$B \to D^* \ell \nu$ decays on the lattice

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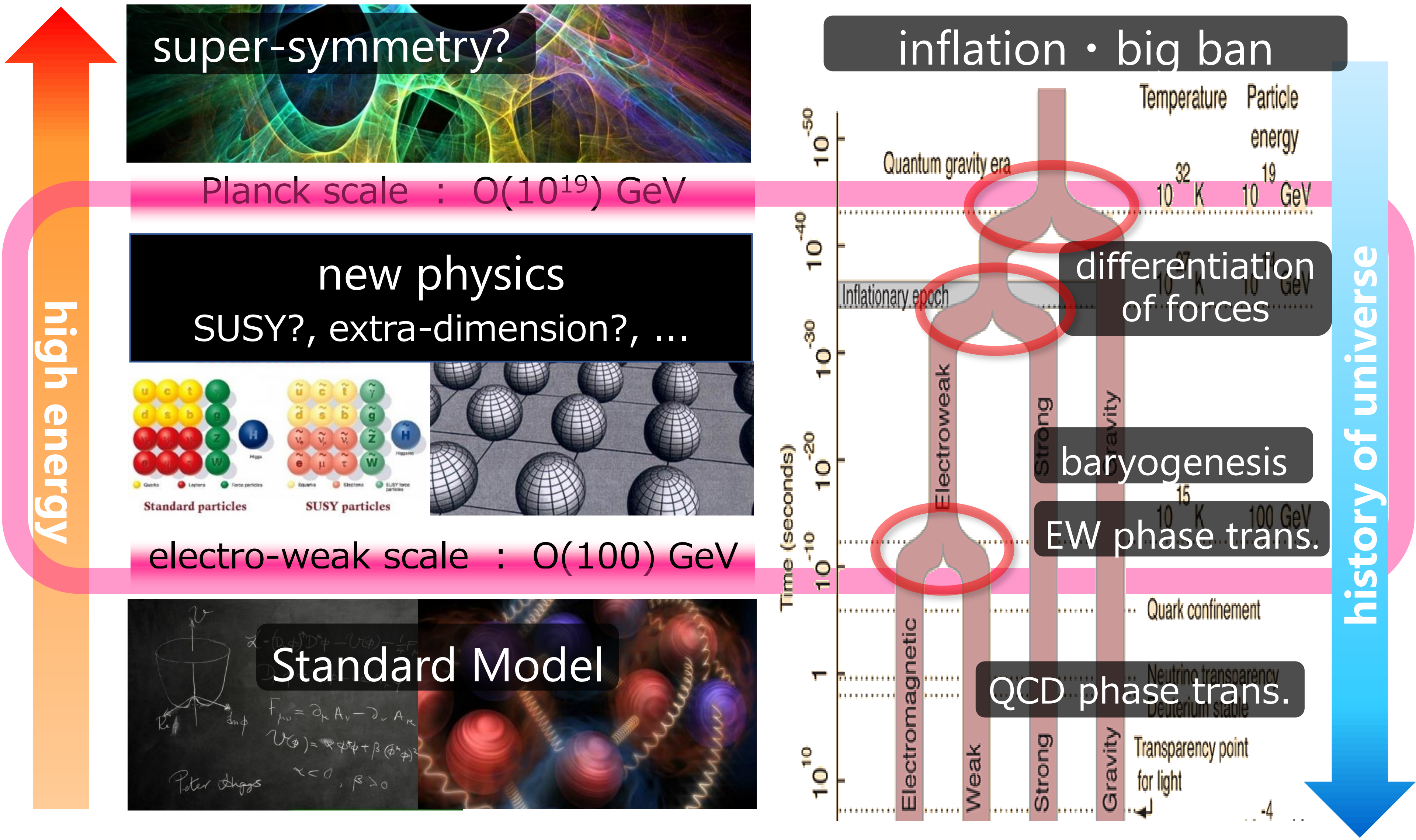
for JLQCD Collaboration

seminar @ RIKEN R-CCS, February 17, 2021

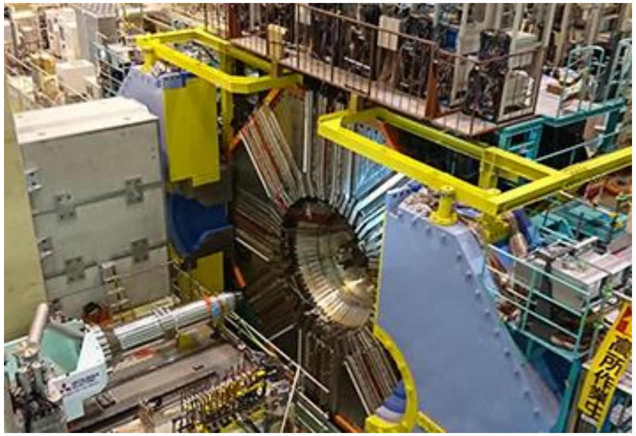
principles and history of universe



search for "new physics" beyond SM

intensity frontier for new physics

experiments vs standard model ⇒ new physics



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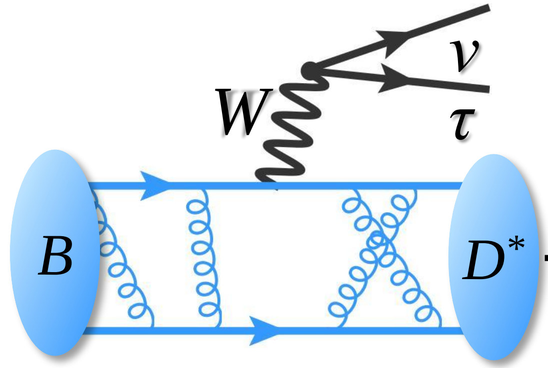
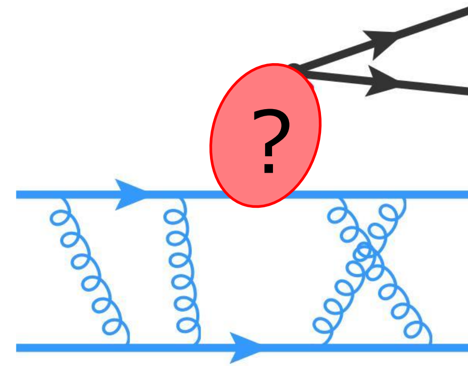


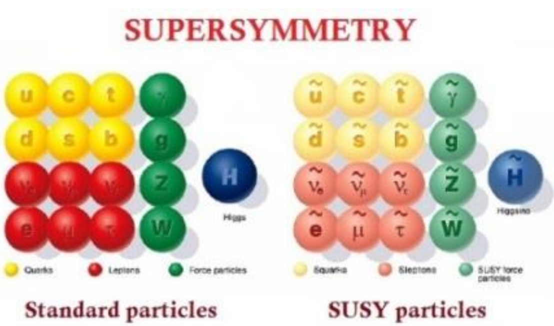
standard model

\+

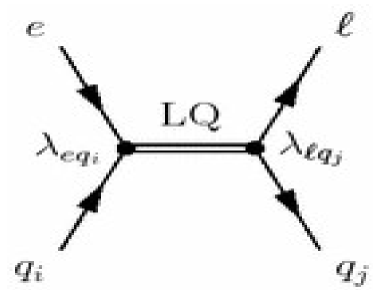


new physics

charged Higgs ?



lepto-quark ?



...

- Higgs sector ⇒ EW symmetry breaking
- grand unification
- ...

Outline

form factors of $B \to D^* \ell \nu$ decays from lattice QCD

- $B \to D^* \ell \nu$ decays and form factors
  - background: flavor physics, "CKM" element $|V_{cb}|$, new physics
  - form factors: definition, phenomenology
- JLQCD 's simulation
  - simulation setup ... extrapolation to the real world
- for determination of $|V_{cb}|$
  - recent determinations
  - indication of our lattice results

please ask questions any time

1. $B \to D^* \ell \nu$ decays and form factors

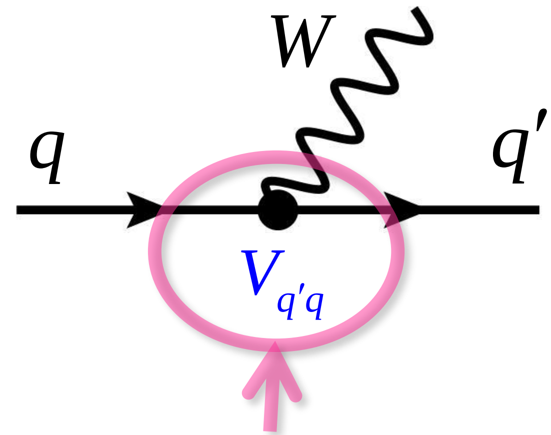
flavor physics and CKM matrix

flavor physics

- processes w/ quark flavor change(s)
- Cabbibo-Kobayashi-Maskawa mechanism
- elements of 3x3 CKM matrix

  ⇒ strength of flavor changes

  ⇒ amplitude of processes

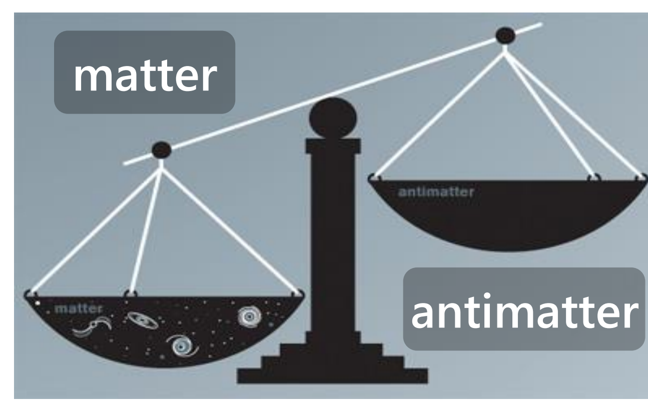


$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

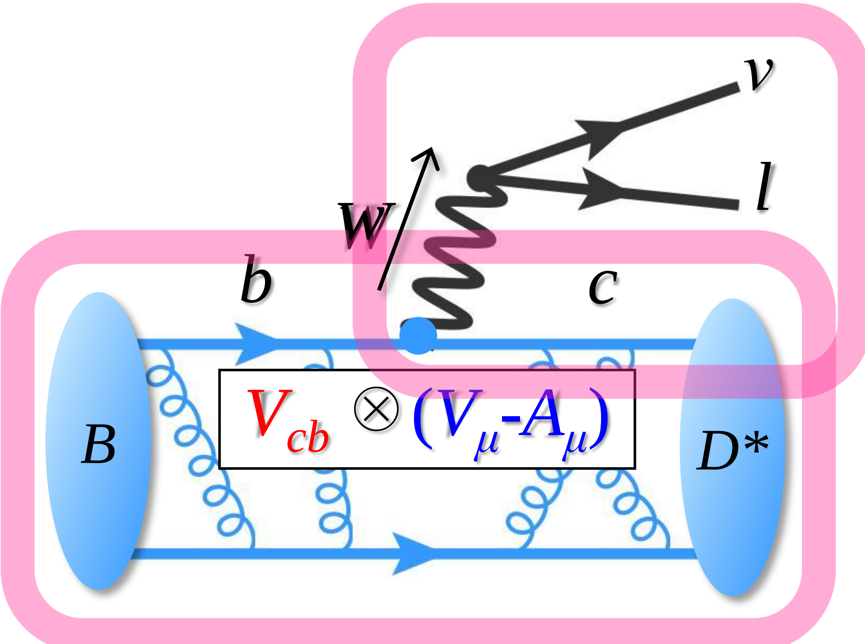
missing anti-matter

- a condition : violation of “CP” symmetry

  C : particle ⇔ anti-particle; P : parity trans.
- in SM : CKM ~~and QCD θ-term~~ nEDM
- CKM is not sufficient to explain →
- other sources (new physics) ???



