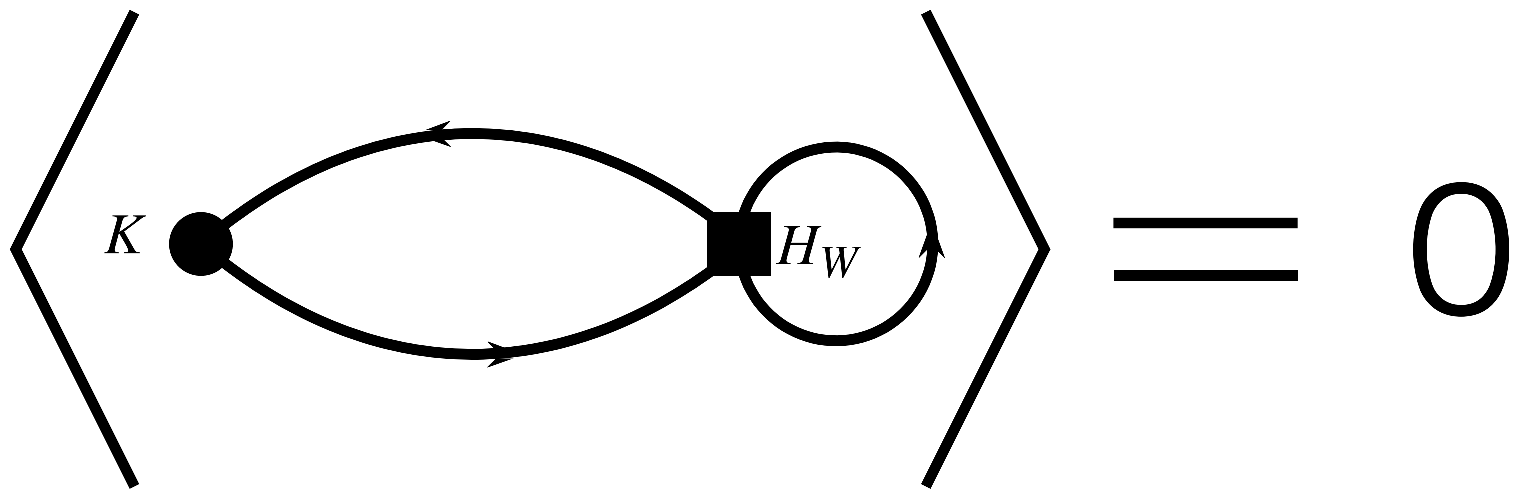




  - due to mixing 4-quark operators with $\mathrm{O}(m/a^2)\bar{s}\gamma_5 d$
  - Remove by subtraction

$$Q_i \to Q_i' = Q_i - \alpha_i \bar{s}\gamma_5 d \quad \text{(mixing w/ parity-even operator } \bar{s}d \text{ is invalid)}$$

