Proton decay matrix element on the lattice with physical pion mass

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Host: Field Theory Research Team(R-CCS)

Overview

Baryon Asymmetry in the Universe(BAU)

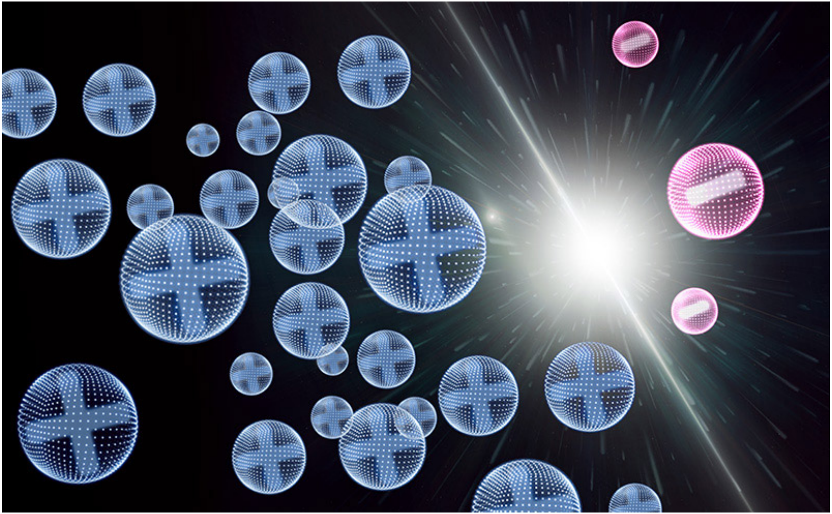


Figure: Matter-antimatter asymmetry [worldsciencefestival.com]

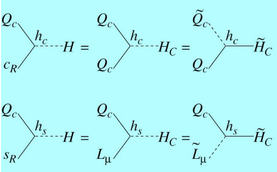
Baryon Asymmetry in the Universe(BAU)

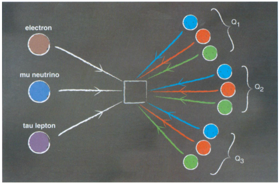
Three possible origins of BAU:

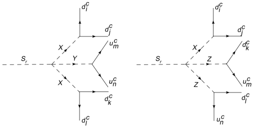
- In the Early Universe
  BV heavy particle decay processes

- During the EW phase transition
  1st order EW phase transition
  2nd order PT based on the Higgs mass

- After the EWPT (post-sphaleron)
  EW scale scalar boson decay to $6q, 6\bar{q}$







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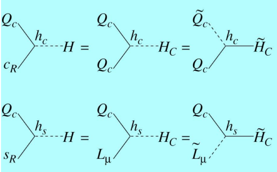
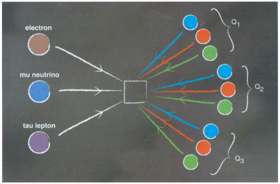
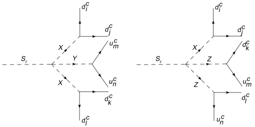
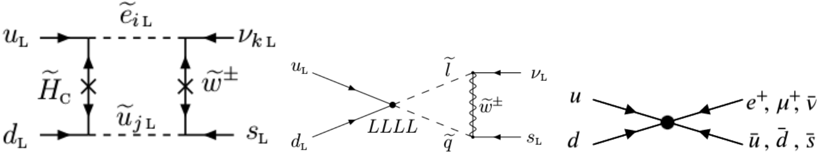






Proton decay

- (SUSY-)GUT provides the effective operator of lowest dimension 6.



(a) $\sim \Lambda_{GUT}$ (b) $\sim \Lambda_{SUSY}$ (c) $\sim \Lambda_{EW}$

Figure: Proton decay operator at different scales

Model parameters come into Wilson coefficients

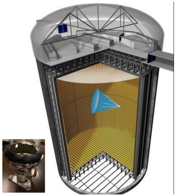
- $Y_{qq}, Y_{ql}, Y_{ud}, Y_{ue}$
- M_{H_C}
- $m_{\tilde{l}}, m_{\tilde{q}}$, triangle loop integrals, ...

whereas the model independent content remains in the effective operator.