$B \to D^* \ell \nu$ semileptonic decay



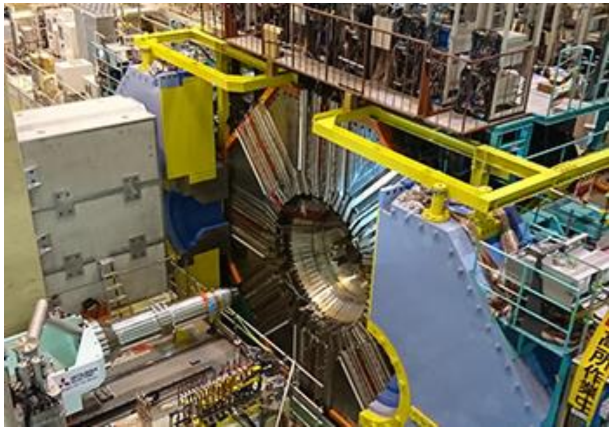
- mediated by weak interaction
  $b \to c\ W(\to \ell \nu) \Rightarrow$ pick up $|V_{cb}|$
- W+lepton part : perturbation
- hadronic QCD effects ⇒ lattice
- recoil : $(v-v')^2 = 2 + w$, $w = vv'$

$$\mathcal{M}\left(B \to D^* \ell \nu\right) = V_{cb} \left\langle \ell \nu \middle| V_\mu - A_\mu \middle| 0 \right\rangle G_W \left\langle D^* \middle| V_\mu - A_\mu \middle| B \right\rangle$$

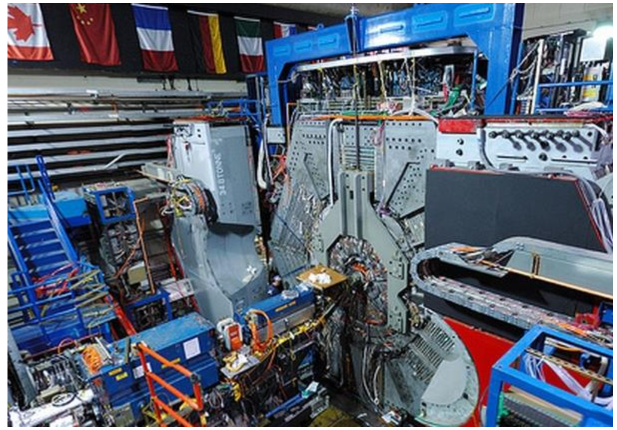
experiment — theory

- lepton+neutrino+W+hadron parts ⇒ determination of $|V_{cb}|$
- $|V_{cb}|$ from another decay, ratio ⇒ search for new physics

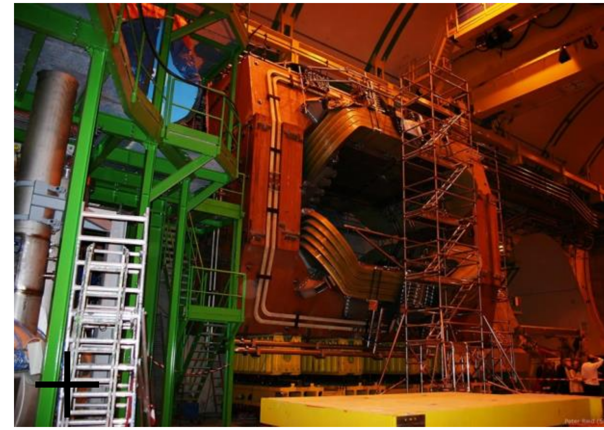
experiments



KEKB / Belle @KEK (-2010)



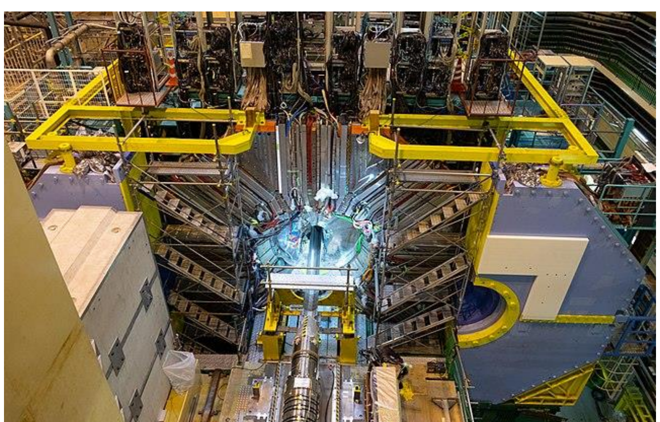
BaBar @ SLAC (-2008)



LHCb @ CERN Run3 2023-

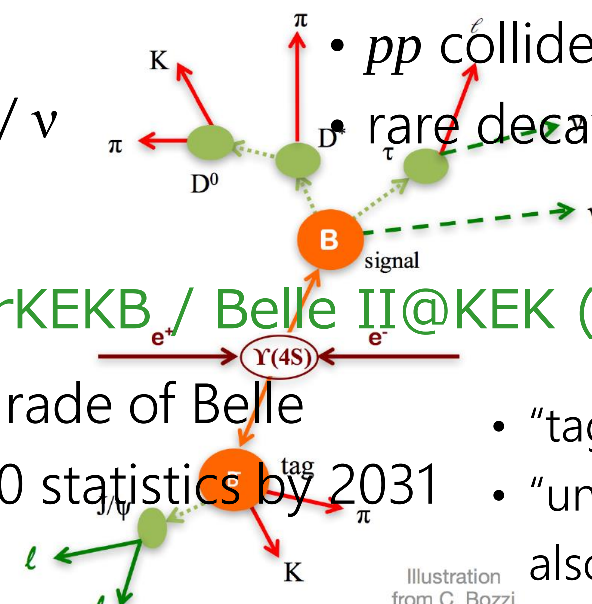
- $e^+e^- \to O(10^9)$ pairs of BB, B^+B^-
- good at semileptonic decays w/ ν

- pp collider
- rare decays



SuperKEKB / Belle II@KEK (2019-)

- upgrade of Belle
- × 50 statistics by 2031

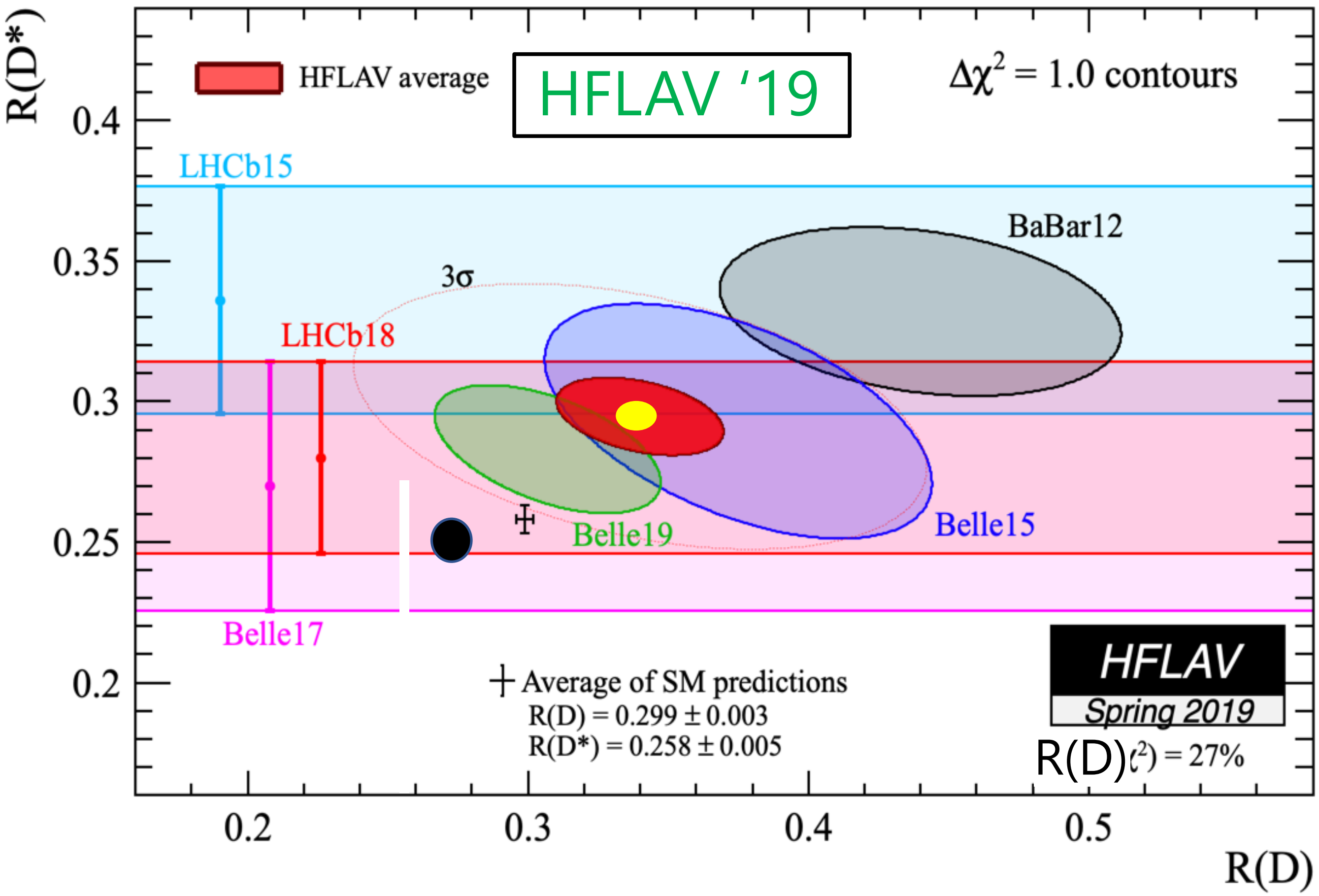


- "tagged" data
- "untagged" is also possible

Illustration from C. Bozzi

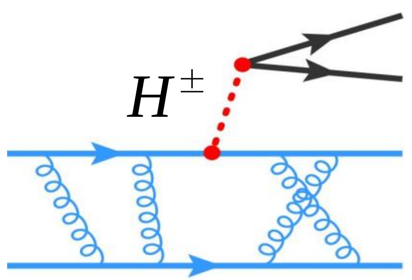
promising probe of new physics

$$R\left(D^{(*)}\right)=\Gamma\left(B\to D^{(*)}\tau\nu\right)/\Gamma\left(B\to D^{(*)}\{e,\mu\}\nu\right)$$



- 3.4 σ from SM
- τ channel via $b \to c\tau\nu$

  ⇒ charged Higgs?

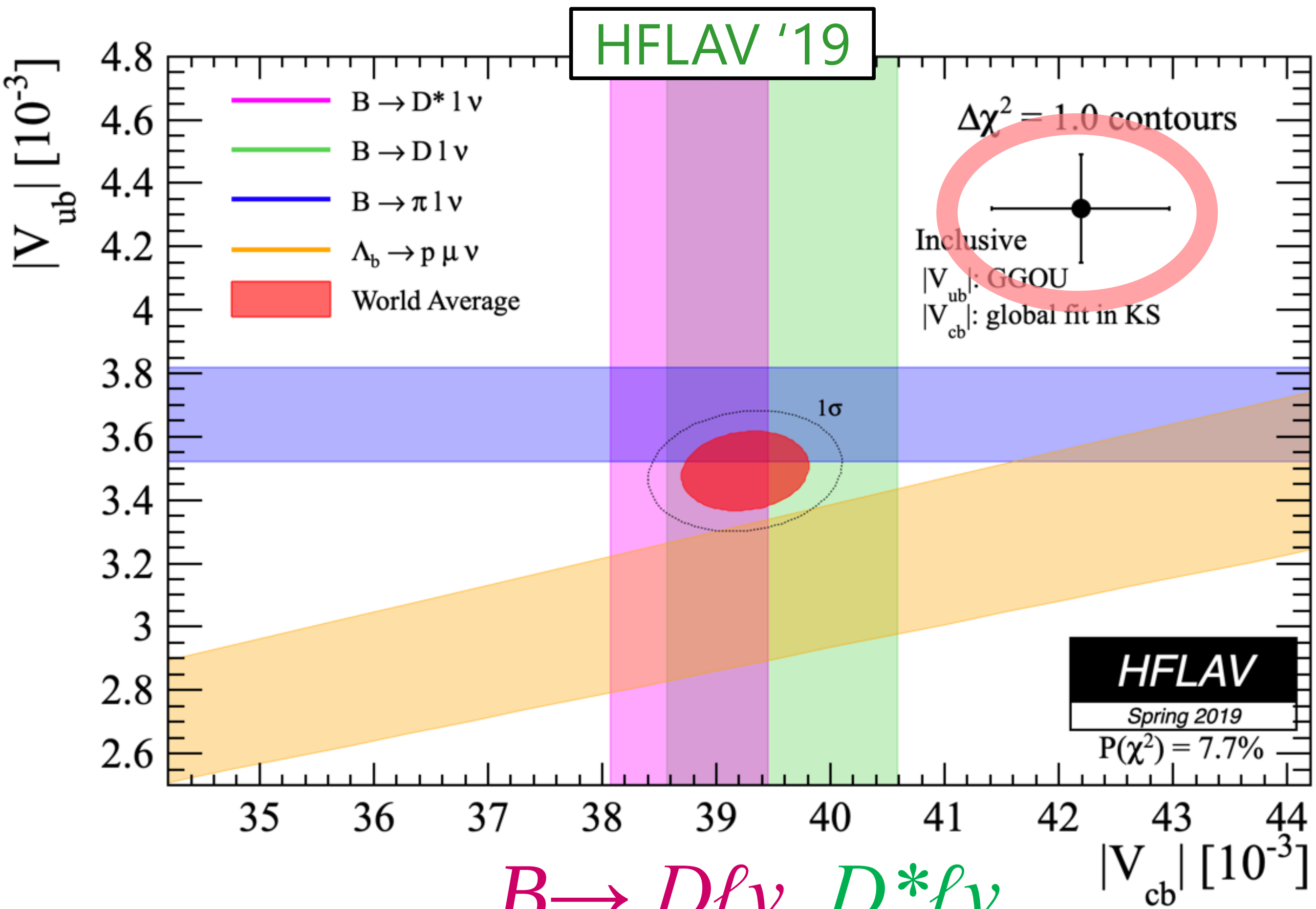


- inconsistent exp's?