Condition: $\langle Q_i'(t_0)K(0)\rangle = 0$ at specific t_0



K → ππ MEs shown earlier are the results after the subtraction

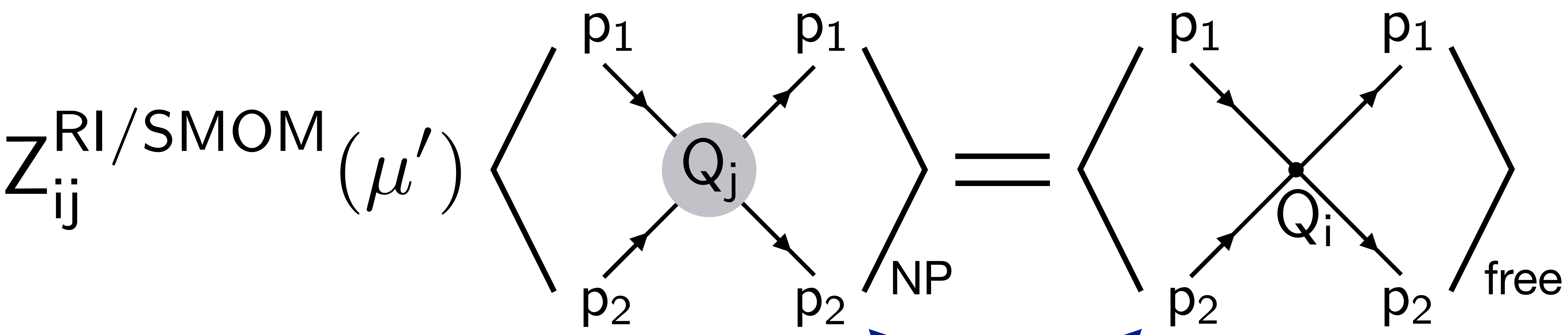
Renormalization

- To remove ln a² divergence
- To construct appropriate Hamiltonian

$$\begin{tikzcd}
w_i^{\overline{\mathrm{MS}}}(\mu) \arrow[r, leftrightarrow, dashed, "\text{perturbative matching}"'] & Z_{ij}^{R}(\mu')Q_j^{\mathrm{lat}} & Q_i^{\mathrm{lat}} \arrow[l, "\text{Renormalization}"']
\end{tikzcd}$$

Wilson coefficients

- RI/SMOM scheme (a common nonperturbative scheme)



$$\mu'^2 = p_1^2 = p_2^2 = (p_1 - p_2)^2$$

Amputated vertex functions

Renormalization

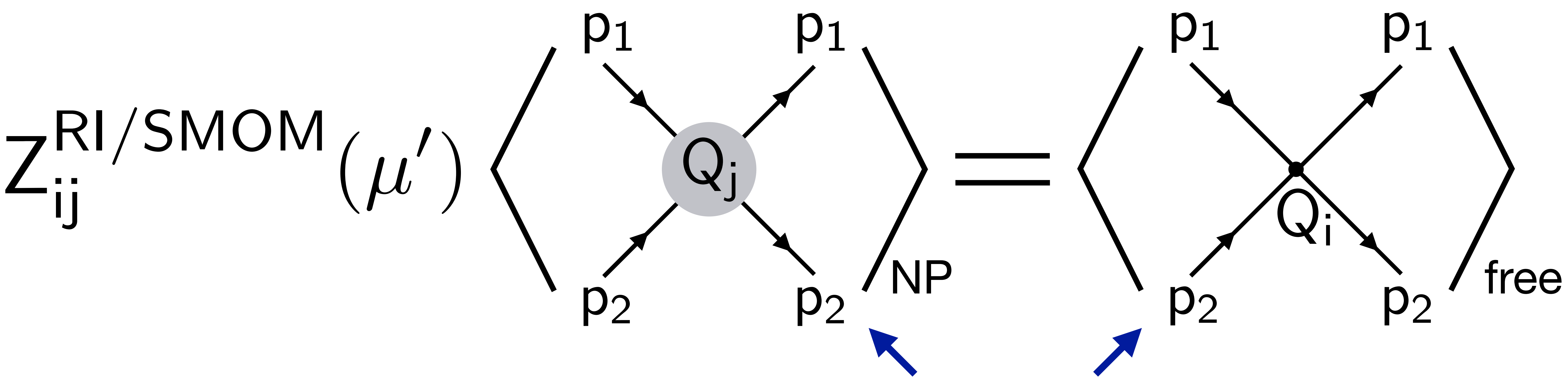
- To remove ln a² divergence
- To construct appropriate Hamiltonian

$$\begin{tikzcd} w_i^{\overline{\mathrm{MS}}}(\mu) \arrow[r, leftrightarrow, dashed, "\text{perturbative matching}"'] & Z_{ij}^{R}(\mu')Q_j^{\mathrm{lat}} & Q_i^{\mathrm{lat}} \arrow[l, "\text{Renormalization}"'] \end{tikzcd}$$