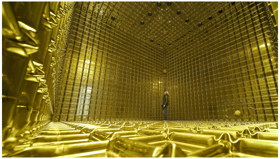
- Each model provides the different list of operators and the lifetime estimates

Experimental measurement of proton lifetime

- Super Kamiokande
  Water Cherenkov detector
  Fiducial Volume 22.5kt

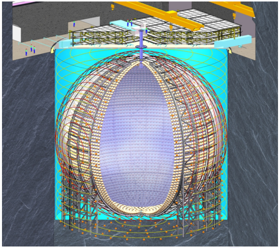


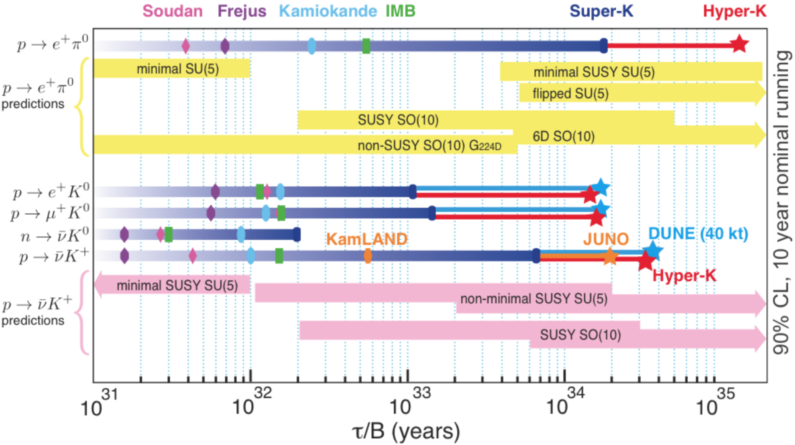
- Hyper Kamiokande
  Water Cherenkov detector
  Fiducial Volume 187kt



- DUNE
  LArTPC
  Fiducial Volume 40kt (10kt x4)

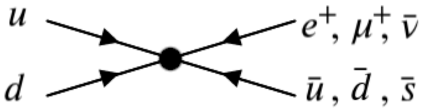
- JUNO
  Liquid Scintillator(LS)
  Fiducial Volume 5kt





Current proton decay bound in SK, [Abe, 2018]

Proton decay amplitude
$\langle \Pi \bar{\ell} | p \rangle_{GUT} \sim C^{\Gamma\Gamma} \langle \Pi \bar{\ell} | \mathcal{O}_{\Gamma\Gamma'} | p \rangle_{SM} = C^{\Gamma\Gamma} \bar{v}_\ell \langle \Pi | (\bar{q}^C q)_\Gamma P_{\Gamma'} q | p \rangle$,
where $C^{\Gamma\Gamma'}$ is a wilson coefficient, Π is a meson, p is a proton and $(XY)_\Gamma = (XP_\Gamma Y)$.



Four-fermion effective theory

$\langle \pi^0 | (\bar{u}^C d)_\chi u_L | p \rangle$
$\langle \pi^+ | (\bar{u}^C d)_\chi d_L | p \rangle$
$\langle K^0 | (\bar{u}^C s)_\chi u_L | p \rangle$
$\langle K^+ | (\bar{u}^C s)_\chi d_L | p \rangle$
$\langle K^+ | (\bar{u}^C d)_\chi s_L | p \rangle$
$\langle K^+ | (\bar{d}^C s)_\chi u_L | p \rangle$
$\langle \eta \, | (\bar{u}^C d)_\chi u_L | p \rangle$

where χ is a chirality L, R of the decay operator.

The decay rate Γ is calculated from the hadronic matrix element,

$$\begin{aligned} &\langle \Pi(p')\bar{\ell}(q)|\mathcal{O}^{\Gamma\Gamma'}|N(p,s)\rangle \\ &= \bar{v}_\ell P_{\Gamma'}\left[W_0^{\Gamma\Gamma'}(q^2) - \frac{i\not{q}}{m_N}W_1^{\Gamma\Gamma'}(q^2)\right]u_N(p,s) \\ &= \bar{v}_\ell P_{\Gamma'}W_0^{\Gamma\Gamma'}(q^2)u_N(p,s) + O(m_l/m_N)\bar{v}_\ell u_N(p,s) \end{aligned} \tag{1.1}$$

where Π a meson, N a nucleon, and $W_{0,1}$ decay form factor[Aoki,1999].
Then the decay rate is

$$\Gamma\left(p \to \Pi + \bar{\ell}\right) = \frac{(m_p^2 - m_\Pi^2)^2}{32\pi m_p^3}\left|\sum_I C_I W_0^I\left(p \to \Pi + \bar{\ell}\right)\right|^2.$$

