Parton physics from Euclidean current-current correlator with a valence heavy quark: pion light-cone distribution amplitude as an example

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THE HOPE COLLABORATION

Outline

- General HOPE strategy

W. Detmold and CJDL, PRD 73 (2006)

- HOPE and the pion light-cone wavefunction

W.Detmold, A.Grebe, I.Kanamori, CJDL, S.Mondal, R.Perry, Y.Zhao,
Contribution to APLAT2020, arXiv:2009.09473.

- Preliminary numerical result

W.Detmold, A.Grebe, I.Kanamori, CJDL, S.Mondal, R.Perry, Y.Zhao,
Contribution to APLAT2020, arXiv:2009.09473.

- Outlook

General strategy

Parton distribution from lattice QCD

The "traditional" approach

Hadronic tensor (PDFs from the twist-2 sector)

$$W_S^{\mu\nu}(p,q) = \int d^4x \, \mathrm{e}^{iq\cdot x} \langle p, S| \left[J^\mu(x), J^\nu(0)\right] |p, S\rangle$$

Parton distribution from lattice QCD

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optical theorem | Imaginary part → challenging in Euclidean QCD

$$T_S^{\mu\nu}(p,q) = \int d^4x \, \mathrm{e}^{iq\cdot x} \langle p, S| T\left[J^\mu(x) J^\nu(0) \right] |p, S\rangle$$

Parton distribution from lattice QCD

The "traditional" approach

Hadronic tensor (PDFs from the twist-2 sector)

$$W_S^{\mu\nu}(p,q) = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle p,S|\, [J^\mu(x), J^\nu(0)]\, |p,S\rangle$$

optical theorem

Imaginary part

challenging in Euclidean QCD

$$T_S^{\mu\nu}(p,q) = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle p,S|T\,[J^\mu(x) J^\nu(0)]\, |p,S\rangle$$

Light-cone OPE

$$T[J^\mu(x)J^\nu(0)] = \sum_{i,n} \mathcal{C}_i\left(x^2,\mu^2\right)\, x_{\mu_1}\ldots x_{\mu_n} \mathcal{O}_i^{\mu\nu\mu_1\ldots\mu_n}(\mu)$$

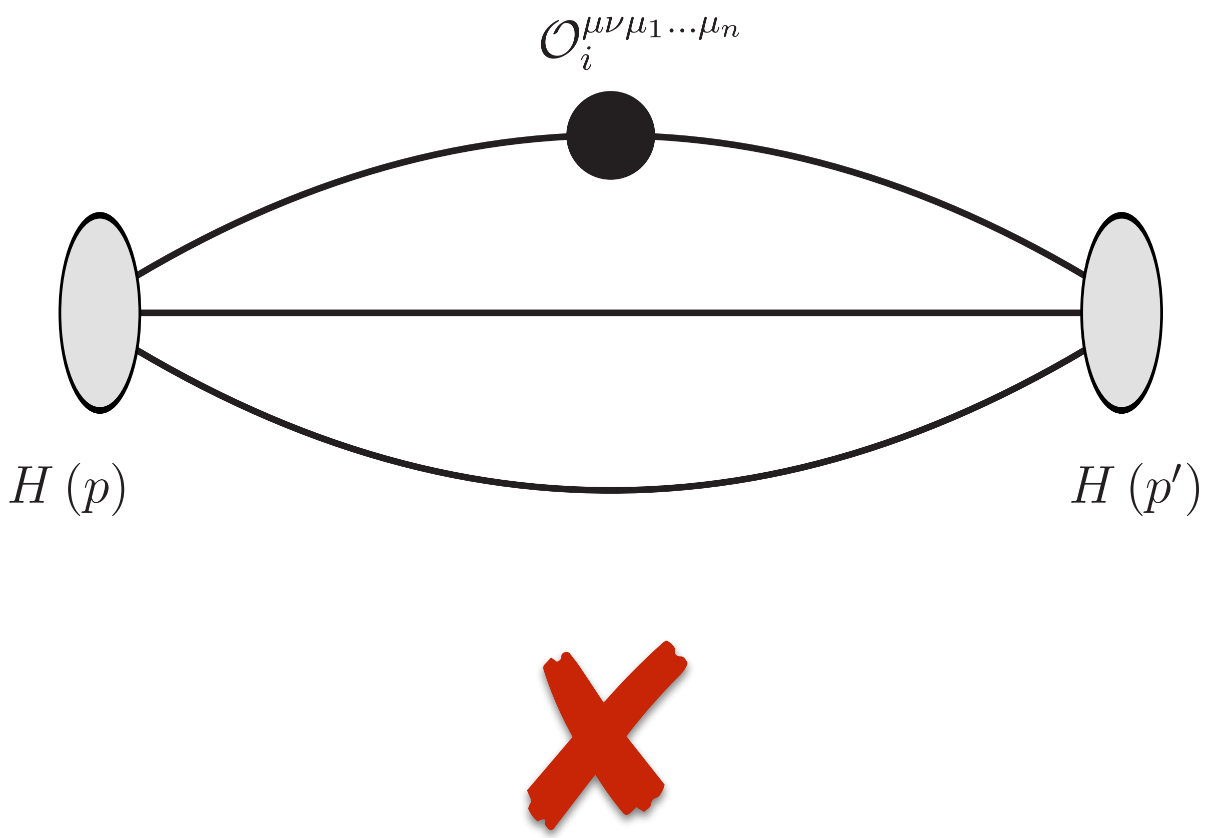
local operators, issue of operator mixing