Wilson coefficients

$\mu' \gg \Lambda_{QCD}$ required $\mu' \ll \pi/a$ required **: caused large sys. error in 2015**

- RI/SMOM scheme (a common nonperturbative scheme)



Amputated vertex functions

$$\mu'^2 = p_1^2 = p_2^2 = (p_1 - p_2)^2$$

Step scaling

- Nonperturbative scale evolution technique

fine lattice ensemble created $\left(\mu_{\text{high}} \ll \pi/a_{\text{fine}}\right)$

$$Z(\mu_{\text{high}}, a_{\text{coarse}}) = \left(\frac{Z(\mu_{\text{high}}, a_{\text{fine}})}{Z(\mu_{\text{low}}, a_{\text{fine}})}\right) \underset{\textbf{used in 2015}}{\underline{Z(\mu_{\text{low}}, a_{\text{coarse}})}}$$

$a_{\text{fine}}^{-1} = 3.148(17)\ \text{GeV}$ $a_{\text{coarse}}^{-1} = 1.378(7)\ \text{GeV}$

$\mu_{\text{high}} \simeq 4.0\ \text{GeV}$ $\mu_{\text{low}} \simeq 1.5\ \text{GeV}$

Final result for ε’

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{SM,2015}} = 1.38(5.15)_{\mathrm{stat}}(4.59)_{\mathrm{sys}} \times 10^{-4}$$

$$\boxed{\begin{aligned} \mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{SM}} &= \mathrm{Re}\left\{\frac{\mathrm{i}\omega e^{\mathrm{i}(\delta_2-\delta_0)}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\} \\ &= 21.7(2.6)_{\mathrm{stat}}(6.2)_{\mathrm{sys}}(5.0)_{\mathrm{EM/IB}} \times 10^{-4} \end{aligned}}$$

🤝

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{exp}} = 16.6(2.3) \times 10^{-4}$$

- ΔI = 1/2 rule also consistent

$(\mathrm{Re}\,A_0/\mathrm{Re}\,A_2)_{\mathrm{SM}} = 19.9(2.3)(4.4)$ 🤝 $(\mathrm{Re}\,A_0/\mathrm{Re}\,A_2)_{\mathrm{exp}} = 22.45(6)$

Breakdown of sys. errors on A_0

Description	2015 Error	2020 Error
Operator normalisation	15%	5%[1]
Wilson coefficients	12%	unchanged
Finite lattice spacing	12%	unchanged
Lellouch - Lüscher factor	11%	1.5%[2]
Residual FV corrections	7%	unchanged
Parametric errors	5%	6%[3]
Excited state contamination	5%	negligible[4]
Unphysical kinematics	3%	5%
Total	**27%**	**21%**

- [1] As a result of step scaling from $\mu = 1.53$ GeV $\rightarrow$ 4.00 GeV.
- [2] Better control of $\pi\pi$ system due to additional operators.
- [3] Largest uncertainty is due to $\tau \sim 5\%$.
- [4] Significantly underestimated in 2015.

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NP treatment on going

finer lattice & continuum limit

- [1] As a result of step scaling from $\mu = 1.53$ GeV → 4.00 GeV.
- [2] Better control of $\pi\pi$ system due to additional operators.
- [3] Largest uncertainty is due to $\tau \sim 5\%$.
- [4] Significantly underestimated in 2015.

Why changed so much?

- Ultimately misestimation of excited-state contaminations

2015: 12,3° from real value
2020, exp: mostly real value

→ Doesn't change too much

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{\frac{\boxed{i\omega e^{i(\delta_2-\delta_0)}}}{\sqrt{2}\epsilon}\underline{\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2}-\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]}\right\}$$

2015: accidental cancellation by order
2020: A0 part increased by x3.5

→ Caused more than 10x difference

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{SM}} = 1.38(5.15)_{\mathrm{stat}}(4.59)_{\mathrm{sys}} \times 10^{-4} \rightarrow 21.7(2.6)_{\mathrm{stat}}(6.2)_{\mathrm{sys}}(5.0)_{\mathrm{EM/IB}} \times 10^{-4}$$

Contents

- ☑ Introduction
- ☑ K → ππ matrix elements
- ☑ Operator renormalization
- On going projects
  - K → ππ in periodic boundary conditions
  - New contraction strategy
  - NP matching of Wilson coefficients from 4 → 3 flavor theory
  - Finer G-parity lattices
  - Introducing QED & IB effects

Why periodic BCs?