Lattice QCD

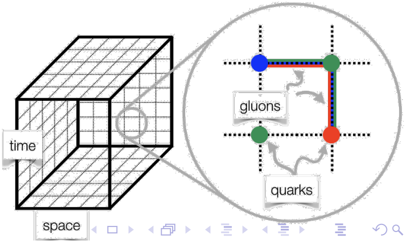
- QCD action implemented on the discretized 4D Euclidean spacetime lattice
- quark fields defined on the lattice site, gluon fields on the link between them
- direct computation of path integral (Monte Carlo method)

  $\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}U\mathcal{D}[\psi,\bar{\psi}]\mathcal{O}e^{-S_E}}{\int \mathcal{D}U\mathcal{D}[\psi,\bar{\psi}]e^{-S_E}} = \frac{1}{N}\sum_i \mathcal{O}_i$

  where $\mathcal{O}_i$ is a random variable distributed along the probability density function $\frac{e^{-S_E}}{\int \mathcal{D}U\mathcal{D}[\psi,\bar{\psi}]e^{-S_E}}$.

Systematic and statistical error sources

- Expensive computational cost
- Discretization error
- Finite volume effect



Previous study

Ref.	JLQCD (2000) [46]	CP-PACS & JLQCD (2004) [47]	RBC (2007) [48]	QCDSF (2008) [50]	RBC/ UKQCD (2008,2014) [51, 52]	RBC/ UKQCD (2017) [3]	This work
Fermion	Wilson	Wilson	DW	Wilson	DW	DW	DW
N_f	0	0	0 and 2	2	3	3	3
Volume (fm^3)	$(2.4)^2 \times 4.1$	$(3.3)^3$	Quench $(1.6)^3$ Two-flavor $(1.9)^3$	$(1.68)^3$	$(2.65)^3$	$(2.65)^3$	$(4.8)^3$ $(4.5)^3$
a (fm)	0.09	0	Quench 0.1 Two-flavor 0.12	0.07	0.11	0.11	0.2 0.14
m_π (GeV)	0.45–0.73	0.6–1.2	Quench 0.39–0.58 Two-flavor 0.48–0.67	0.42–1.18	0.34–0.69	0.34–0.69	0.14 (physical)
Renorm. scale	One-loop $1/a, \pi/a$	One-loop 2GeV	NPR 2GeV	NPR 2GeV	NPR 2GeV	NPR 2GeV	NPR 2GeV
α (GeV^3)	-0.015(1)	-0.0090	Quench -0.0100*(19) Two-flavor -0.0118*(21)	-0.0091(4)	-0.0119*(26)	-0.0144(15)	-0.0131(20)
β (GeV^3)	0.014(1)	0.0096	Quench 0.0108*(21) Two-flavor 0.0118*(21)	0.0090(4)	0.0128*(28)	0.0144(15)	0.0162(22)

Problems in achieving physical point

For a lot of simulation on the lattice, non-physical quark mass m_{ud} is adopted for computational cost reasons, resulting in $m_\pi^{latt} > 140\,\mathrm{MeV}$.

- Inversion in gauge configuration production (RHMC) cost is highly dependent on the quark mass

$$\mathrm{Cost} \sim \left(\frac{L}{fm}\right)^5 L_s \left(\frac{\mathrm{MeV}}{m_\pi}\right)\left(\frac{fm}{a}\right)^7 \left(\frac{\mathrm{MeV}}{m_K}\right)^2 \left(C_1 + C_2\left(\frac{a}{fm}\right)^3\left(\frac{m_K}{m_\pi}\right)^2\right)$$

- Inverting Dslash with physical mass - high computational cost
  $S = M^{-1}b$, where M is an implementation of Dirac op. $\not{D} = \not{\partial} + m$.
- Finite volume effect - systematic errors of $O(m_\pi L)$
  The size of the hadron cloud around the hadron, does not fit into the lattice volume, and it shifts the spectrum.
- Noise controlled by quark mass - Nucleon and 3/2 pions
  [Lepage] the correlation function of nucleon has noise due to pions $O(e^{-m_N t - \frac{3m_\pi t}{2}})$.

We define the hadron interpolating operators,

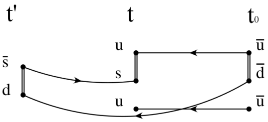
$$\begin{aligned} J_N &= \epsilon^{abc}(u^{a\,T} C\gamma_5 d^b)u^c \\ J_{\pi^+} &= \bar{d}^a\gamma_5 u^a \\ J_{K^+} &= \bar{s}^a\gamma_5 u^a, \end{aligned}$$

where u, d, s are quark fields with up, down, and strange flavors.