Belle II : 3 σ tension ⇒ 10 σ evidence !?

determination of CKM element $|V_{cb}|$

long-standing discrepancy (6-15%)



inclusive decay

= sum up over final state hadron(s)

$B \to X_u \ell \nu$

$= \pi \ell \nu + \rho \ell \nu + \ldots$

exclusive decay

$B \to \pi \ell \nu$

$B \to D \ell \nu$, $D^* \ell \nu$

to be revolved for precision new physics search

hadronic matrix elements

"relativistic" convention

$$\langle D^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle = ig\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p'_\beta p_\gamma,$$

$$\langle D^*(p',\epsilon)|A^\mu|\bar{B}(p)\rangle = f\epsilon^{*\mu} + (\epsilon^*\cdot p)\left[a_+(p+p')^\mu + a_-(p-p')^\mu\right]$$

"heavy quark" convention $\left(\sqrt{M_B}|B\rangle \to |B\rangle\right)$

$$\langle D^*(v',\varepsilon')|V^\mu|\bar{B}(v)\rangle = ih_V(w)\,\epsilon^{\mu\nu\alpha\beta}\,\varepsilon'^*_\nu v'_\alpha v_\beta$$

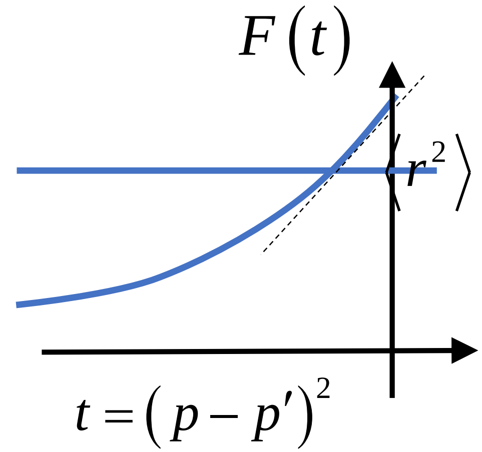
$$\langle D^*(v',\varepsilon')|A^\mu|\bar{B}(v)\rangle = h_{A_1}(w)\,(w+1)\,\varepsilon'^{*\mu} - \left[h_{A_2}(w)\,v^\mu + h_{A_3}(w)\,v'^\mu\right]\varepsilon^*\cdot v$$

- 4 form factors (FFs) w/ $\varepsilon_{D^*}p_{D^*} = 0$
- function of momentum transfer / recoil parameter
- $q^2 = t = (p - p')^2$, $q^2 \le (M_B\text{-}M_{D^*})^2 = q^2_{\rm max} = t_-$ $(\mathbf{p}=\mathbf{p}'=\mathbf{0})$
- $v = p/M_B$, $w = v\,v' = (M_B{}^2+M_{D^*}{}^2\text{-}q^2)/2M_BM_{D^*}$, $1 \le w$

"form" factors (FFs)

Lorentz invariant function of Lorentz invariant variable describing MEs

cf. pion electro-magnetic form factor



- form factor of $\pi \to \pi\gamma$
- shape w.r.t charge distribution probed by γ
  - point-like particle $\Rightarrow$ t-indep. constant
  - slope = charge radius

$B \to D^* \ell \nu$ form factors

- shape of B probed by W boson
- shape of QCD bound state $\Rightarrow$ essentially non-perturbative

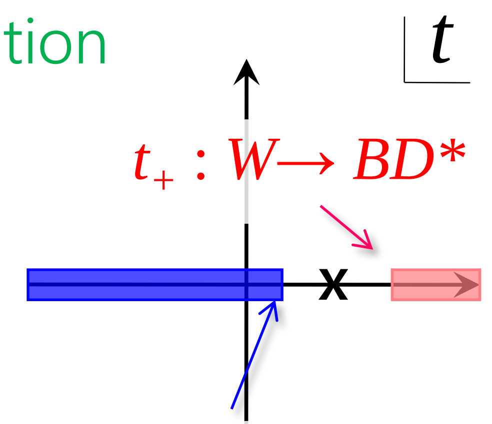
how can we describe FFs analytically?

recoil parameter dependence

small parameter expansions

zero recoil expansion in $(w - 1)$

- heavy quark convention, near $w = 1$, interpolation

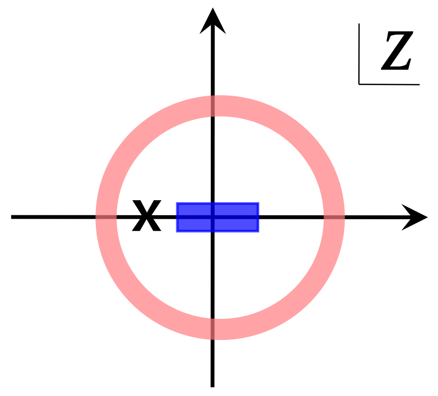


z-parameter expansion

- map semileptonic region to short segment
- factor out singularity due to resonances

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$F(t) = \frac{1}{P(t)\phi(t;t_0)} \sum_{n=0}^{\infty} a_n\, z(t;t_0)^n \quad P(t) = t - M^2_{\rm reso}$$



- model-independent ⇒ phenomenology

quark mass dependences

heavy quark effective theory (HQET)

- based on heavy quark symmetry w.r.t heavy hadron's flavor and spin
- FFs: $1/m_Q{}^n$ expansion around the $m_Q = \infty$ limit
- NLO is known but with a set of unknown parameters (Isgur-Wise func.)
- unknown higher order corrections MAY be small for FF rarios h_X / h_Y

chiral perturbation theory (ChPT)

- based on spontaneous breaking of chiral symmetry w.r.t. light flavors
- non-analytic correction "chiral logarithm" $\ln[M_\pi{}^2/\mu^2]$
- NLO but with a set of unknown parameters (LECs)
- heavy quark symmetry $\Rightarrow$ suppressed by D^*-D mass splitting

w/ unknown param.s but useful in LQCD

2. JLQCD collabortion's study

lattice QCD

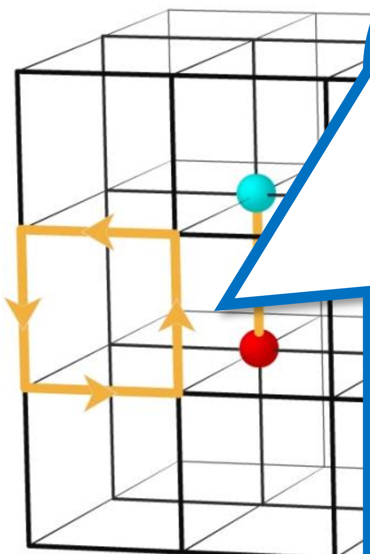
numerical and first-principles calculation

massively paralle

e.g. on Oakfores

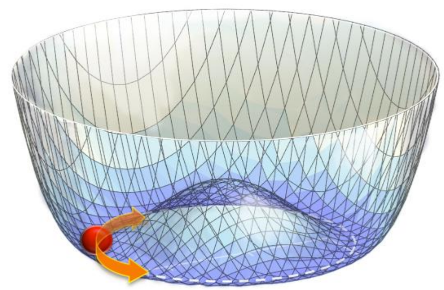
HPCI: hp170106,

QCD on a space-ti

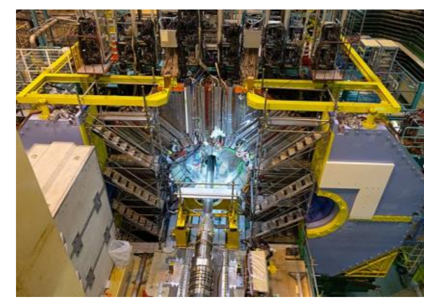


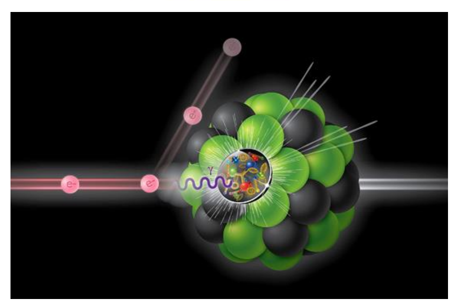
wide applications

nature of QCD

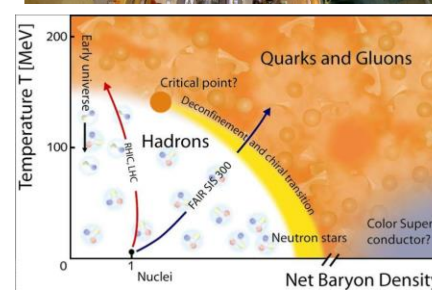


new physics search





nuclear physics



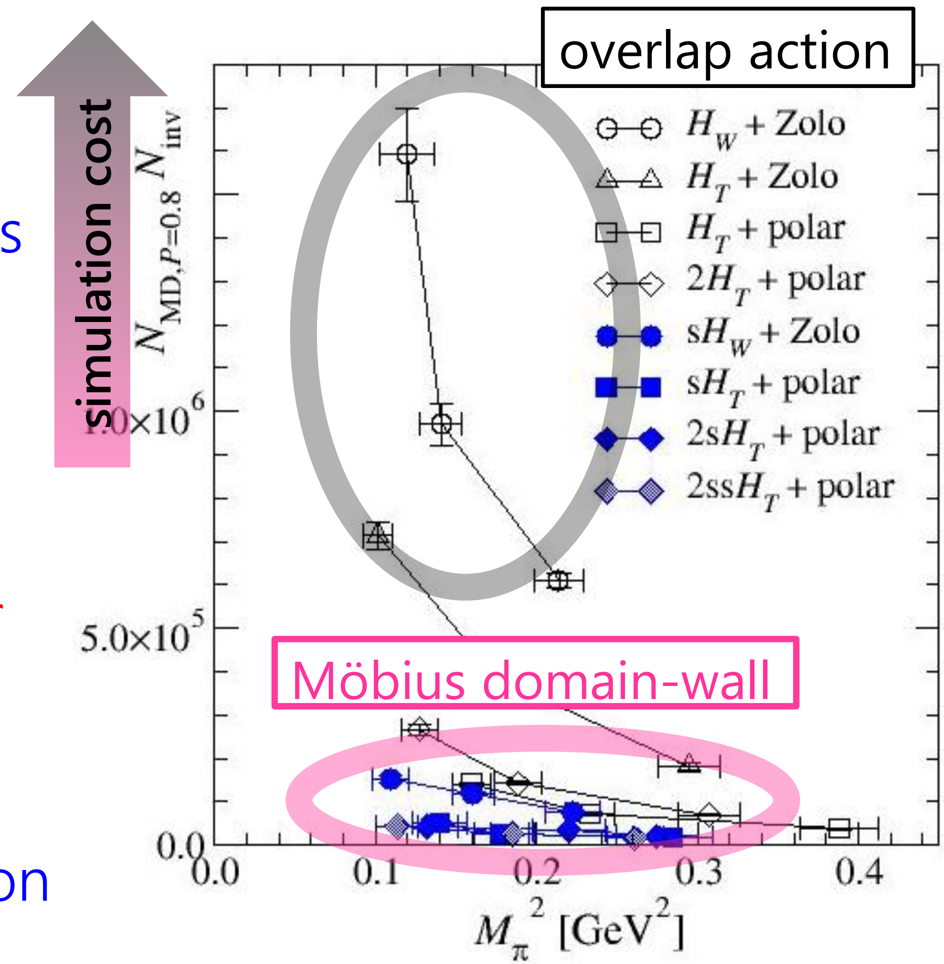
phase structure

JLQCD's simulation of QCD ①

preserving chiral symmetry

chiral symmetry

- spontaneous breaking:
  characterize low-energy dynamics
  ⇔ Wilson types : explicitly violated
  ⇔ staggered types : partly, ~~locality~~



for heavy quark physics

- forbid leading discretization error
- simplified renormalization
  - lattice : removed ∞'s
  - lattice ⇔ MS$^{\text{bar}}$: finite correction
- ChPT to describe m_q dependence

computationally demanding ⇒ speed up x 20

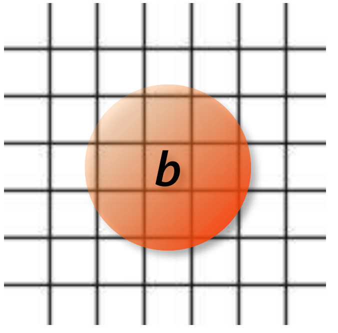
JLQCD's simulation of QCD ②

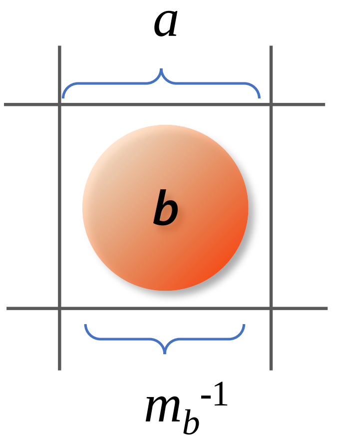
relativistic QCD

relativistic QCD

- QCD as it is
- lattice fine enough for m_b?
  - Compton wavelength $1/m_b$
  - discretization error as $O((am_b)^n)$