- Already have lattice ensembles with physical pion mass
  - 1 GeV, 24^3 x 64, 1.4 GeV, 32^3x64 and …
  - Continuum limit possible

- Hope to introduce QED/IB effects near future
  - Difficult with G-parity boundary conditions
  - Periodic BC study valuable

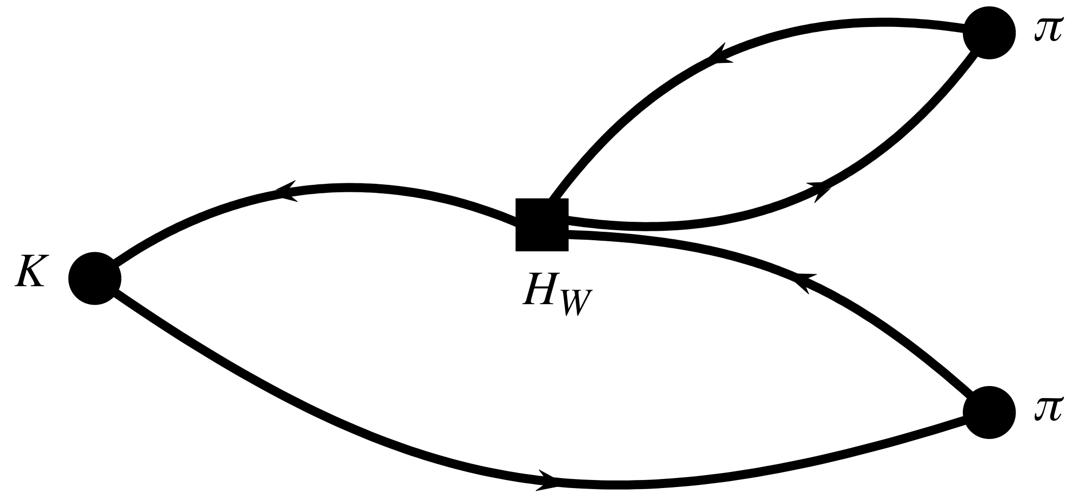
- Presence of $E_{\pi\pi} = 2m_\pi$ state challenging
  - S/N ratio of $E_{\pi\pi} = m_K$ state should be the same as G-parity BC

type 4 dominates stats. error

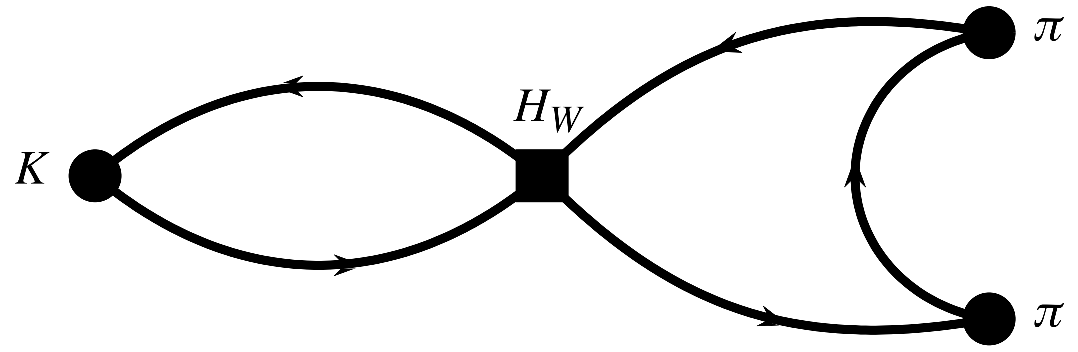
- G-parity calculation
  - types 1,2: averaged over every 8 time translations
  - types 3,4: averaged over every time translation