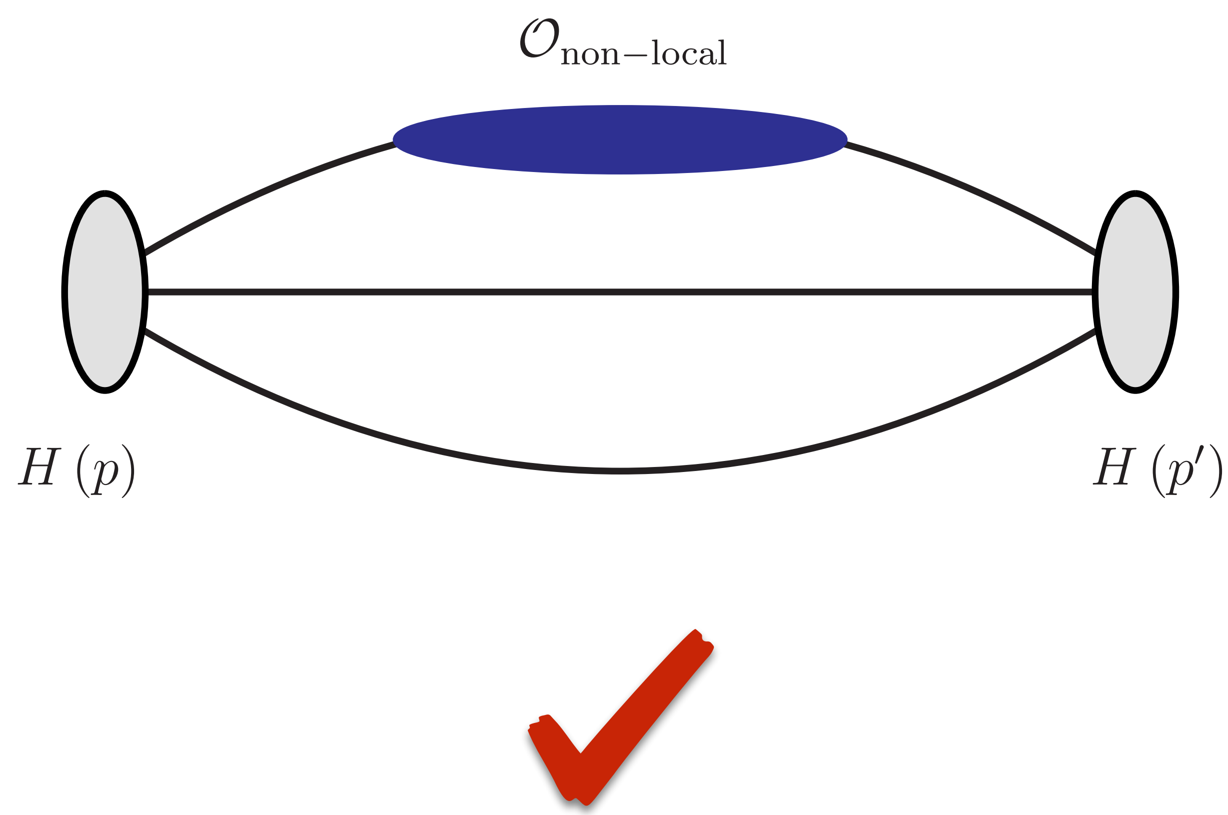
leading moments in practice

Power divergences arising from Lorentz symmetry breaking

Parton distribution from lattice QCD

The “new” approach to avoid difficulties in renormalisation

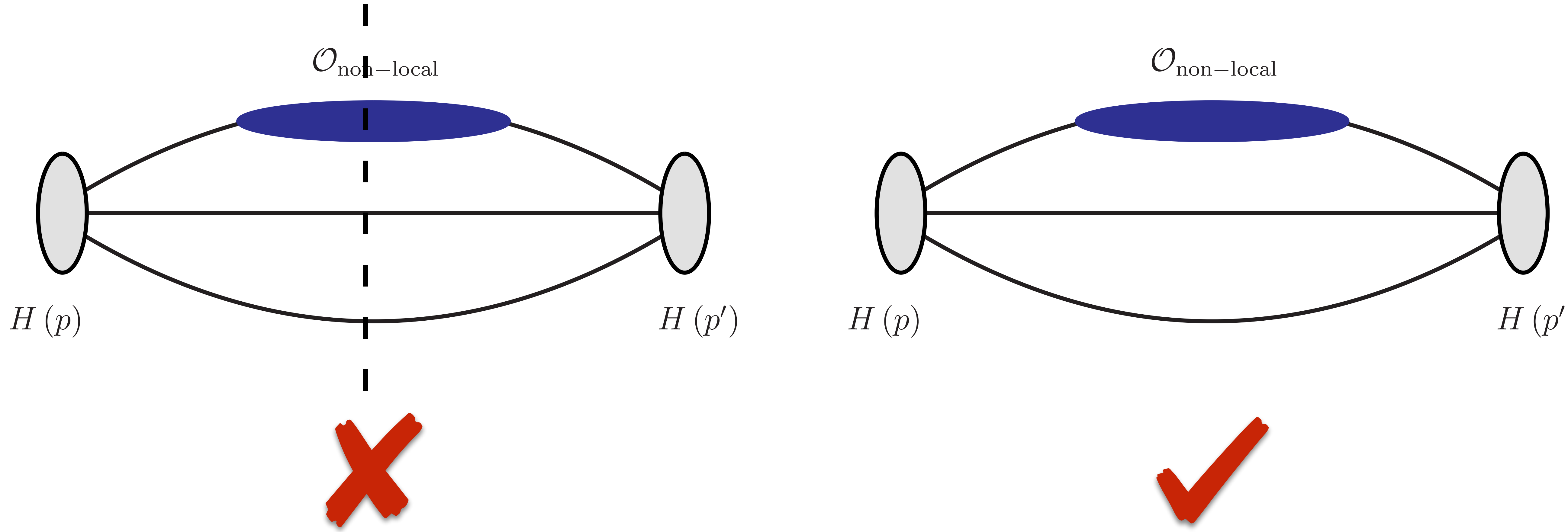




General idea: Inserting non-local, instead of local, operator

Parton distribution from lattice QCD

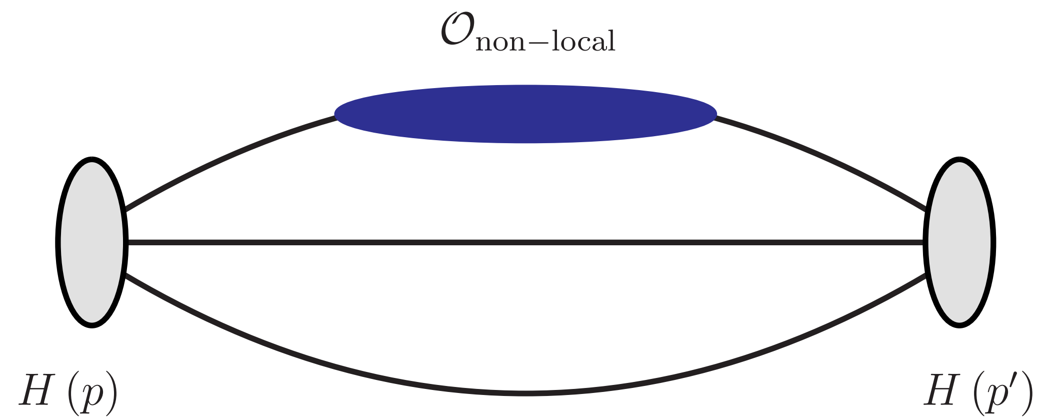
The "new" approach to avoid difficulties in renormalisation



Make certain the absence of on-shell states for analytic continuation

Parton distribution from lattice QCD

The “new” approach to avoid difficulties in renormalisation



Typical examples of the non-local operator

A space-like Wilson line (quasi-PDF and pseudo-PDF)
X. Ji, PRL 110 (2013); A. Radyushkin, PRD 96 (2017)