  $\Rightarrow$ unphysically small m_b + extrapolation to $m_{b,\text{phys}}$





effective theory (ET) approach

- NRQCD / HQET action for heavy quarks
- can simulate $m_{b,\text{phys}}$
- need matching : parameter tuning of ET to reproduce QCD

independent calculations w/ different systematics

previous and on-going studies

only limited number of studies

published studies

- Fermilab / MILC '14 staggered light quarks + ET-based heavy quarks
- HPQCD '17 staggered light quarks + relativistic heavy quarks
  - only h_{A1} @ w=1 ⇔ four FFs h_{A1}, h_{A2}, h_{A3}, h_V

need to calculate all FFs

on-going studies

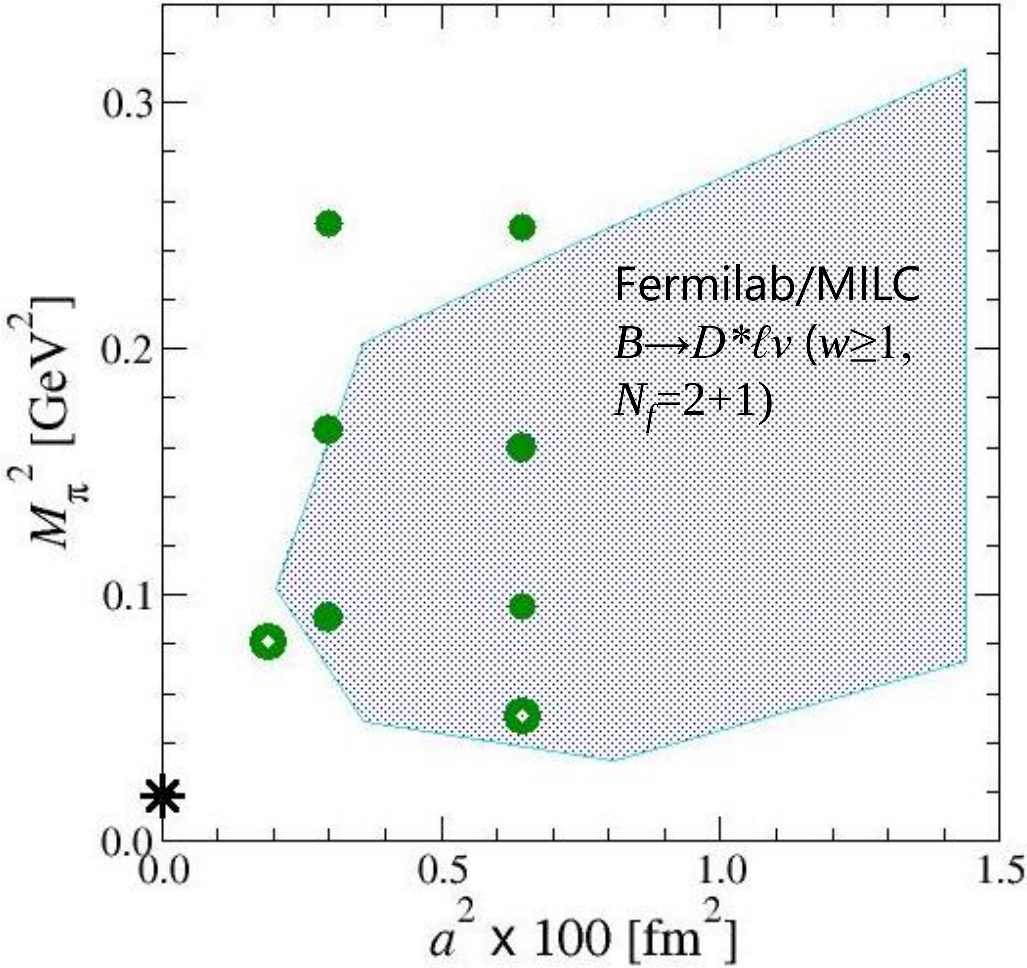
- Fermilab / MILC staggered light quarks + ET-based heavy quarks
- JLQCD chiral light quarks + reltativistic heavy quarks
  - calculating all FFs w/ very different systematics

gauge ensembles

clean simulation w/ chiral symmetry

simulation parameters

- $N_f = 2+1$
- a^{-1} ~ 2.5, 3.6, 4.5 GeV
- M_π ~ 230, 300, 400, 500 MeV
- 5,000 HMC traj. for each
- $M_\pi L \geq 4$



similar parameter regions w/ chiral symmetry

measurements

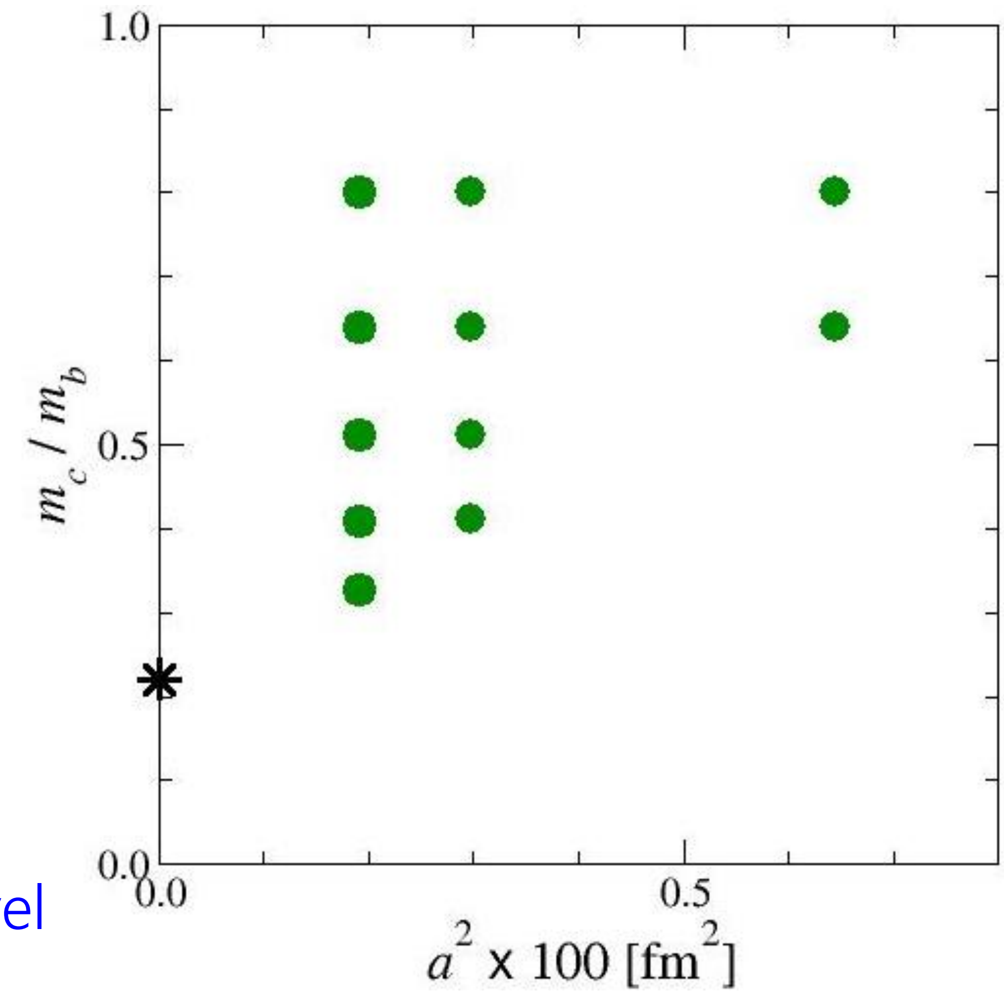
relativistic approach for hadron correlators

chiral fermion action
also for heavy quarks

- m_b = {1.00, 1.25, 1.25^2, ...} $\times\, m_c$
- $m_b \leq 0.7a^{-1}$
- extrapolation to $m_{b,\text{phys}}$

$\Leftrightarrow$ no $O(am_b)$ error
small $O((am_b)^2)$ error

$\Leftrightarrow$ discretization effects
suppressed to a few % level

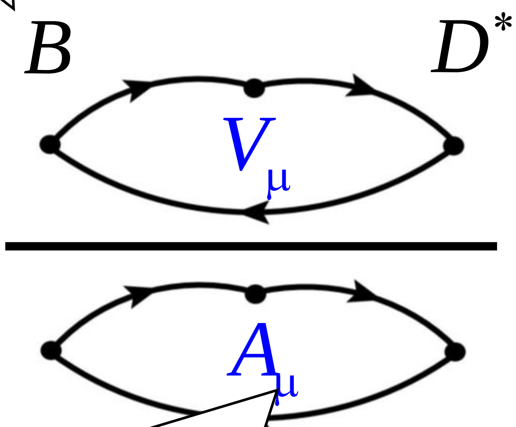


extrapolation is controllable

ratio method (Hashimoto et al. '99)

$\mathbf{p} = \mathbf{0}$

$|\mathbf{p}'|^2 = 0, 1, 2, 3, 4$ in units of $(2\pi/L)^2$, physical m_c



$$= \frac{\langle D^* | V_\mu^{\text{(lat)}} | B \rangle}{\langle D^* | A_\mu^{\text{(lat)}} | B \rangle} \rightarrow \frac{h_V(w)}{h_{A_1}(w)}$$

$\langle B | O_B^\dagger \rangle$, $\exp[-M_B \Delta t]$, $\cdots$ cancel

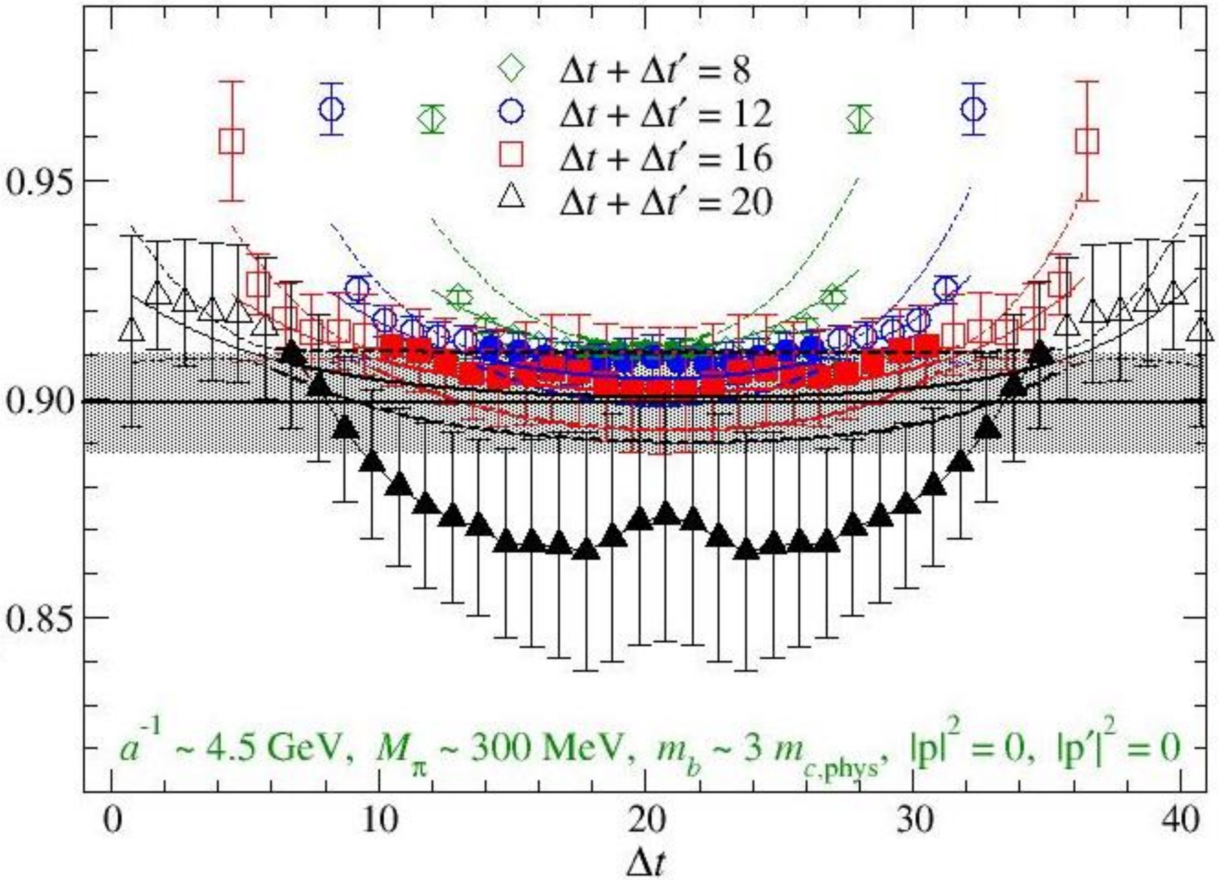
Z_A, Z_V cancel

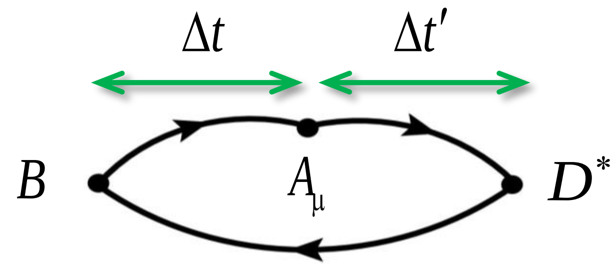
can calculate SM FFs w/o explicit renormalization

$$\{h_{A_2}(w), h_{A3}(w), h_V(w)\} / h_{A_1}(w), \quad h_{A_1}(w) / h_{A_1}(1), \quad h_{A_1}(1) / \sqrt{F_B^{\text{EM}}(1) F_{D^*}^{\text{EM}}(1)}$$

ground state contribution

$\langle D^* | A_1 | B \rangle \langle B | A_1 | D^* \rangle / \langle D^* | V_4 | D^* \rangle \langle B | V_4 | B \rangle \;\rightarrow\; h_{A1}(1)$