- types 1,2 still expensive but no need of such precision

  → cost reduction?



type 1



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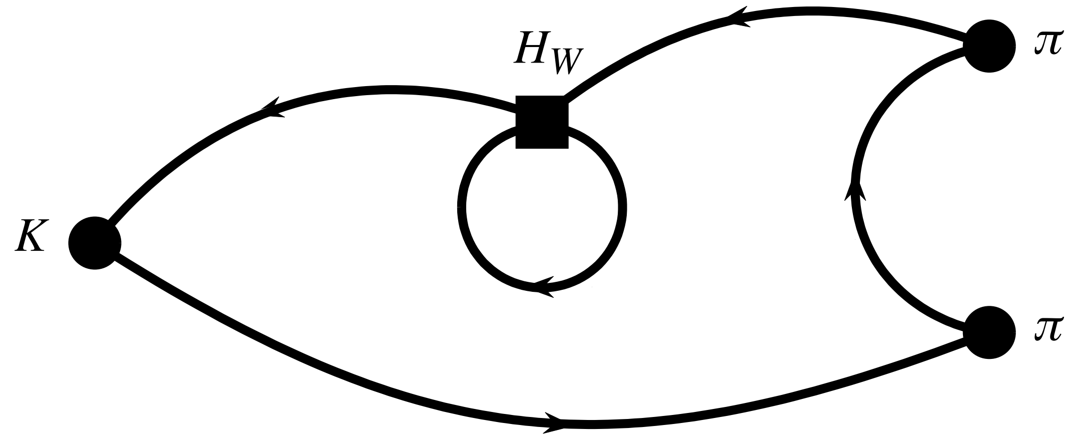

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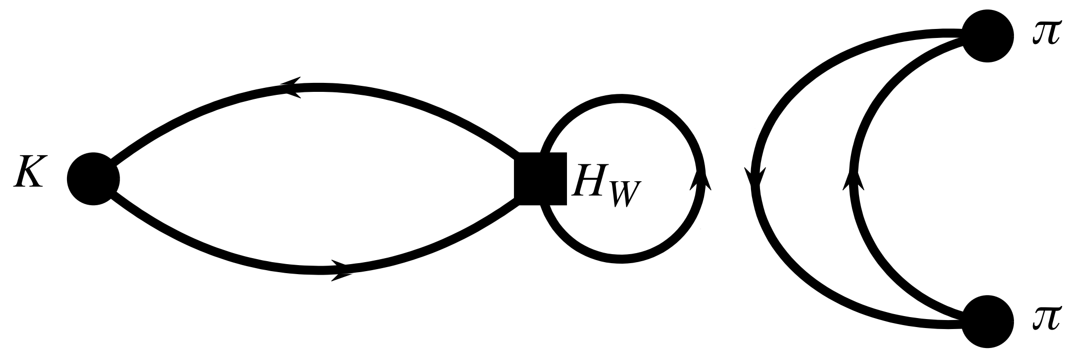