Smeared “local” operators
Z. Davoudi and M. Savage, PRD 86 (2012)

Two currents separated by space-like distance
V. Braun and D. Mueller, EPJC 55 (2018)

Two flavour-changing currents with valence heavy quark
W. Detmold and CJDL, PRD 73 (2006)

And other proposals
A. Chambers *et al.*, PRL 118 (2017); Y. Ma and J.-W. Qiu, PRL 120 (2018);……

Introducing the valence heavy quark

W. Detmold and CJDL, PRD 73 (2006)

Valence → Not in the action

The “heavy quark” is relativistic → propagating in both space and time

The current for computing the even moments of the PDF

$$J^{\mu}_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^{\mu}\psi(x) + \overline{\psi}(x)\gamma^{\mu}\Psi(x)$$

↓ Compton tensor

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_{S} \langle p,S|t^{\mu\nu}_{\Psi,\psi}(q)|p,S\rangle = \sum_{S} \int d^4x\ \mathrm{e}^{iq\cdot x} \langle p,S|T\left[J^{\mu}_{\Psi,\psi}(x) J^{\nu}_{\Psi,\psi}(0)\right]|p,S\rangle$$

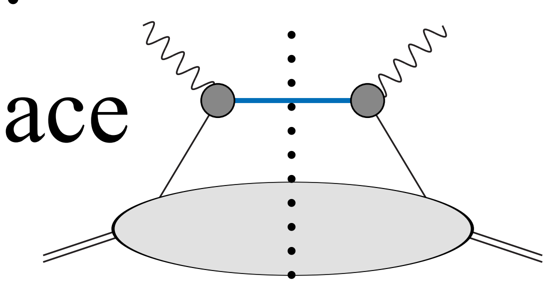
Strategy for extracting the moments

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_S \langle p,S|t^{\mu\nu}_{\Psi,\psi}(q)|p,S\rangle = \sum_S \int d^4x\ \mathrm{e}^{iq\cdot x}\langle p,S|T\left[J^{\mu}_{\Psi,\psi}(x)J^{\nu}_{\Psi,\psi}(0)\right]|p,S\rangle$$

$$J^{\mu}_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^\mu\psi(x) + \overline{\psi}(x)\gamma^\mu\Psi(x)$$

- Simple renormalisation for quark bilinears.

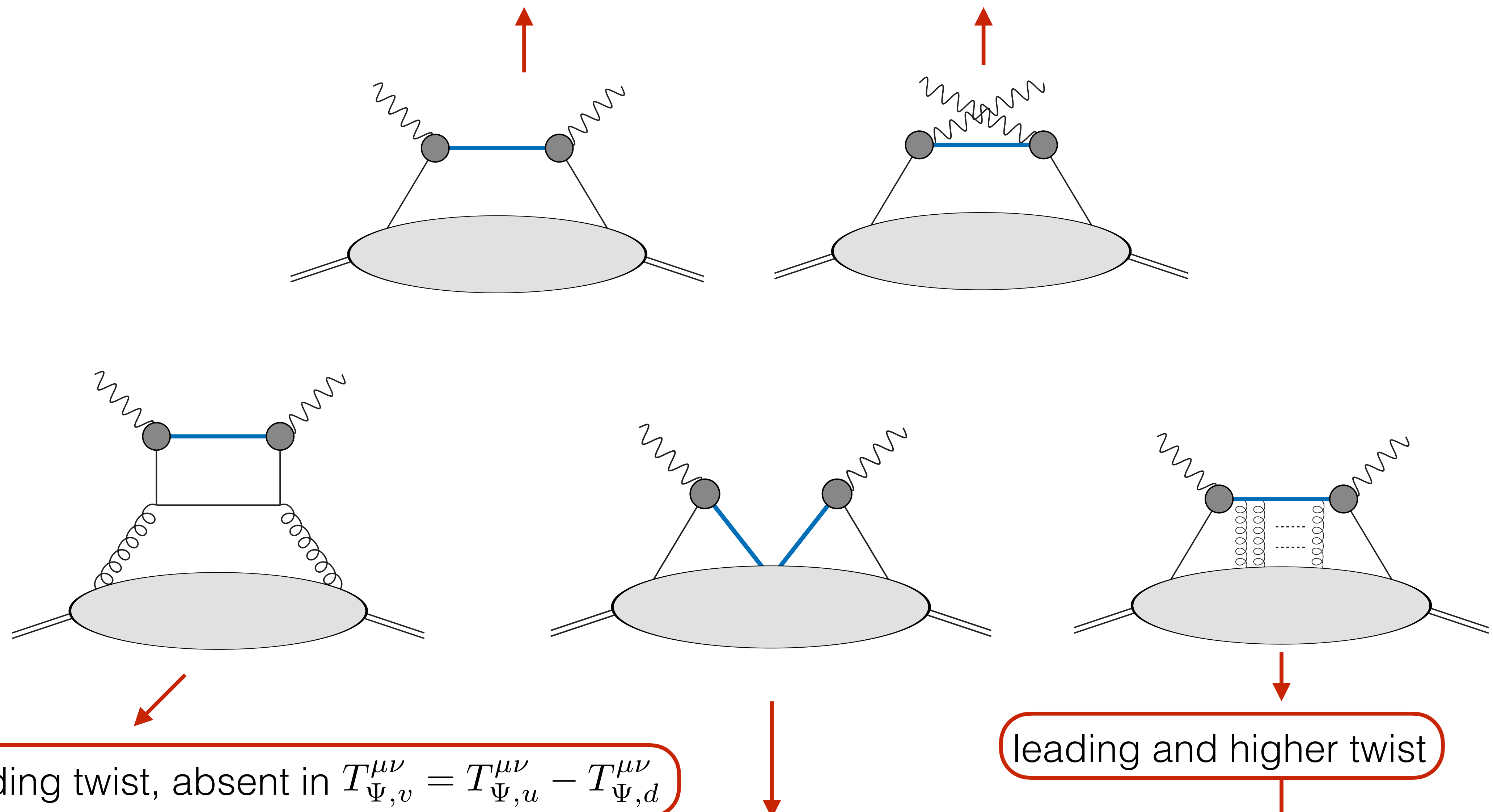
- Work with the hierarchy of scales $\Lambda_{\mathrm{QCD}} << \sqrt{q^2} \leq m_\Psi << \frac{1}{a}$
  - → Heavy scales for short-distance OPE.
  - → Avoid branch point in Minkowski space at $(q+p)^2 \sim (m_N + m_\Psi)^2$



- Extrapolate $T^{\mu\nu}_{\Psi,\psi}(p,q)$ to the continuum limit first.
  - → Then match to the short-distance OPE results.
  - → Extract the moments without power divergence.