$$\sum_{\vec{x'},\vec{x}} e^{-i\vec{k}\cdot(\vec{x'}-\vec{x})}\mathrm{Tr}[P_+\langle 0|J_N(\vec{x'},t)\bar{J}_N(\vec{x},0)|0\rangle] = \sum_i Z_N^{(i)}\frac{m_N+E_N}{2E_N}e^{-E_N^{(i)}t}$$

$$\sum_{\vec{x'},\vec{x}} e^{-i\vec{p}\cdot(\vec{x'}-\vec{x})}\langle 0|J_\Pi(\vec{x'},t)\bar{J}_\Pi(\vec{x},0)|0\rangle = \sum_i \frac{Z_\Pi^{(i)}e^{-E_\Pi^{(i)}t}}{2E_\Pi^{(i)}},$$

(Meson)-(Decay Operator)-(Proton)



$$C^{3pt}(t,t') = \sum_{\vec{x},\vec{x}'} e^{i(\vec{p}'\cdot\vec{x}'-\vec{q}\cdot\vec{x})}\langle 0|J_\Pi(x')\mathcal{O}(x)\bar{J}_N(x_0)|0\rangle$$

Lattice paramters

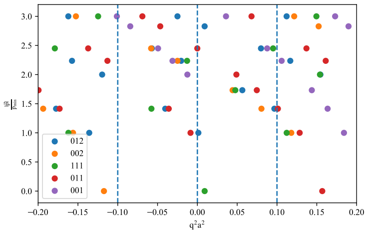
lattice size	$24^3 \times 64 \times 24$
gauge action	Iwasaki-DSDR
fermion	DWF
β	1.633
lattice cutoff	$a^{-1} = 1.0230(20)$GeV[1]
$m_l a$	0.00107
$m_h a$	0.0850
$m_\pi a$	0.1387
$m_K a$	0.5051
m_{res}	0.0022824(70)
$m_\pi L$	3.3
Deflated CG	2000+1000
AMA	32+2
N_{cfg}	129

lattice size	$32^3 \times 64 \times 12$
gauge action	Iwasaki-DSDR
fermion	DWF
β	1.75
lattice cutoff	$a^{-1} = 1.3787(48)$GeV[1]
$m_l a$	0.0001
$m_h a$	0.0450
$m_\pi a$	0.1046
$m_K a$	0.3602
m_{res}	0.0018915(75)
$m_\pi L$	3.4
Deflated CG	2000+250
AMA	32+1
N_{cfg}	75(out of 112)

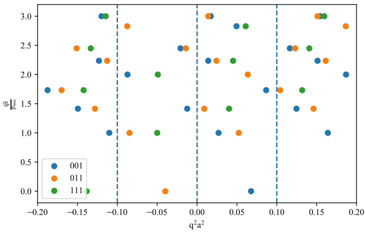
Kinematic Choice

- Energy-momentum conservation
- $q^2 = -m_l^2 \sim 0$ on-shell condition
- Discrete momenta insertion on the lattice

π



K



Kinematic Choice

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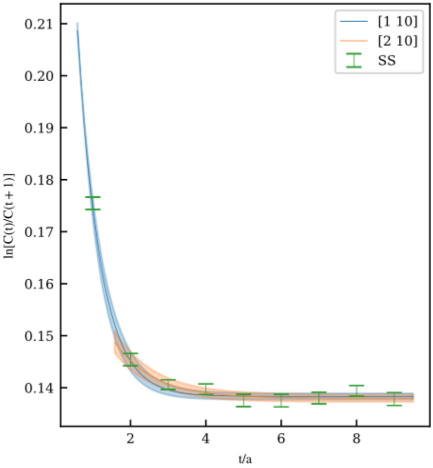
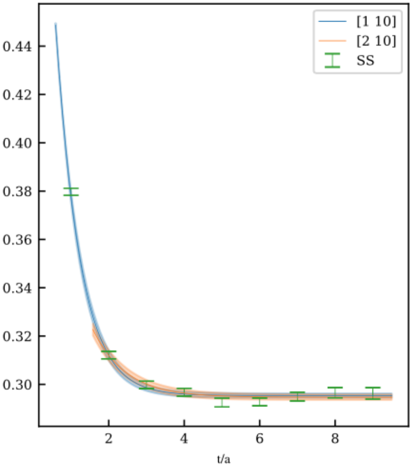
Π	$\vec{n}_\Pi$	$\vec{n}_N$	$q^2(\mathrm{GeV}^2)$ 24c, 32c
π	[1 1 1]	[0 0 0]	0.010, -0.012
		[0 1 0]	0.113, 0.095
	[0 0 2]	[0 0 0]	-0.116, -0.14
K	[0 1 1]	[0 0 0]	-0.034, -0.042
		[0 1 0]	0.058, 0.056
	[0 0 1]	[0 0 0]	0.075, 0.074