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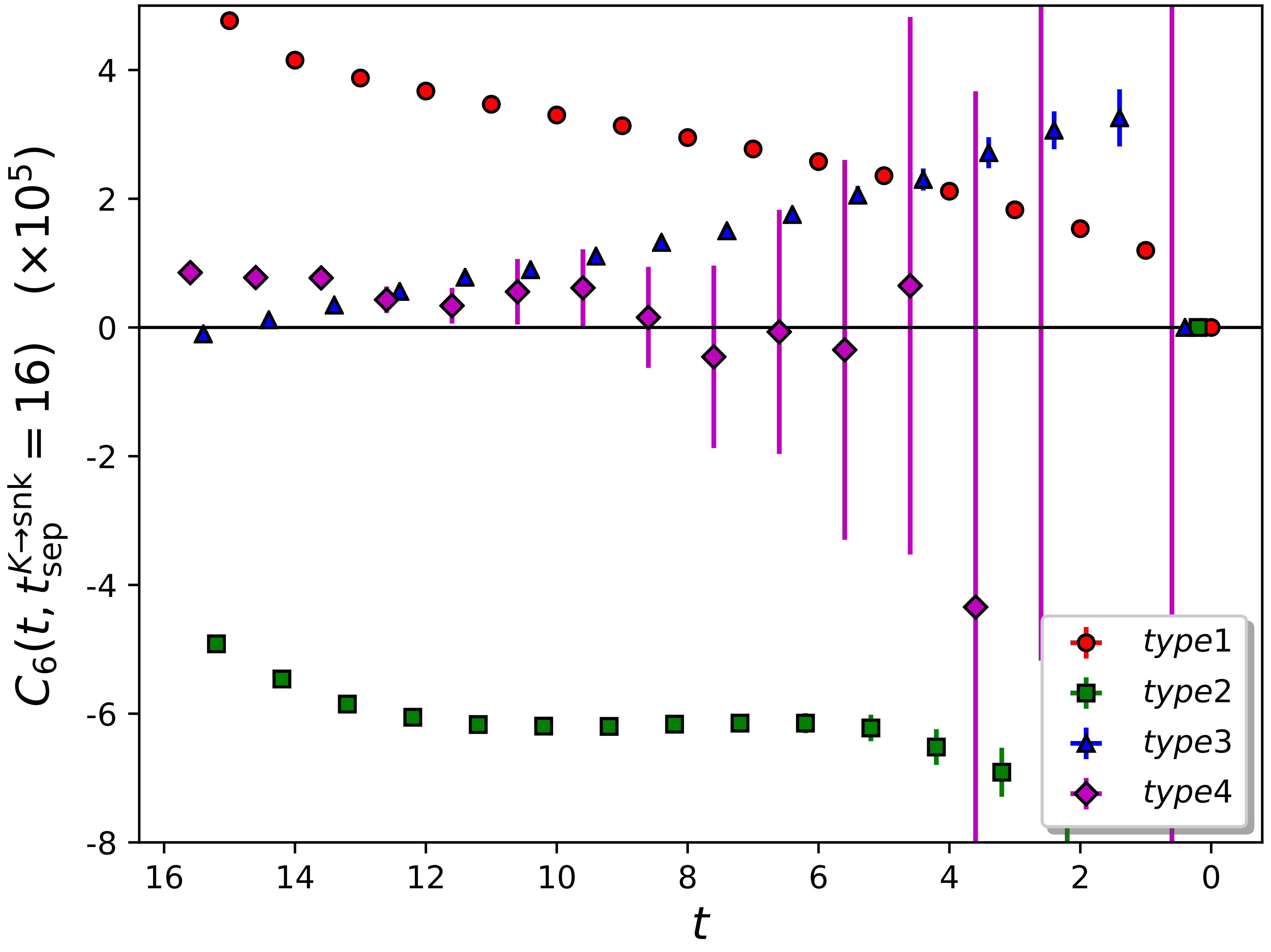
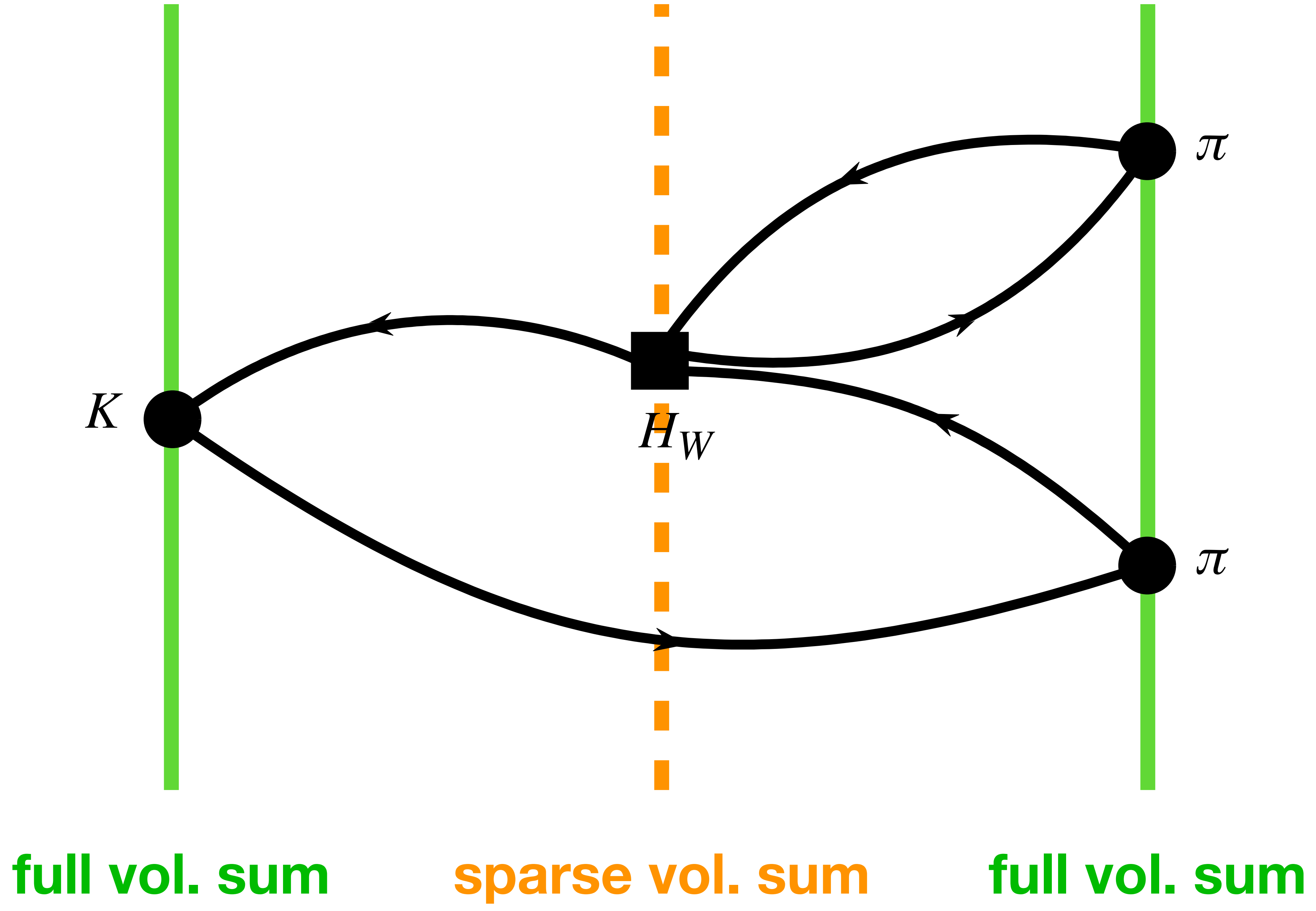