Short-distance OPE & valence heavy quark

These are the leading-twist contributions that we are after.



leading twist, absent in $T^{\mu\nu}_{\Psi,v} = T^{\mu\nu}_{\Psi,u} - T^{\mu\nu}_{\Psi,d}$

higher twist, absent

leading and higher twist

ambiguity in heavy quark mass

HOPE
and pion light-cone distribution amplitude

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Pion light-cone wavefunction

$$\langle 0|\overline{\psi}(z_2 n)\not{n}\gamma_5 W[z_2 n, z_1 n]\psi(z_1 n)\left|\pi^+(\mathbf{p})\right\rangle$$

$$= i f_\pi (p\cdot n)\int_0^1 dx\, e^{-i(z_1 x + z_2(1-x))p\cdot n}\phi_\pi(x,\mu^2)$$

$$\langle \xi^n\rangle_{\mu^2} = \int_{-1}^{1} d\xi\, \xi^n \phi_\pi(\xi,\mu^2)$$

Mellin moments

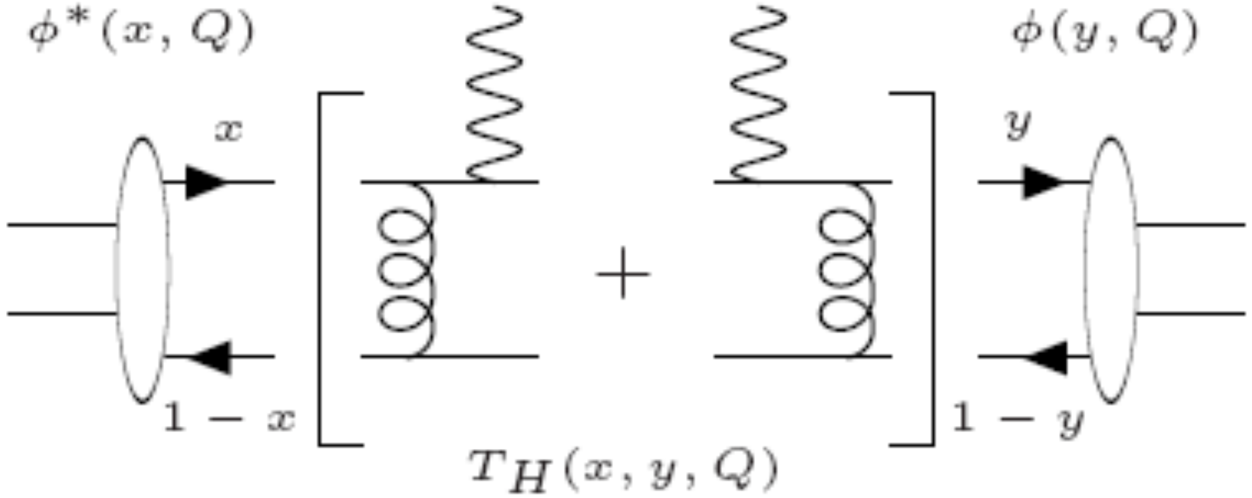
OPE

$$\langle 0|O_\psi^{\mu_1\ldots\mu_n}|\pi(p)\rangle = f_\pi \langle\xi^{n-1}\rangle [p^{\mu_1}\ldots p^{\mu_n} - \text{traces}]$$

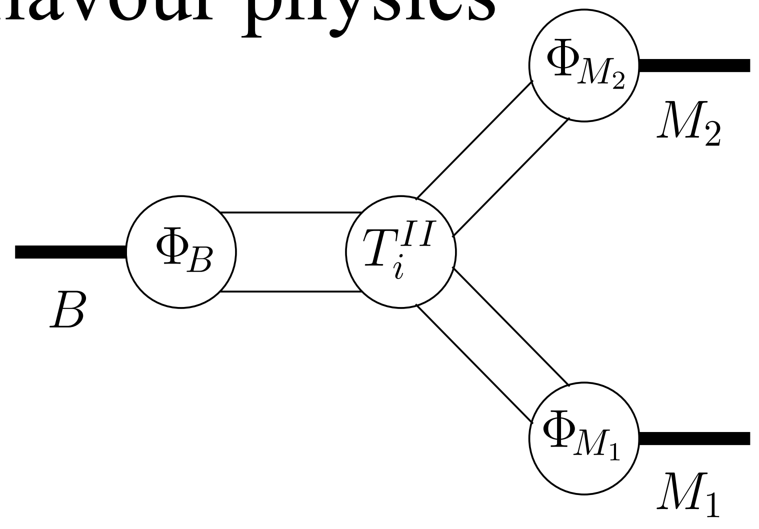
$$O_\psi^{\mu_1\ldots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu_1}(iD^{\mu_2})\ldots(iD^{\mu_n\}})\psi - \text{traces}$$

Phenomenological relevance

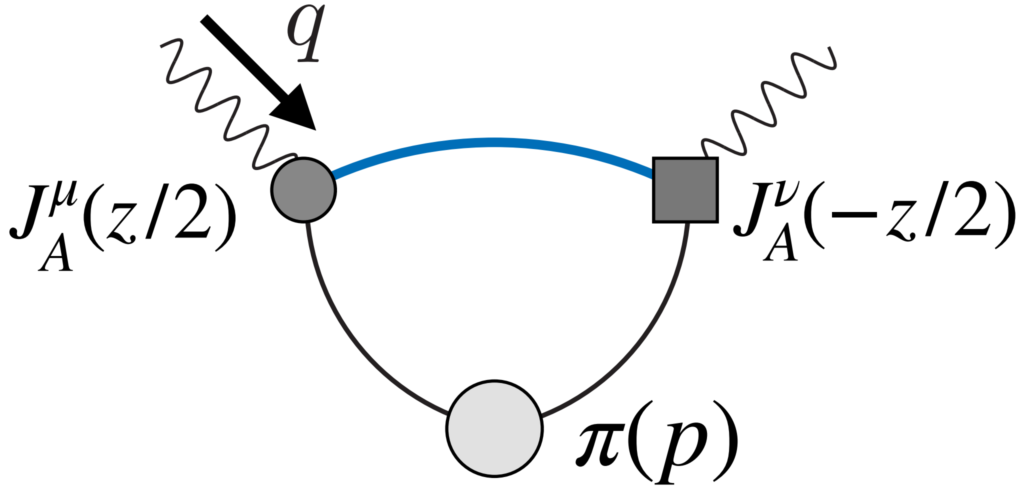
Pion form factor in QCD exclusive processes



Important input for flavour physics



Hadronic tensor for computing pion LCDA



+ the cross term

$$T^{\mu\nu}(p,q) = \int d^4 z\, e^{iq\cdot z} \langle 0| \mathcal{T}[J_A^\mu(z/2) J_A^\nu(-z/2)] |\pi(\mathbf{p})\rangle$$

$$J_A^\mu = \bar{\Psi}\gamma^\mu\gamma^5\psi + \bar{\psi}\gamma^\mu\gamma^5\Psi$$