4 values of total separation $\Delta t+\Delta t'$

$$\left|\langle D^* | A | B \rangle\right|^2 \times \left\{1 + c\, e^{-\Delta E_B \Delta t} + c' e^{-\Delta E_{D^*} \Delta t'}\right\}$$

multiple $(\Delta t+\Delta t')$'s $\Rightarrow$ ~~excited state contamination~~

good statistical accuracy

extrapolation to the real world

NLO HMChPT (Randall-Wise '92, Savage '01)

- non-analytic log term (but small)
- polynomial in light quark masses

$$\xi_\pi = \frac{M_\pi^2}{(4\pi f_\pi)^2}, \quad \xi_{\eta s} = \frac{M_{\eta s}^2}{(4\pi f_\pi)^2}$$

$$h_{A1}(w) = c(m_c) + \frac{g_{D^*D\pi}^2}{16\pi^2 f_\pi^2} \Delta_c^2 b_{\log} \bar{F}_{\log}(M_\pi, \Delta_c, \Lambda_\chi) + c_\pi \xi_\pi + c_{\eta s} \xi_{\eta s}$$

$$+ c_w (w-1) + d_w (w-1)^2 + c_b \varepsilon_b + c_a \xi_a + c_{am_b} \xi_{amb}$$

0 recoil expansion

- interpolation w/ quad.

HQET

- in $\varepsilon_b = \frac{\bar{\Lambda}}{2m_b}$

discretization effects

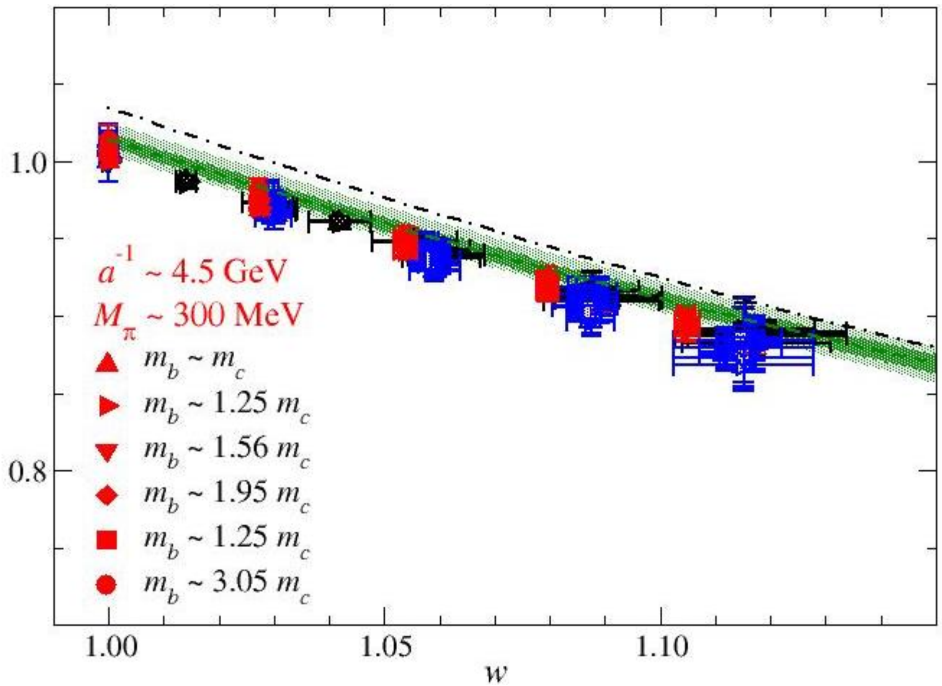
- in $\xi_a = (a\Lambda_{\rm QCD})^2$, $\xi_a = (am_b)^2$

- systematic error of this form : adding higher order / remove terms
- large covariance matrix, input $g_{D^*D\pi}$,...

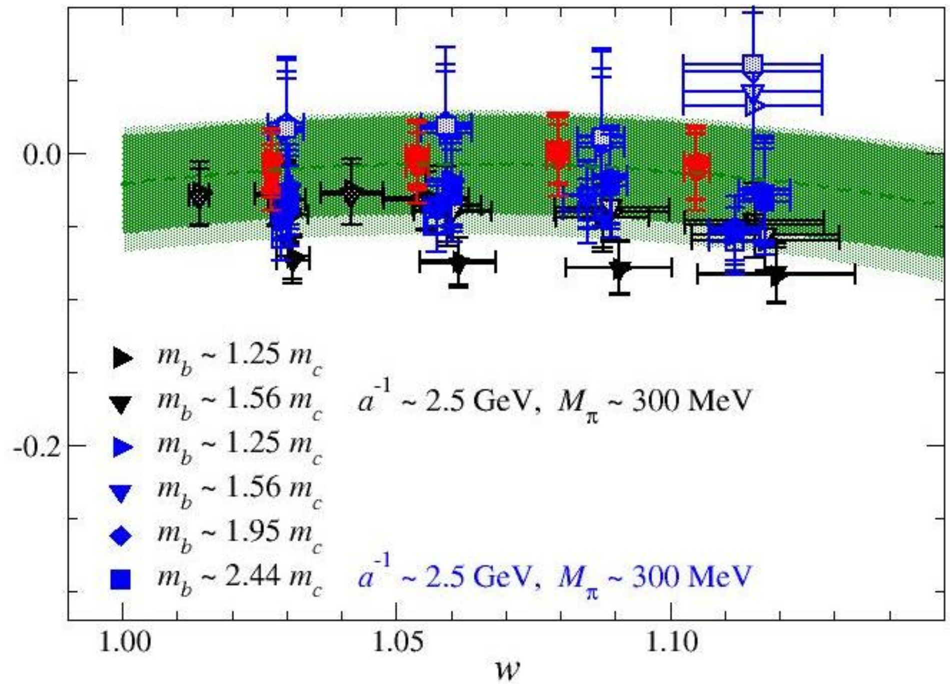
analysis is being finalized $\Rightarrow$ preliminary results in this talk

$B \to D \ell \nu$ form factors

h_+ vs w



h_- vs w

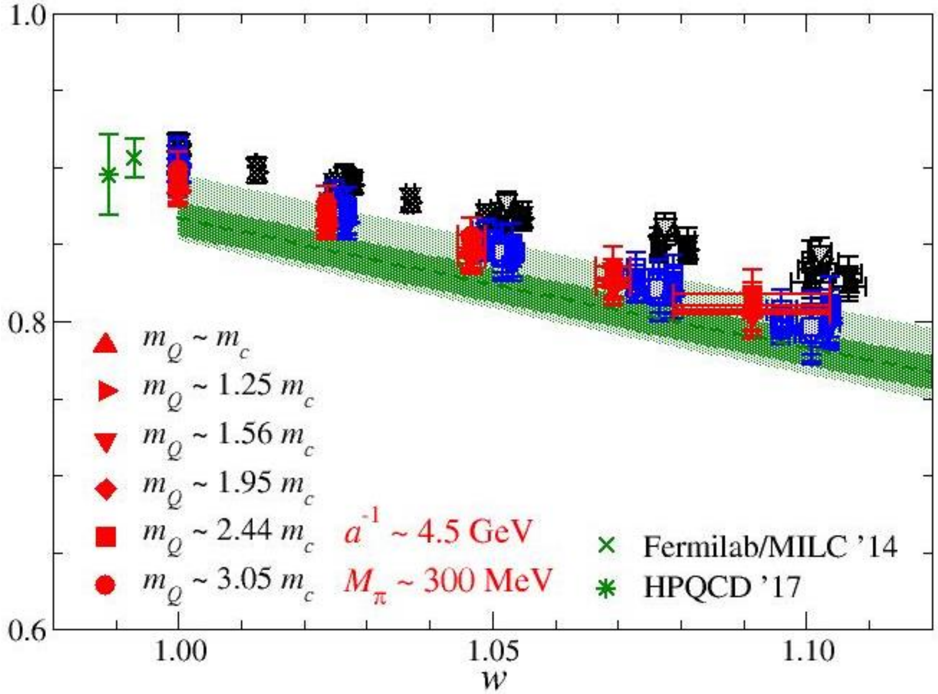


- mildly depend on a, M_π, m_s, m_b $\Rightarrow$ $\geq$ 50% error except c, $c_{w'}$ c_b (h_+)

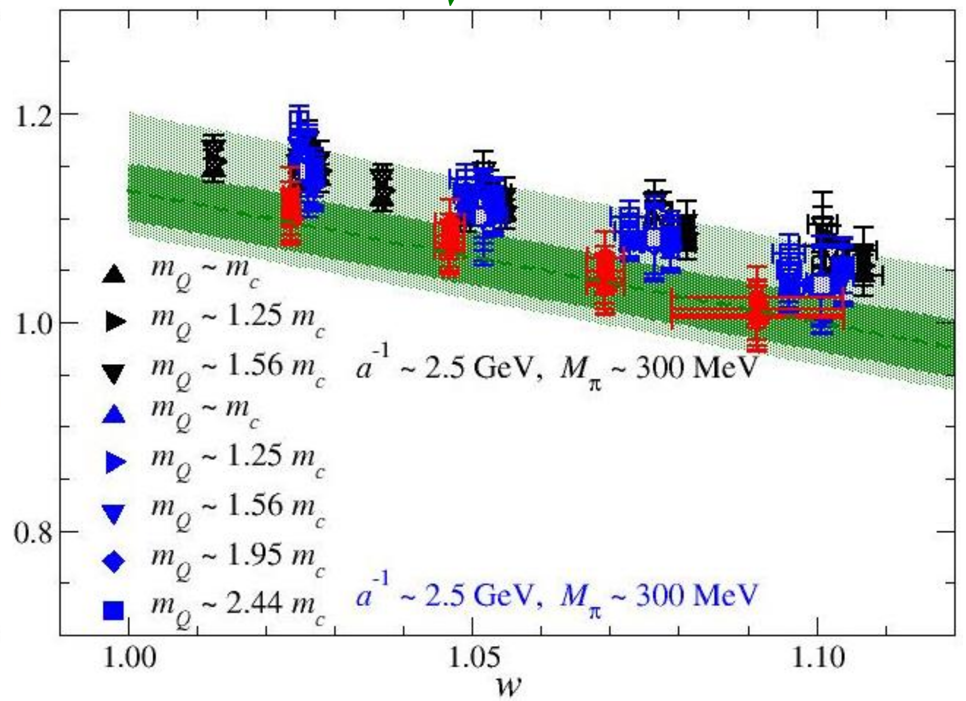
  $\Rightarrow$ consistent w/ world average (FLAG4 '19) within 2σ
- h_+ : ~ 1% stat / 1% systematic errors @ $a = 0$, $m_{q,\text{phys}}$