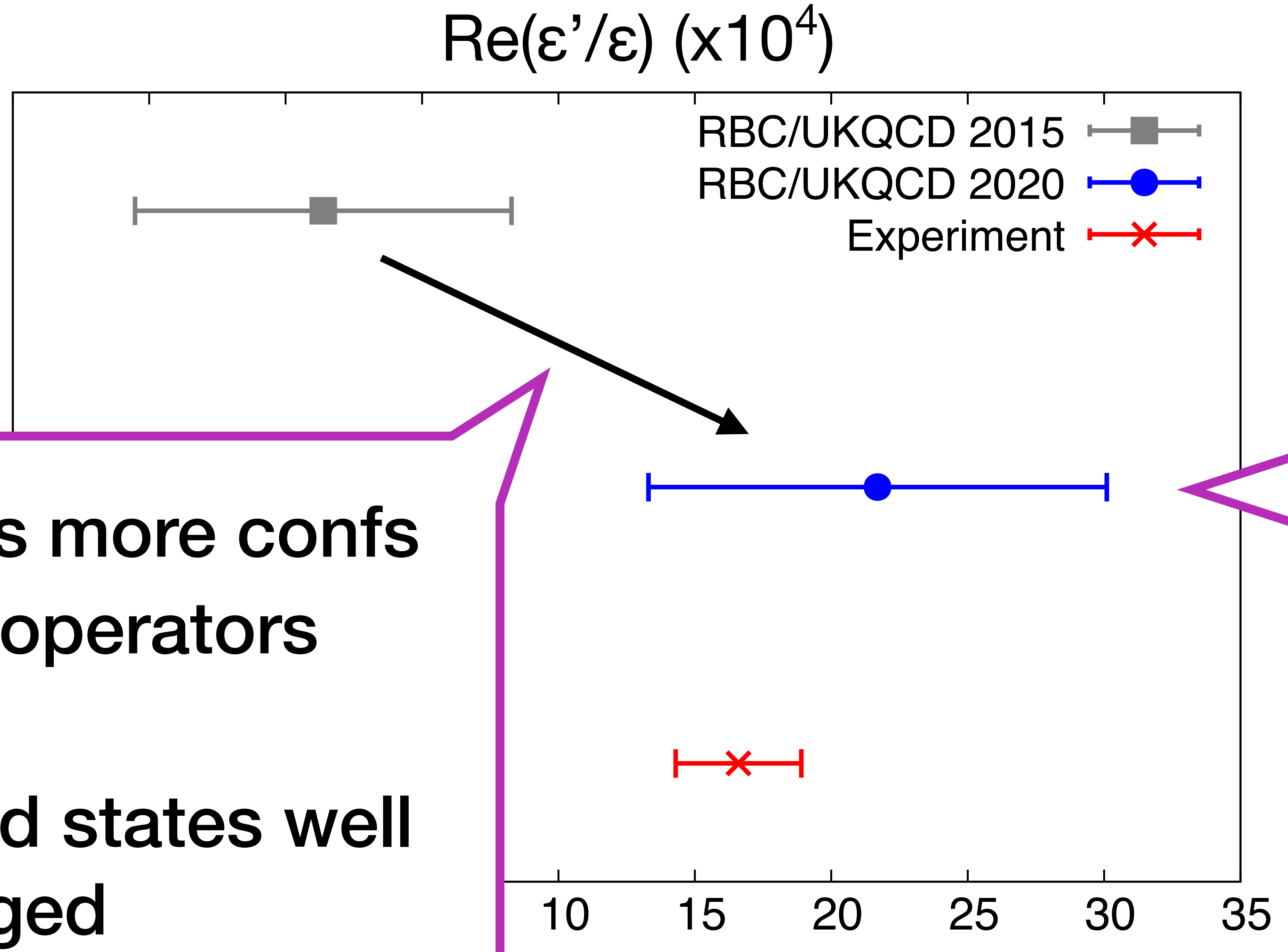
RBC/UKQCD, PRD 102,054509

Sparsening H_W

- Cost mostly promotional to volume of H_W
- G-parity calculation: summed H_W over whole 3D volume
- Plan for this time: reduce the volume of H_W (32^3 → 8^3: 64x speed up) for types 1 & 2



Summary



- 3+ times more confs
- # of ππ operators
  - 1 → 3
  - excited states well managed
- Step scaling in NPR

$$21.7(2.6)_{\mathrm{stat}}(6.2)_{\mathrm{sys}}(5.0)_{\mathrm{EM/IB}} \times 10^{-4}$$

- More independent calculations desired
- Systematic error
  - Isospin breaking effects
  - Truncation error of Wilson coefficients
  - Finite lattice cutoff

RBC/UKQCD working hard to figure out these & conclude K → ππ story