Table: Momenta choice for physical kinematics

Two states fit of two point functions

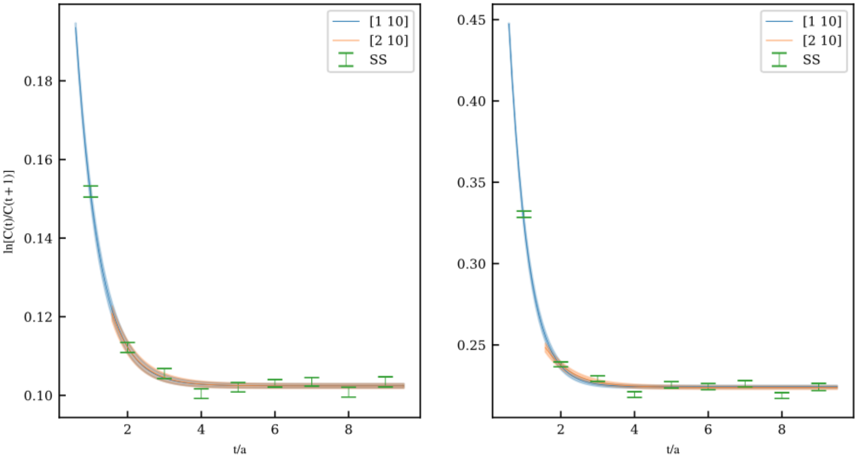
Pion





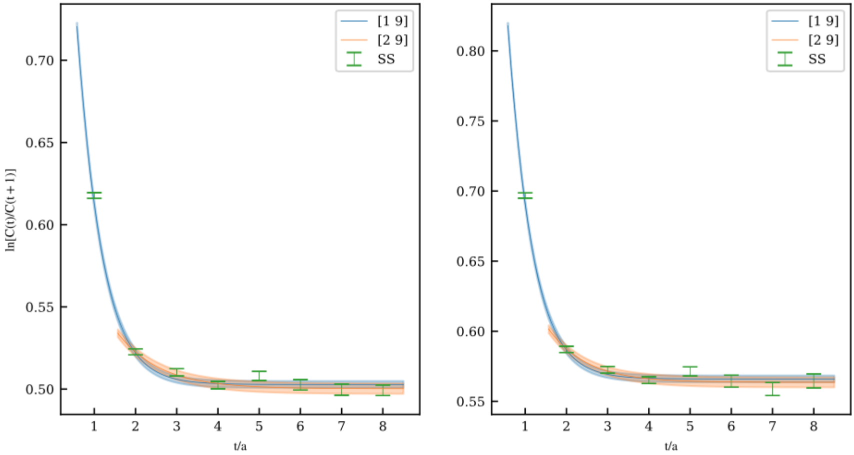
$24^3 \times 64$ lattice ($a^{-1} = 1.0230$ GeV)

Pion



$32^3 \times 64$ lattice ($a^{-1} = 1.3787$ GeV)

Kaon



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Two states fit of two point functions

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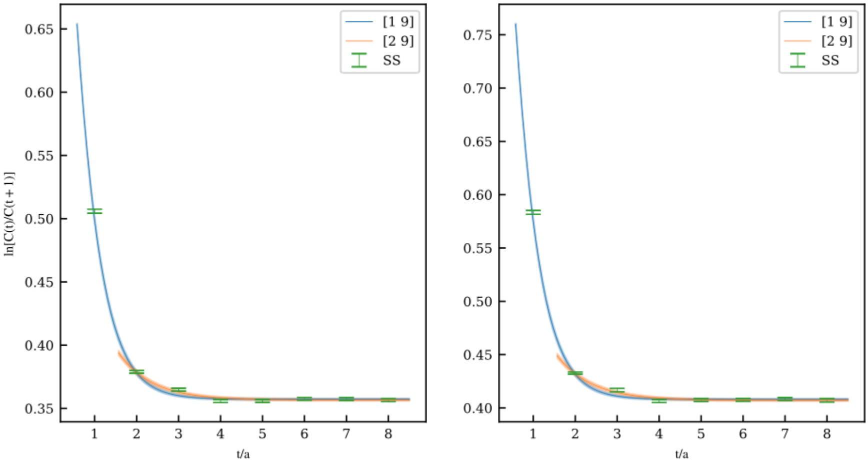
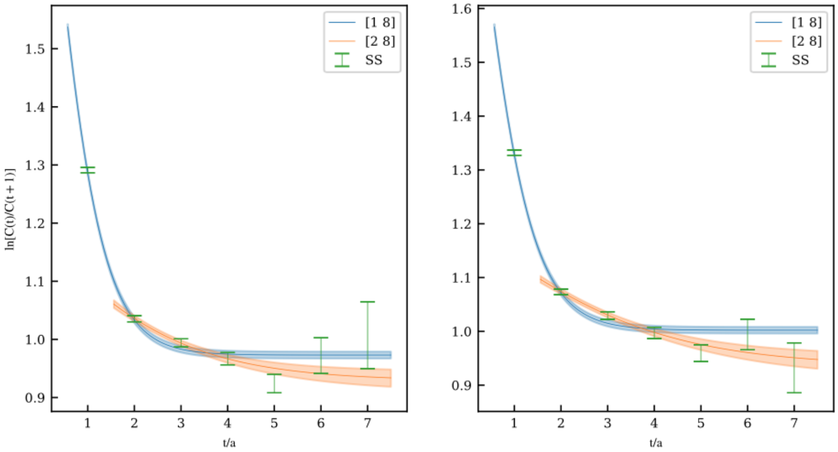