$B \to D^* \ell \nu$ form factors

h_{A1} vs w



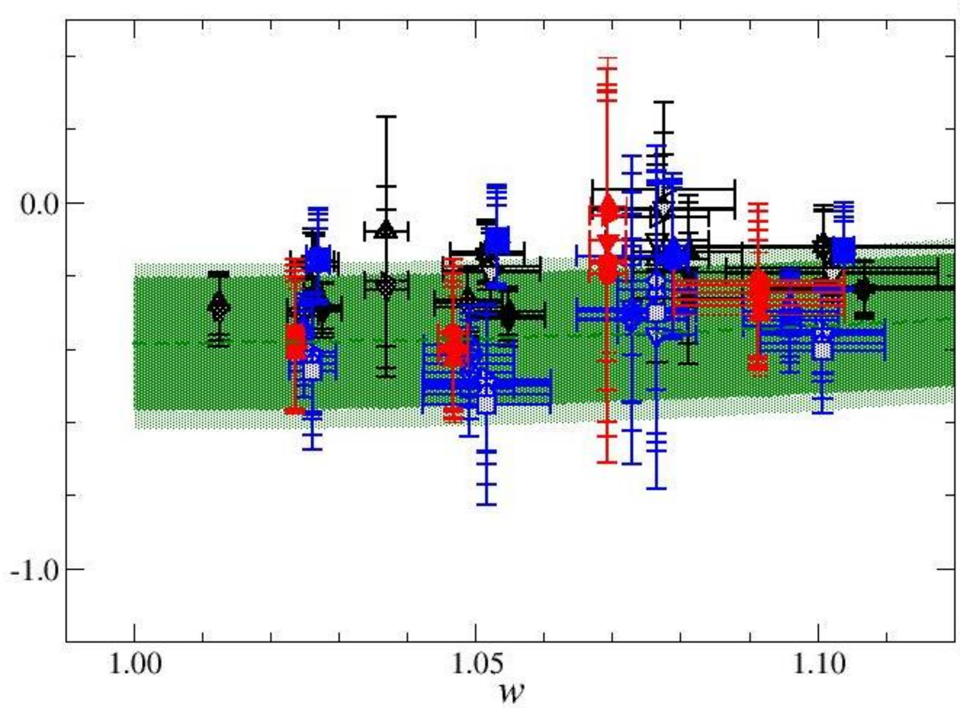
h_V vs w



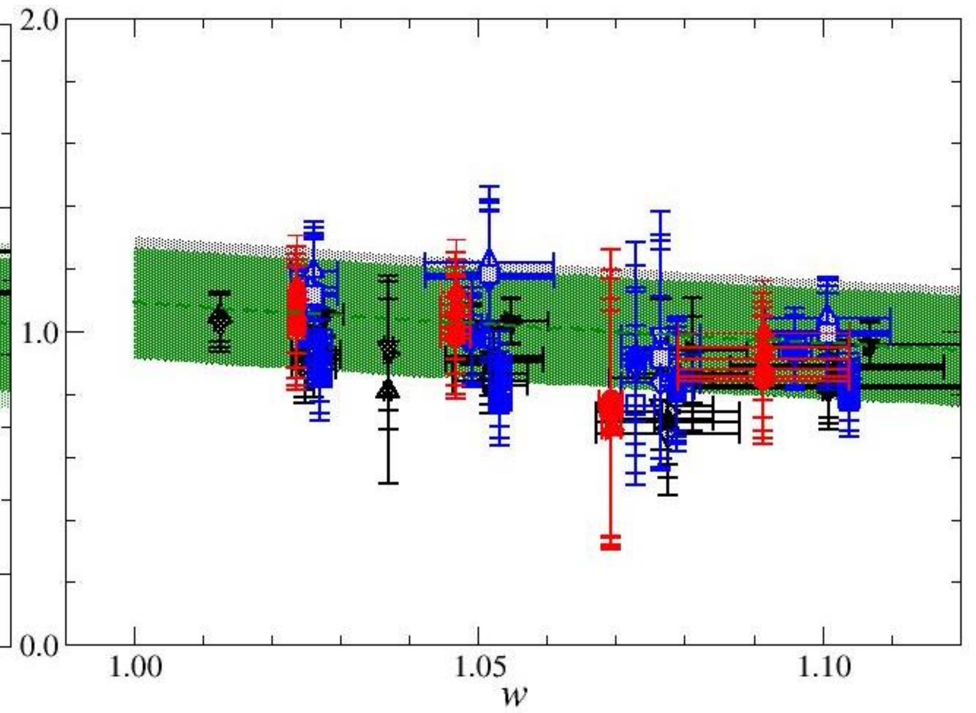
- mild a, M_π, m_s, m_b dependence
- only $h_{A1}(1)$ from previous studies [consistent]
- h_{A1}, h_V ~ 1 - 2 % stat / 3 – 6 % systematic ($a \neq 0$)

$B \to D^* \ell \nu$ form factors

h_{A2} vs w



h_{A3} vs w



- h_{A2}, h_{A3} ~ 20 % stat. / sys. errors $\Leftrightarrow$ $\Delta[d\Gamma/dw]$ ~ Belle

$$\left\langle D^*(p',\varepsilon')\middle|A_\mu\middle|B(p)\right\rangle \Rightarrow \left\{h_{A_1}(w), h_{A_2}(w), h_{A_3}(w)\right\}$$

3. determination of $|V_{cb}|$

$B \to D^* \ell \nu$ differential decay rate

$B \to D^* \{e,\mu\}\ \nu$ in the limit $m_l^2 = 0$

expr't

CKM matrix element

EW : perturbation

$$\frac{d\Gamma}{dw} = |V_{cb}|^2 \frac{G_F^2}{48\pi^3}\left(M_B - M_{D^*}\right)^2 M_{D^*}^3 \sqrt{w^2-1}\,(w+1)^2\,\eta_{EW}$$

$$\times \left[2\frac{1-2wr+r^2}{(1-r)^2}\left\{1+\frac{w-1}{w+1}R_1(w)\right\} + \left\{1+\frac{w-1}{1-r}\left(1-R_2(w)\right)\right\}^2 \right] h_{A1}(w)^2$$

QCD : non-perturbative

- described by h_{A1}, $\quad R_1 = \dfrac{h_V}{h_{A1}}$, $\quad R_2 = \dfrac{r h_{A2} + h_{A3}}{h_{A1}}$ (functions of w)

$|V_{cb}|$: a relative normalization by comparing exp. and th.

conventional determination of $|V_{cb}|$

$$\frac{d\Gamma}{dw} \propto |V_{cb}|^2 \left[2\frac{1-2wr+r^2}{(1-r)^2}\left\{1+\frac{w-1}{w+1}R_1(w)\right\}+\left\{1+\frac{w-1}{1-r}\left(1-R_2(w)\right)\right\}^2\right] h_{A1}(w)^2$$

- no published results for FFs except $h_{A1}(1)$!!

① assume a parametrization of FFs as a function of w

② fix $|V_{cb}|$ and unknown parameters by the fit to exp. and th. data

⇒ estimate FFs from experimental data !!

Boyd-Grinstein-Lebed (BGL) parametrization '97

- model-independent z expansion based on analyticity
- many parameters ... can not be determined by previous exp. data

$$g(z) = \frac{1}{P_{1-}(z)\phi_g(z)}\sum_{n=0}^{\infty} a_n^g z^n \quad f(z) = \frac{1}{P_{1+}(z)\phi_f(z)}\sum_{n=0}^{\infty} a_n^f z^n \quad \mathcal{F}_1(z) = \cdots$$

conventional determination of $|V_{cb}|$

$$\frac{d\Gamma}{dw} \propto |V_{cb}|^2 \left[2\frac{1-2wr+r^2}{(1-r)^2}\left\{1+\frac{w-1}{w+1}R_1(w)\right\} + \left\{1+\frac{w-1}{1-r}(1-R_2(w))\right\}^2 \right] h_{A1}(w)^2$$

Caprini-Lellouch-Neubert (CLN) parametrization '97

- BGL + NLO HQET constraints $\Rightarrow$ (too much?) less parameters
  - HQET for $h_{A1}(w)$ / $h_{A1}(1)$, R_1, R_2 expecting small correction to FF ratios

$$h_{A_1}(w) = h_{A_1}(1)\left(1 - 8\rho^2_{D*} z + \left(53\rho^2_{D*} - 15\right) z^2 - \left(231\rho^2_{D*} - 91\right) z^3\right)$$

fit parameters: $h_{A_1}(1)$, ρ^2_{D*}, $R_1(1)$, $R_2(1)$

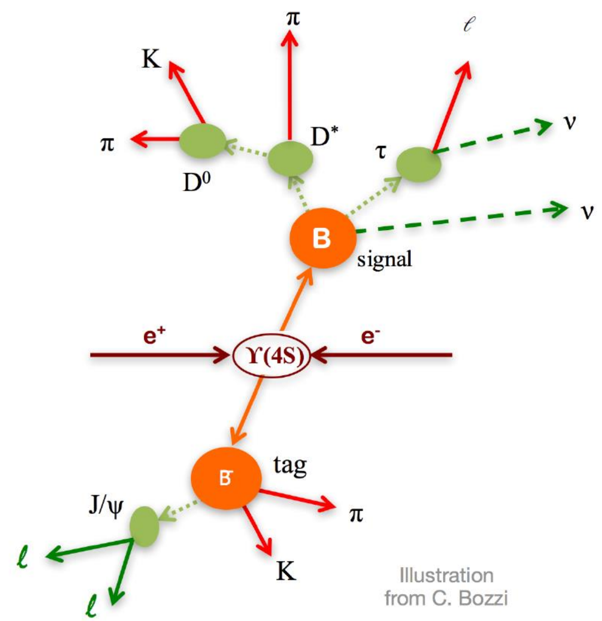
NLO HQET: the remaining coefficients

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$
$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

determinations w/ Belle data

Belle '17 and '18 (near full statistics)

- full kinematical distribution (q^2 and 3 θ's)
  - '17: tagged (less miss-identification of ν)
  - '18: untagged (more statistics)



conventional CLN analysis w/ tagged data

- Bernlochner - Ligeti - Papucci - Robinson '17

"model-independent" BGL analyses w/ tagged data

- Bigi – Gambino - Schacht '17 : $|V_{cb}| \times 10^3 = 41.7\ (+2.0/-2.2)$
- Grinstein – Kobach '17 : $|V_{cb}| \times 10^3 = 41.9(+2.0/-1.9)$
- consistent w/ inclusive determination 42.0 (0.5)

HQET constraints for CLN are source of the $|V_{cb}|$ tension?

determinations w/ Belle data

it is plausible that a tension is resolved by removing phenomenological assumptions ....

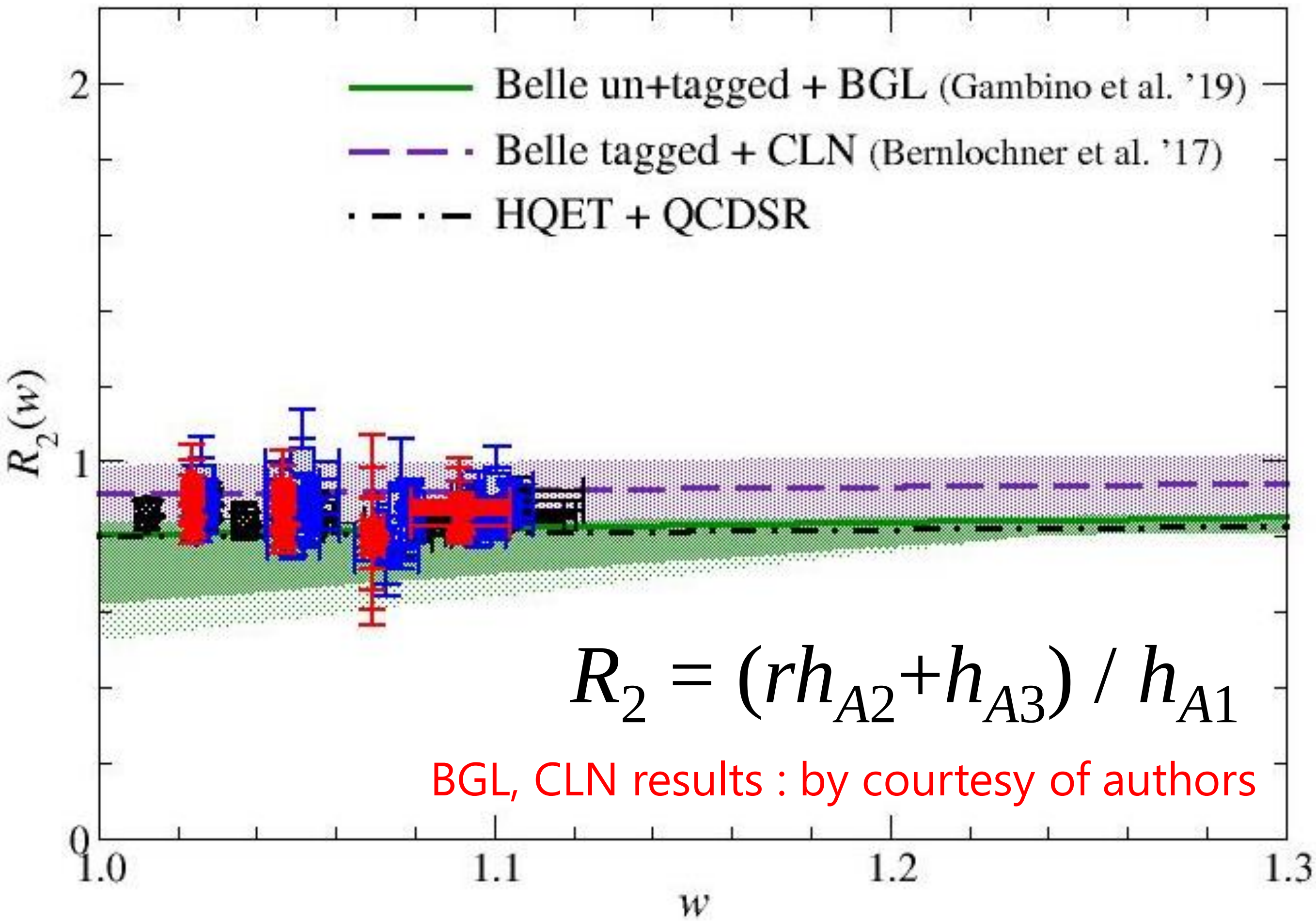
form factors should be predicted from QCD

the CLN and BGL analyses gave inconsistent results for

$h_{A1}(\mathrm{w})$ / $h_{A1}(1)$, R_1

$\Rightarrow$ let us compare w/ our lattice results from first-principles

FF ratio R_2



no big difference among CLN, BGL, HQET...