Ψ is the valence, relativistic heavy quark

$$U^{\mu\nu}(p,q) = \frac{1}{2}\Big(T^{\mu\nu}(p,q) - T^{\nu\mu}(p,q)\Big)$$

OPE for the hadronic tensor: Euclidean result

$$U^{\mu\nu}(p,q) = \frac{2\epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2} \sum_{n \text{ even}}^{\infty} \frac{\zeta^n \mathcal{C}_n^2(\eta)}{2^n(n+1)} C_W^{(n)}(\tilde{Q}^2) f_\pi \langle \xi^n \rangle + \mathcal{O}(1/\tilde{Q}^3)$$

higher-twist

$\mu = 2$ GeV in this talk

$$\tilde{Q}^2 = q^2 + m_\Psi^2$$

$$\eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}, \quad \zeta = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2}$$

$\mathcal{C}_n^2(\eta)$: target-mass effect

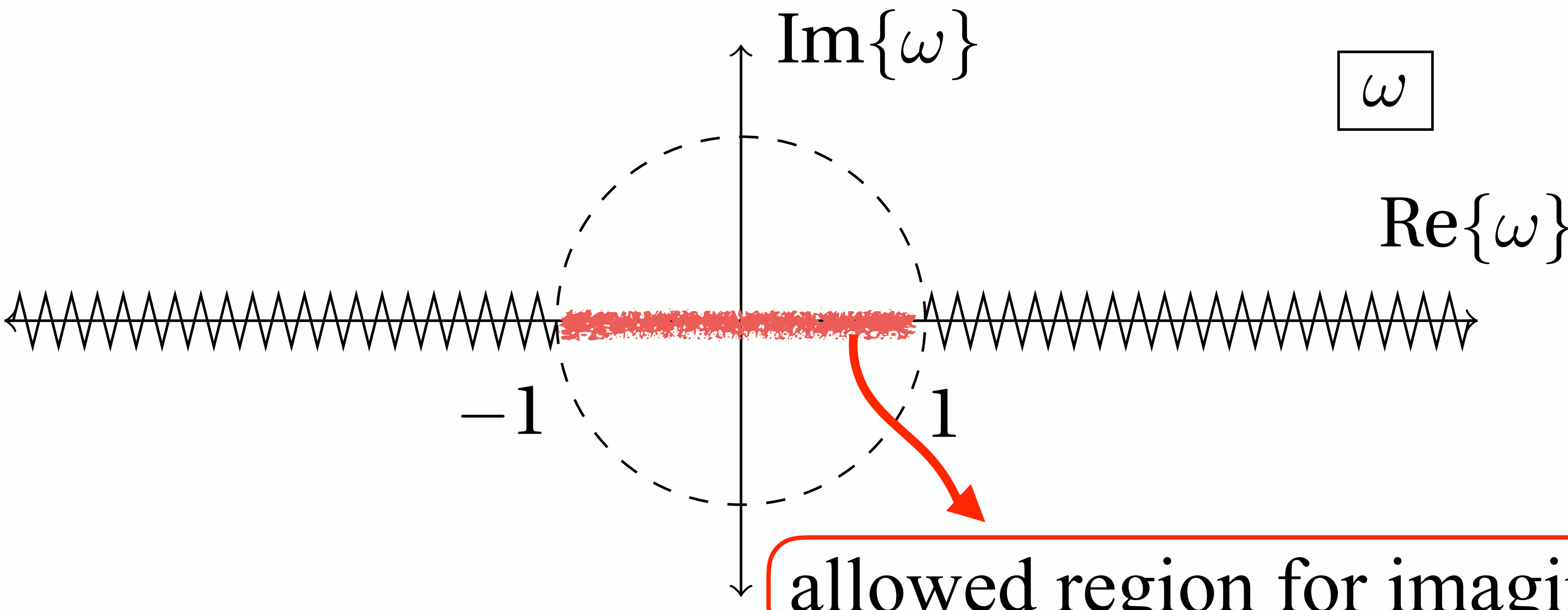
- tree-level OPE
- one-loop
- fit lattice data

OPE for $U^{\mu\nu}$: issue in fitting higher moments

$$U^{\mu\nu}(p,q) \sim \sum_{n=0}^{\infty} \langle \xi^n \rangle \, \omega^n, \quad \omega = \frac{2p\cdot q}{\tilde{Q}^2} = \frac{2\mathbf{p}\cdot\mathbf{q}}{q_4^2 + \mathbf{q}^2 + m_Q^2} + \frac{2iE_\pi q_4}{q_4^2 + \mathbf{q}^2 + m_Q^2}$$