$32^3 \times 64$ lattice ($a^{-1} = 1.3787$ GeV)

Two states fit of two point functions

Proton



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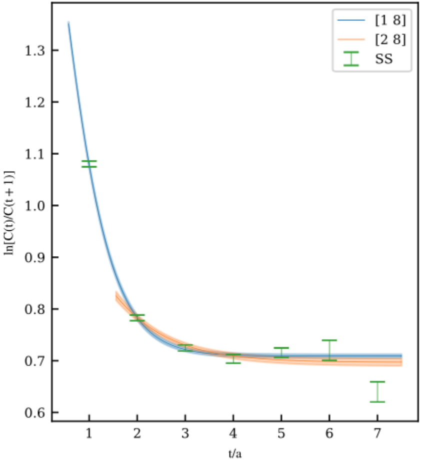
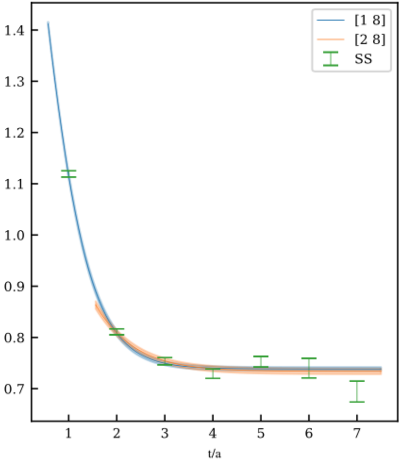
Proton





$32^3 \times 64$ lattice ($a^{-1} = 1.3787$ GeV)

Dispersion relation

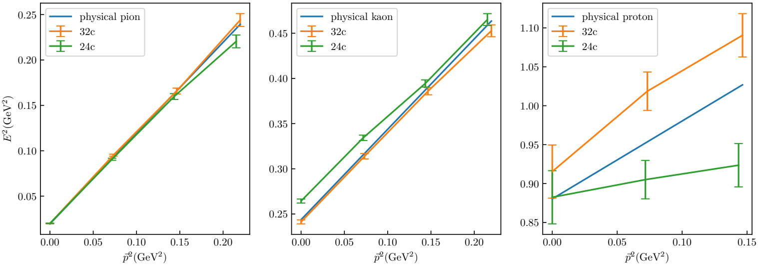


Figure: Dispersion relation compared with experimental measurement

Two states fit of three point functions

For linear combinations

$$\mathrm{Tr}[\mathcal{P} C_X] = \begin{cases} \mathrm{Tr}\ [P_+ C_{3\mathrm{pt}}^{\Gamma\Gamma'}] - \vec{q} \cdot \mathrm{Tr}\ [P_+ i\vec{\gamma} C_{3\mathrm{pt}}^{\Gamma\Gamma'}] \frac{(E_N - E_\pi)(m_N + E_N)}{\vec{q}^2} \\ \frac{1}{\vec{q}^2} \mathrm{Tr}\ [P_+ \gamma_j C_{3\mathrm{pt}}^{\Gamma\Gamma'}] \end{cases}$$

- Plateau method

$$R^{\Gamma\Gamma'}(t', t, t_0) = \frac{\mathrm{Tr}[\mathcal{P} C_X(t', t, t_0)] \sqrt{Z_\Pi(\vec{p}') Z_N(\vec{p})}}{C_\Pi^{2\mathrm{pt}}(\vec{p}', t' - t) \mathrm{Tr}[\mathcal{P}_+ C_N^{2\mathrm{pt}}(\vec{p}, t - t_0)]}$$