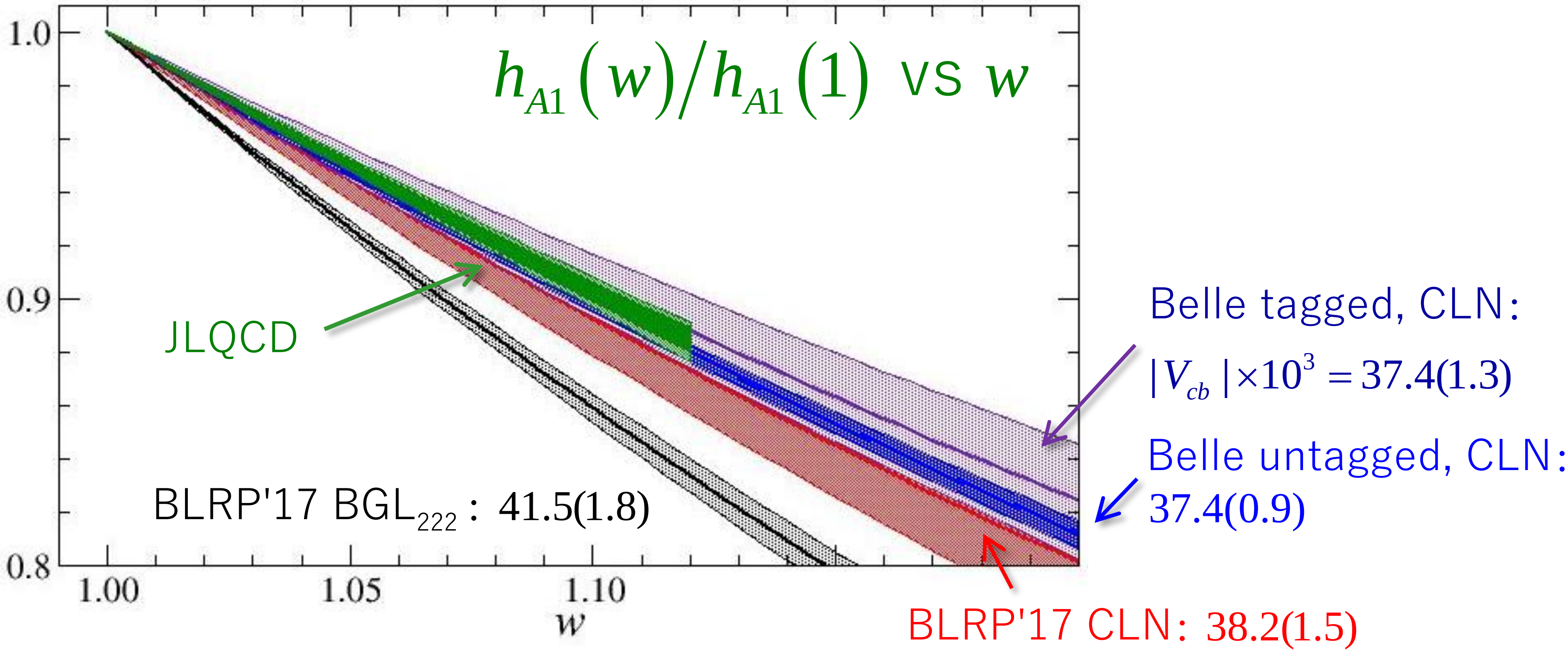
JLQCD

nicely consistent w/ phenomenological analyses

reduced stat.+sys. error in the ratio

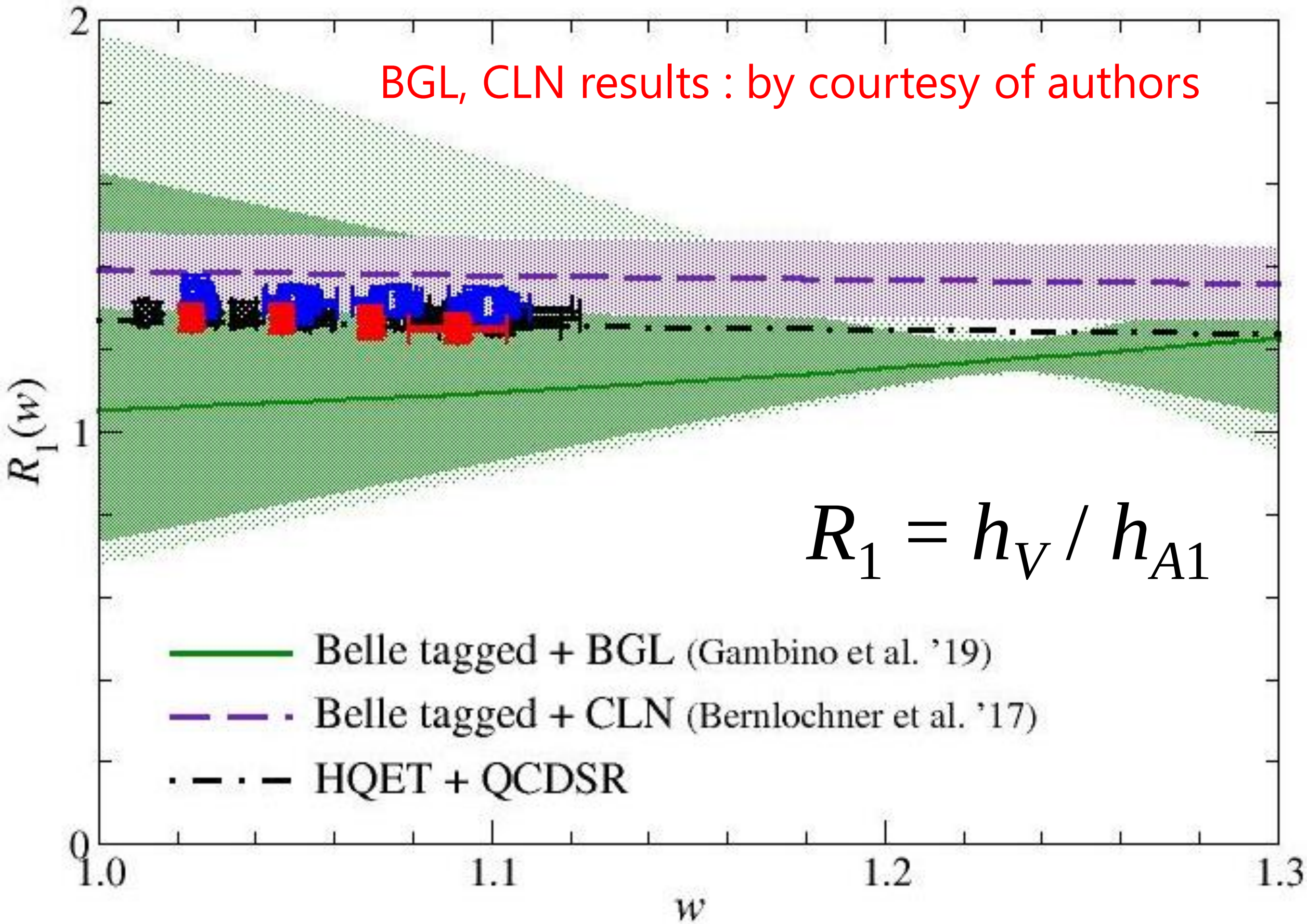
- good consistency among BGL / CLN / HQET / LQCD

FF shape of $h_{A1}(w)$



- '17: tension b/w CLN and BGL
- recent Belle tagged / untagged data : consistent CLN analyses
- our lattice estimate : 1st principles th. prediction, favor CLN

FF ratio R_1



in 2017

BGL vs CLN, HQET

JLQCD

strongly favor

CLN and HQET

Gambino et al '19

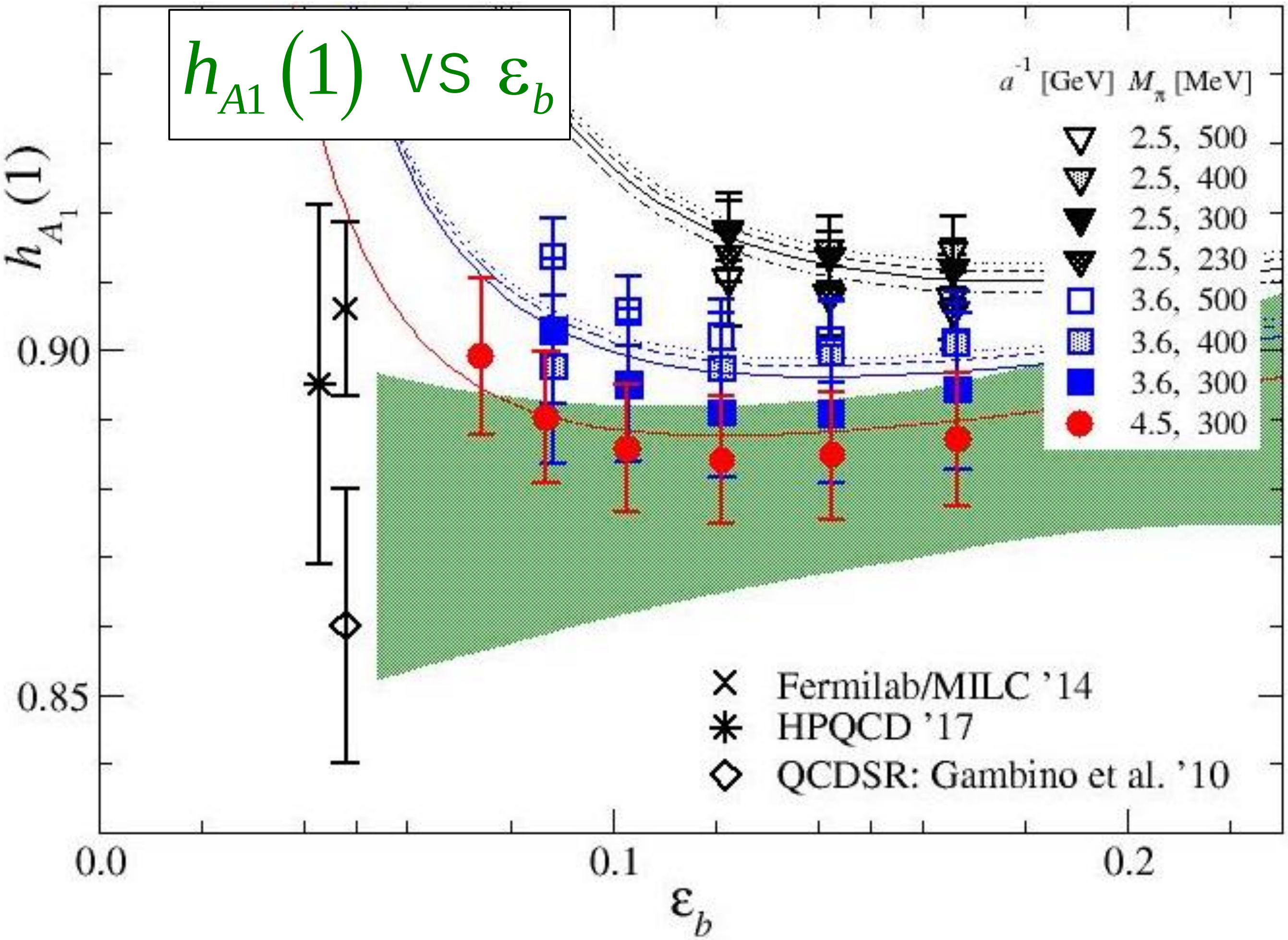
tagged+untagged

$\Rightarrow$ consistent

w/ CLN, LQCD

- systematic uncertainty of BGL analyses was underestimated
- LQCD could be helpful to stabilize "model-independent" BGL fit

normalization $h_{A1}(1)$



HQET expansion

$\varepsilon_b = \bar{\Lambda}/M_{\eta_b}$

$\bar{\Lambda} = 0.5\text{GeV}$

largest but 1-3% uncertainty from discretization

- $c_a a^2 + c_{amb}(am_b)^2$
- $c_{amb}(am_b)^2$
- $c_a a^2$

give good χ^2's

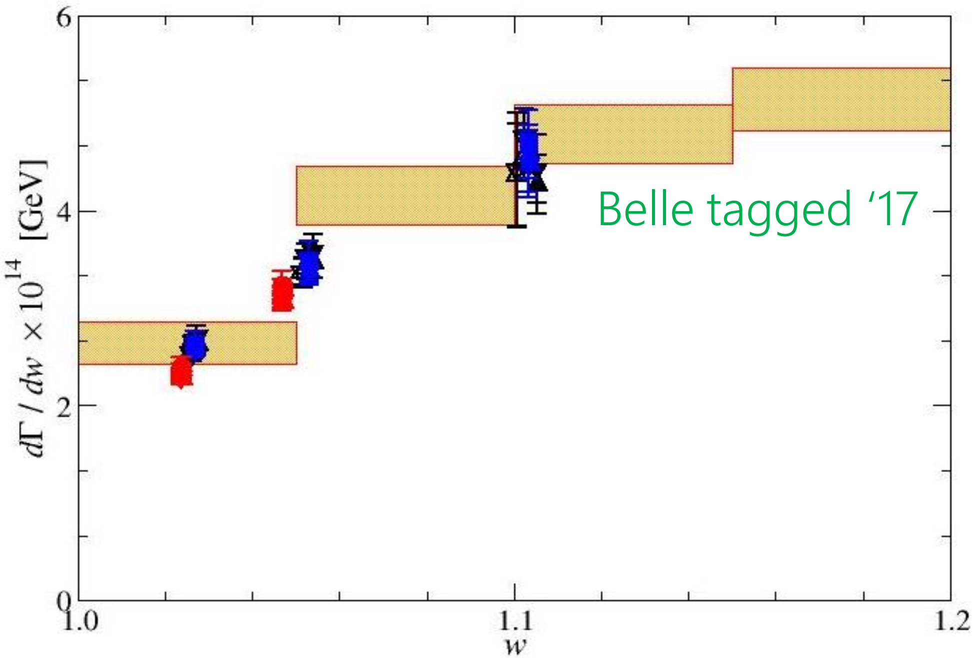
- closer to QCDSR $\Rightarrow$ $|V_{cb}| \times 10^3$ = 38.4 – 41.1 $|_{\pm 1\sigma}$ $\Leftrightarrow$ 42.2(0.8) incl
- consistent both with previous exclusive and inclusive @ w=1