suppressing higher-moments

need large **p** to make $\omega \to 1$

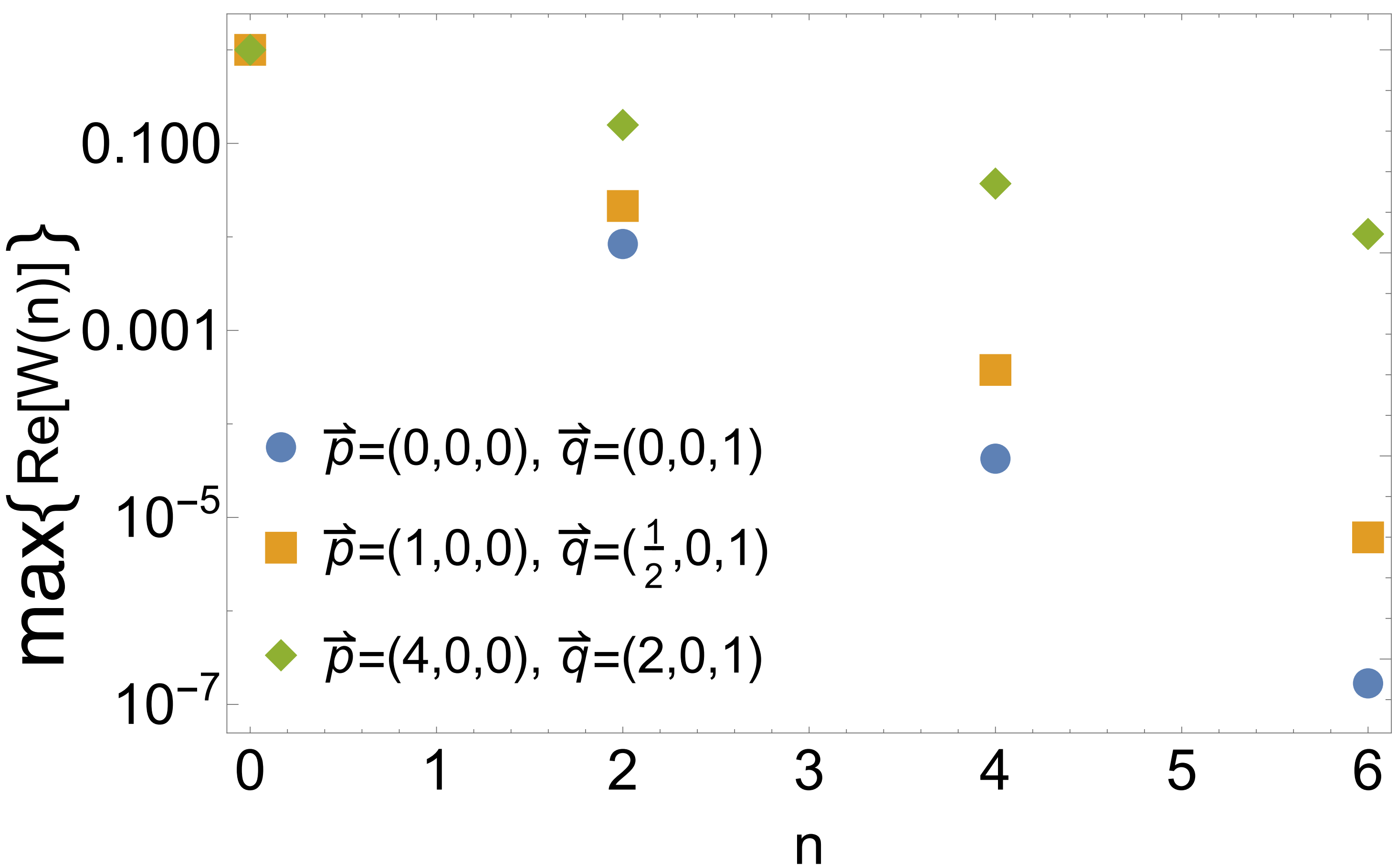


allowed region for imaginary q_4

OPE for $U^{\mu\nu}$: issue in fitting higher moments

$$U^{\mu\nu}(p,q) = \frac{2\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta}{\tilde{Q}^2}\sum_{n\ \text{even}}^{\infty}\frac{\zeta^n \mathcal{C}_n^2(\eta)}{2^n(n+1)}C_W^{(n)}(\tilde{Q}^2)f_\pi\langle\xi^n\rangle + \mathcal{O}(1/\tilde{Q}^3)$$

$$= \frac{2\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta}{\tilde{Q}^2}\sum_{n\ \text{even}}^{\infty}W(n)C_W^{(n)}(\tilde{Q}^2)f_\pi\langle\xi^n\rangle + \mathcal{O}(1/\tilde{Q}^3)$$



In general, need large **p** to access non-leading moments

Strategy for fitting $\langle \xi^2 \rangle$ at low pion momentum

$$U^{12}(p,q) = \frac{2\epsilon^{12\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2} \sum_{n \text{ even}}^{\infty} \frac{\zeta^n \mathcal{C}_n^2(\eta)}{2^n(n+1)} C_W^{(n)}(\tilde{Q}^2) f_\pi \langle \xi^n \rangle + \mathcal{O}(1/\tilde{Q}^3)$$

$$= \frac{2(q_3 p_4 - q_4 p_3)}{\tilde{Q}^2} \left[C_W^{(0)}(\tilde{Q}^2) f_\pi + \frac{6(p\cdot q)^2 - p^2 q^2}{6(\tilde{Q}^2)^2} C_W^{(2)}(\tilde{Q}^2) f_\pi \langle \xi^2 \rangle + \ldots \right] + \mathcal{O}(1/\tilde{Q}^3)$$

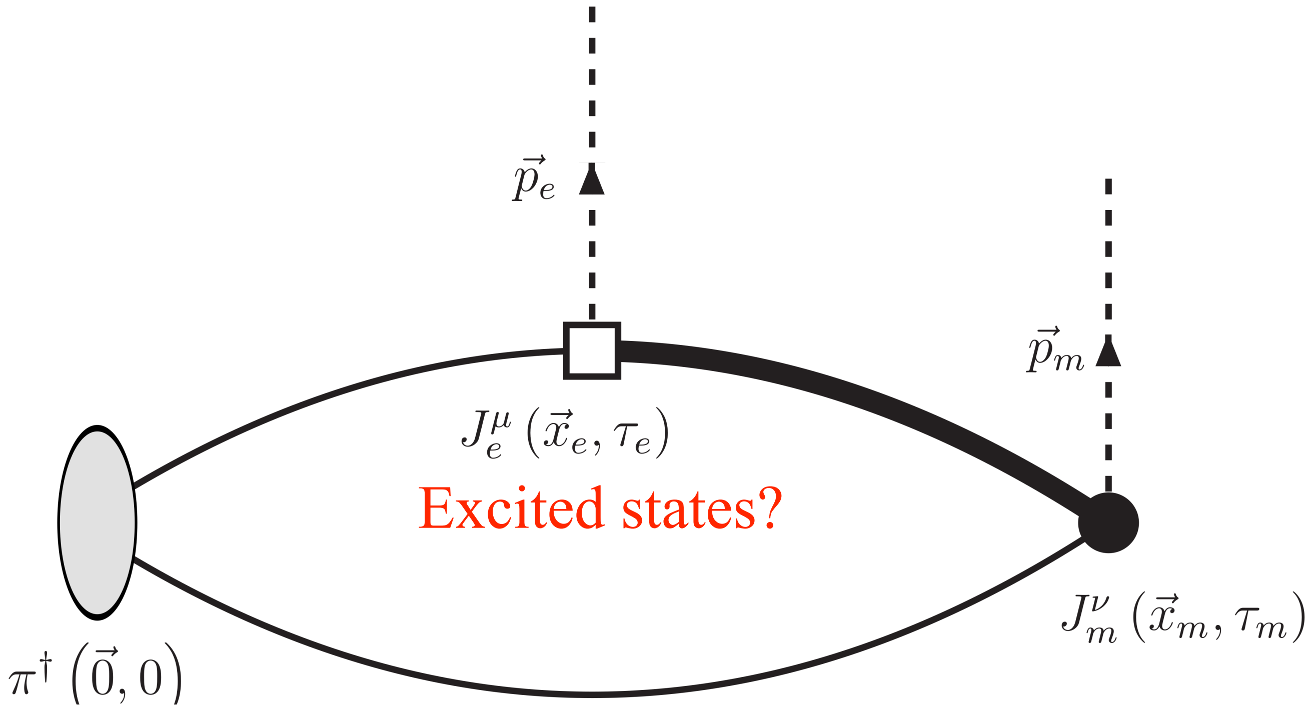
choose **p·q**≠0 while p_3=0, $q_3 \neq 0$ and q_4 being real