- Two states fit

$$\mathrm{Tr}[\mathcal{P} C_X] = \sum_{i,j=0,1} \frac{\sqrt{Z_\Pi^i Z_N^j}}{2E_{\Pi_i}} e^{-E_\Pi^i(t'-t) - E_N^j(t-t_0)} \frac{m_N^j + E_N^j}{2E_N^j} W_C^{(ij)}$$

where

$$W_C(q^2) = \begin{cases} W_0^\Gamma(q^2)(m_N + E_N) + \frac{(E_N - E_\pi)(m_N + E_N) - (\vec{k} \cdot \vec{q})}{m_N} W_1(q^2) & (\mathcal{P} = P_+) \\ -ik_j W_0(q^2) + \frac{ik_j(E_N - E_\pi) - iq_j(m_N + E_N) \pm (\vec{q} \times \vec{k})_j}{m_N} W_1(q^2) & (\mathcal{P} = P_+ \gamma_j) \end{cases}$$

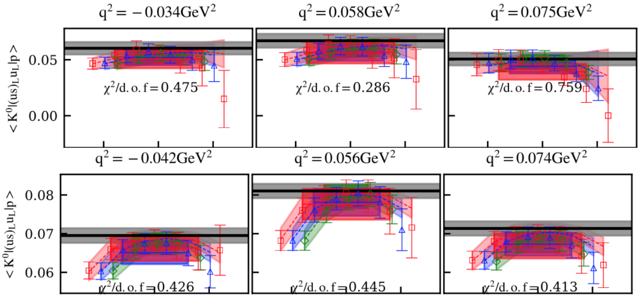


Figure: Bare $W_0(q^2)$ form factor for the matrix element $\langle K^0|(us)_L u_L|p\rangle$ on the (top) 24c (bottom) 32c lattice.

- Source-sink separation of 8,9,10 (Green, Blue, Red)
- Plateau method in horizontal line for each separation(color)
- Two-states fit as dashed line and curved range
- Ground states extraction as Black horizontal line with grey shades

Three quark operator

$$\mathcal{O}_{uds}^{\Gamma\Gamma'} = \epsilon^{abc}(\bar{u}^{c\ a}\Gamma d^b)\Gamma' s^c,$$

The Green's function in Landau gauge

$$G_{uds}^{\Gamma\Gamma'}(p,q,r) = \sum_{xyz} e^{-ipx-iqy-irz}\langle\mathcal{O}_{uds}^{\Gamma\Gamma'}(0)\bar{s}(x)\bar{d}(y)\bar{u}(z)\rangle,$$

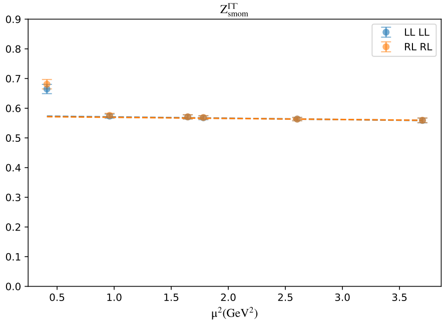
$$G_{udd}^{\Gamma\Gamma'}(p,q,r) = \sum_{xyz} e^{-ipx-iqy-irz}\langle\mathcal{O}_{udd}^{\Gamma\Gamma'}(0)\bar{d}(x)\bar{d}(y)\bar{u}(z)\rangle,$$

where the momentum configurations meet $p^2 = q^2 = r^2 = \mu^2, p+q+r=0$.

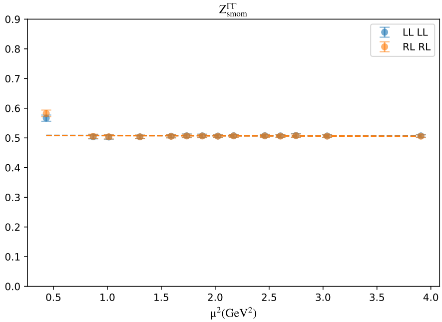
Two-loop matching, Three-loop perturbative running [Gracey, 2012]

$$U^{\overline{MS}\leftarrow latt}(\mu) = U^{\overline{MS}}(\mu,\mu')\frac{Z^{\overline{MS}}(\mu')}{Z^{MOM}(\mu')}Z_{latt}^{MOM}(\mu')$$