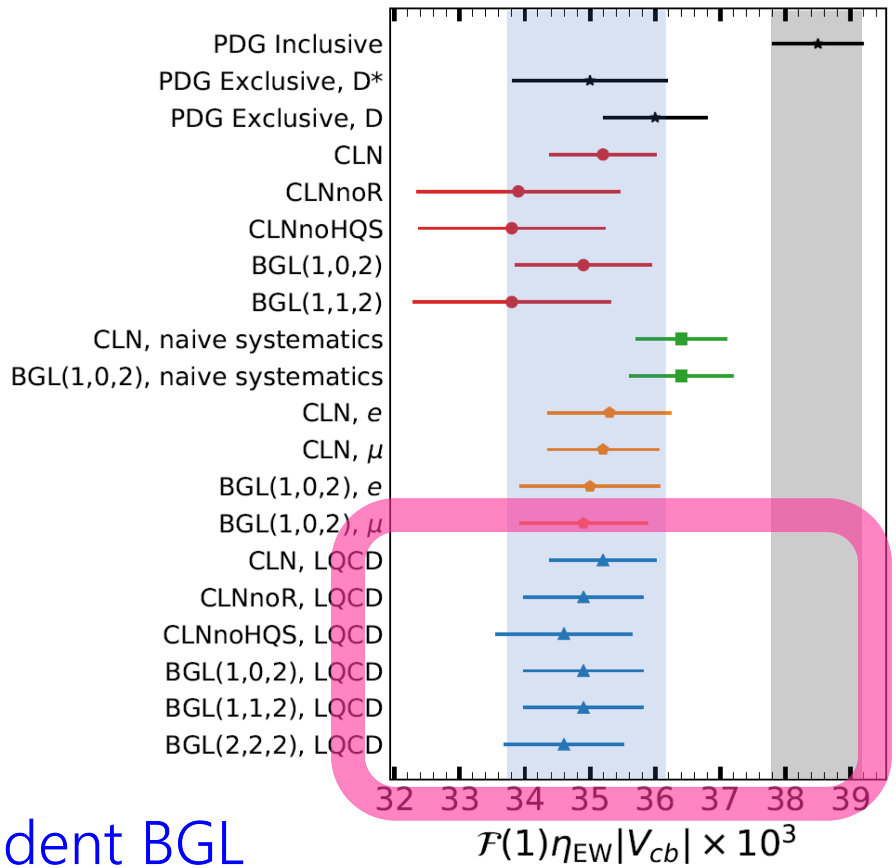
towards resolution of $|V_{cb}|$ tension

collaboration w/ Belle using data in full w region

in full w region

Ferlewicz et al., ‘20



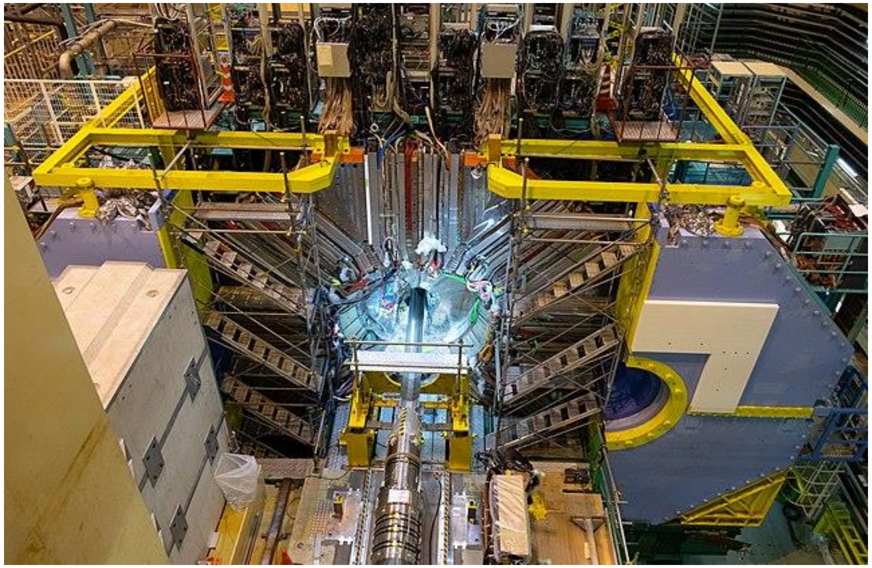


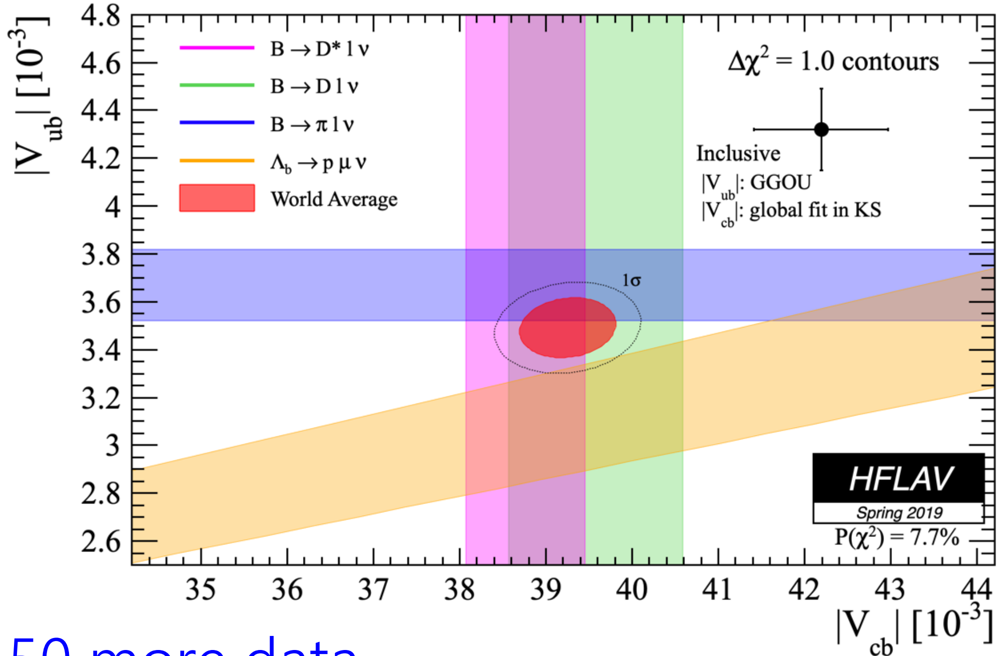
lattice data not only w=1 but w≠1

⇒ stable fit even w/ model-independent BGL

in progress

with Fugaku and Belle II



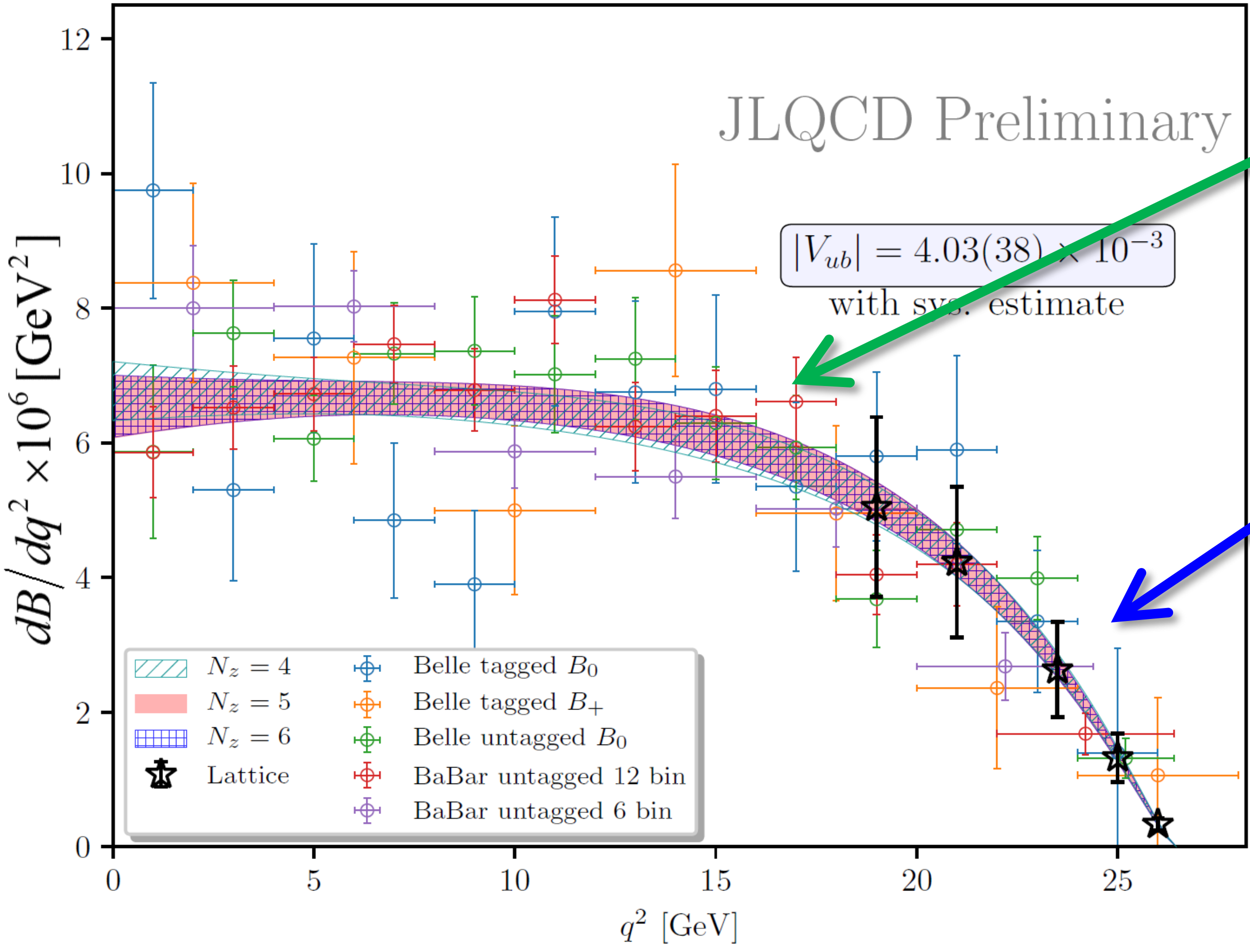


- physics run 2019 – 2031 ⇒ x 50 more data
  - $B \to D^{(*)}\ell\nu$: slight improvement by good control of systematics
    [systematics cancel in the ratio $B \to D^{(*)}\tau\nu$ / $B \to D^{(*)}\{e,\mu\}$]
  - $B \to \pi\ell\nu$: big improvement in next 5 years
- expect progress for $|V_{cb}|$ tension, but how about $|V_{ub}|$???

$B \to \pi\ell\nu$ to resolve $|V_{ub}|$ tension / new physics search

current status

e.g. differential branching fraction



experimental data

- Belle, BaBar ~ 20%
- Belle II ≤ 5% (-2031)

our lattice QCD

- Priority Issue 9
  ~ 20%

Program for Promoting Researches : 10% in a few years

Summary

$B \to D^* \ell \nu$ decays in relativistic lattice QCD

- promising probe of new physics, but a tension in $|V_{cb}|$
- all form factors @ zero and non-zero recoils
  - relativistic lattice QCD w/ chiral symmetry : ~~$O(a)$~~, ~~NPR~~
- indication to $|V_{cb}|$ determination
  - CLN w/ HQET can reasonably describe FF ratios
  - LQCD would be helpful to obtain a stable BGL analysis
- on-going analysis
  - collaborative analysis with Belle
  - BSM form factors for new physics search (need NPR)