$$= \frac{2 i q_3 E_\pi}{\tilde{Q}^2} \left[C_W^{(0)}(\tilde{Q}^2) f_\pi + \frac{6(p\cdot q)^2 - p^2 q^2}{6(\tilde{Q}^2)^2} C_W^{(2)}(\tilde{Q}^2) f_\pi \langle \xi^2 \rangle + \ldots \right] + \mathcal{O}(1/\tilde{Q}^3)$$

imaginary — real — complex

→ The largest contribution to Re[U^{12}] is from $\langle \xi^2 \rangle$

Correlators for lattice calculation



$$C_3^{\mu\nu}(\tau_e, \tau_m; \mathbf{p}_e, \mathbf{p}_m) = \int d^3\mathbf{x}_e \, d^3\mathbf{x}_m \, e^{i\mathbf{p}_e\cdot\mathbf{x}_e} e^{i\mathbf{p}_m\cdot\mathbf{x}_m} \langle 0|\mathcal{T}\left[J_A^\mu(\mathbf{x}_e, \tau_e) J_A^\nu(\mathbf{x}_m, \tau_m)\mathcal{O}_\pi^\dagger(\mathbf{0}, 0)\right]|0\rangle$$

$$C_2(\tau_\pi, \mathbf{p}) = \int d^3\mathbf{x} \, e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\mathcal{O}_\pi(\mathbf{x}, \tau_\pi)\mathcal{O}_\pi^\dagger(\mathbf{0}, 0)|0\rangle$$

$R^{\mu\nu}$ and the Fourier transform for $U^{\mu\nu}$

From $C_3^{\mu\nu}$ and C_2 , one can construct

$$R^{\mu\nu}(\tau; \mathbf{p}, \mathbf{q}) = \int d^3\mathbf{z}\, e^{i\mathbf{q}\cdot\mathbf{z}} \langle 0|\mathcal{T}\left[J^\mu\left(\frac{z}{2}\right) J^\nu\left(-\frac{z}{2}\right)\right]|\pi(\mathbf{p})\rangle$$

$$z = x_e - x_m, \quad \mathbf{p} = \mathbf{p}_e + \mathbf{p}_m, \quad \mathbf{q} = \frac{1}{2}(\mathbf{p}_m - \mathbf{p}_e)$$

Then the hadronic tensor can be obtained *via*

$$U^{\mu\nu}(p,q) \equiv \int d\tau\, e^{iq_4\tau} R^{[\mu\nu]}(\tau; \mathbf{p}, \mathbf{q})$$

Exploratory quenched calculation @ $M_\pi \approx 560$ MeV

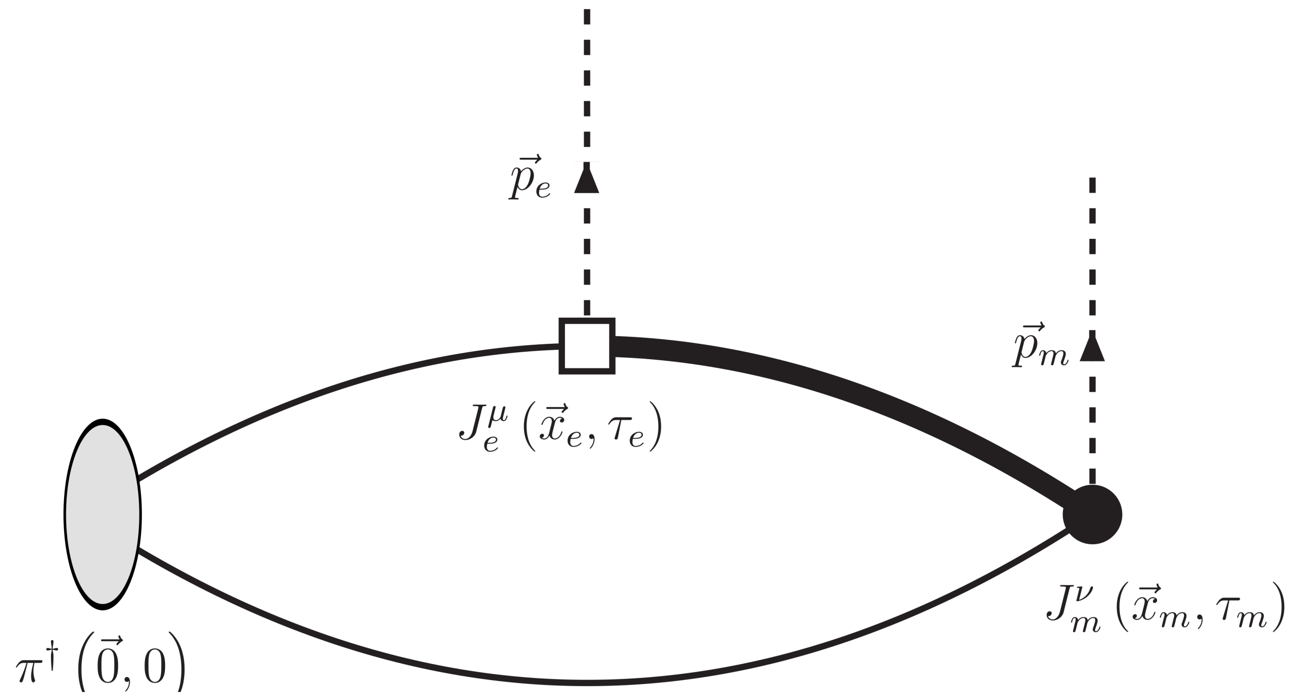
Wilson plaquette and non-perturbatively improved clover actions