Renormalization



(a) 24c lattice



(b) 32c lattice

Renormalization

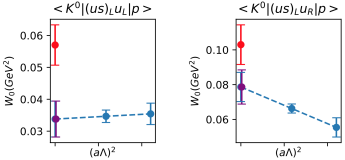
Z_{ND}	lattice	flavor	$am_q^{NPR,1}$	$am_q^{NPR,2}$	$am_q^{NPR,3}$
LL	24c	uds	0.62(2)	0.573(24)	0.577(12)
RL	24c	uds	0.61(2)	0.568(21)	0.577(11)
LL	32c	uds	0.506(13)	*	0.508(7)
RL	32c	uds	0.504(10)	*	0.508(7)
LL	24c	udu	0.62(2)	*	*
RL	24c	udu	0.61(2)	*	*
LL	32c	udu	0.500(12)(10)	*	0.508(7)
RL	32c	udu	0.506(11)(16)	*	0.508(7)

Table: NPR result

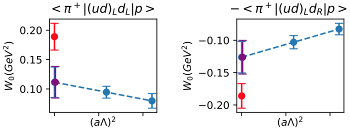
Continuum limit ($a = 0$)
as an extrapolation of two lattice spacing 32c($a = 0.14$fm) and 24c($a = 0.2$fm).

$$W((a\Lambda)^2) = w^{(0)} + w^{(1)}(a\Lambda)^2 + w^{(2)}(a\Lambda)^4 + \mathcal{O}((a\Lambda)^6)$$

Comparison with earlier study, [Aoki, 2017]



kaon channel



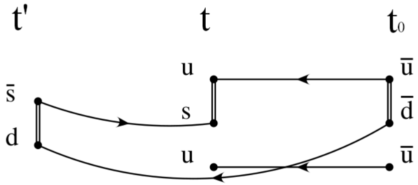
pion channel

Possible extensions

Proton decay matrix elements can be investigated futher to see:

- Vector meson channels from proton decay
- Three body decay channel from proton decay
- Induced Nucleon Decay from Dark matter

Vector meson channels



- Same computation with different Γ structures
- Different form factor decomposition
- Asymptotic vector meson channel should be there.

$$\langle K^{*i}(Q)\ell(p')|O_{d=6}|p(p,s)\rangle = \epsilon^{i}_{\mu}\bar{v}^{c}_{\ell}[F_1\gamma_5\gamma^{\mu} + F_2 i\gamma_5\sigma^{\mu\nu}Q_{\nu} + F_3\gamma_5 Q^{\mu} + F'_1\gamma^{\mu} + F'_2 i\sigma^{\mu\nu}Q_{\nu} + F'_3 Q^{\mu}]u_N$$

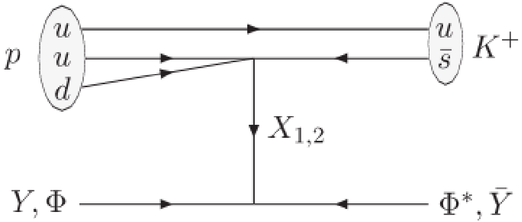
Three body decay

- Resonant to vector meson channels : $p \to K^* + \bar{\ell} \to (K\pi)\bar{\ell}$
- Decay rate ratio $(\Gamma(p \to \pi\pi e^+)/\Gamma(p \to \pi e^+))$ estimates to $\sim$ 24–150% [Wise,1980]
- Prime channel of next generation experiment
- Numerically cheapest among three body decay channels

Decay Mode	Water Cherenkov		Liquid Argon TPC	
	Efficiency	Background	Efficiency	Background
$p \to K^+\bar{\nu}$	19%	4	97%	1
$p \to K^0\mu^+$	10%	8	47%	< 2
$p \to K^+\mu^-\pi^+$			97%	1
$n \to K^+e^-$	10%	3	96%	< 2
$n \to e^+\pi^-$	19%	2	44%	0.8

DUNE proton decay efficiency

Induced Nucleon Decay model [Davoudiasl, 2010]



- DM can annihilate the nucleon
- $\mathcal{L}_{eff} \sim (1/\Lambda^3) u_R d_R d_R Y_R \Phi +$ h.c.
- $\langle \Pi(p') | O^{\Gamma\Gamma'}(q) | N(p,s) \rangle = P_{\Gamma'} \left[W_0^{\Gamma\Gamma'}(q^2) + \frac{m_Y}{m_N} W_1^{\Gamma\Gamma'}(q^2) \right] u_N(p,s)$

Conclusion

- Proton decay matrix elements on the two lattice ensemble with chiral fermions at physical scale
- Two-loop NPR, with Three-loop perturbative running is done
- Three quark op. non perturbative renormalization
- New channels(vector meson, three body)
- Induced nucleon decay by DM

The End