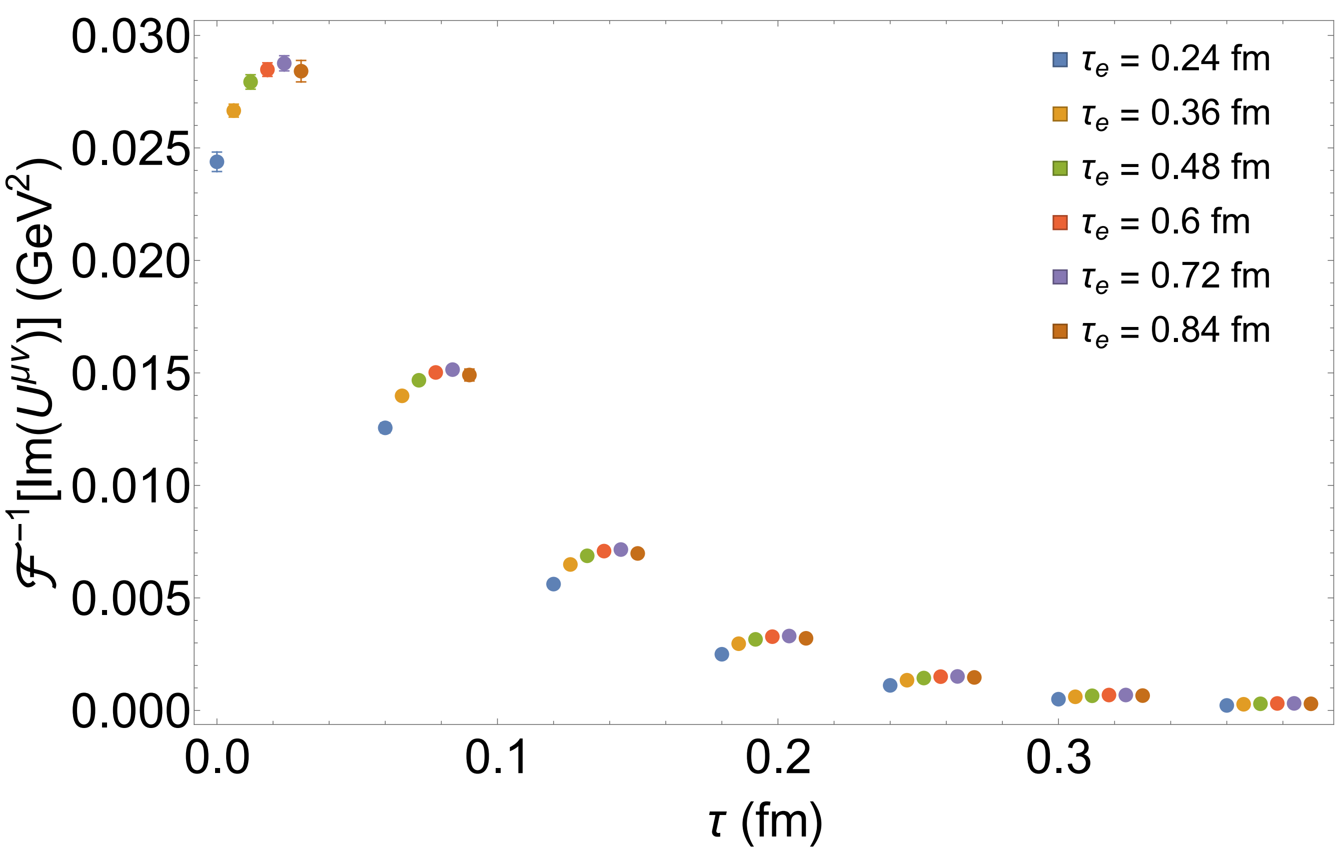
a (fm)	$\hat{L}^3 \times \hat{T}$	$N_{\rm config}$	$N_{\rm src}$
0.081	$24^3 \times 48$	650	2
0.060	$32^3 \times 64$	450	3
0.048	$40^3 \times 80$	250	3
0.041	$48^3 \times 96$	341	3

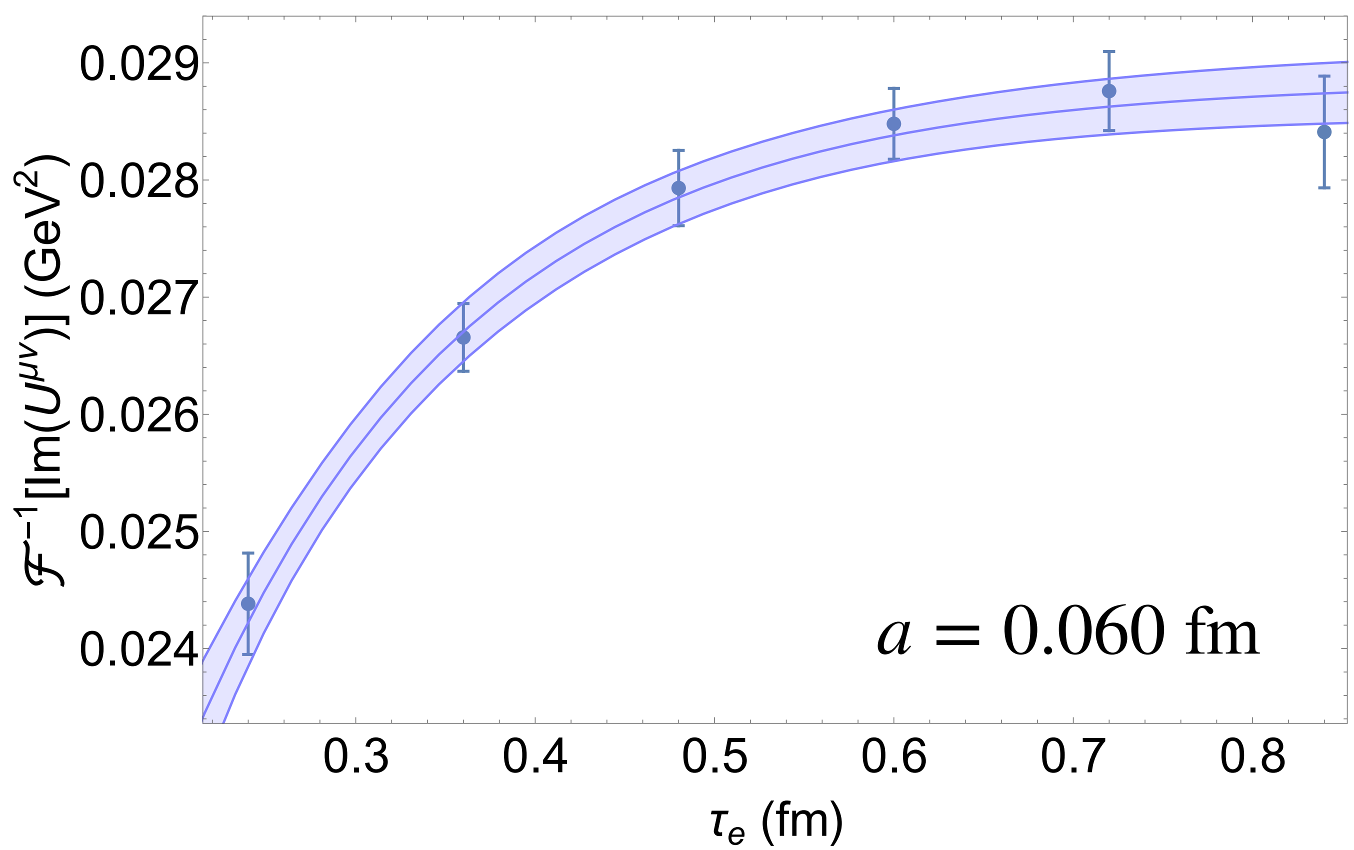
bare m_Ψ	fitted m_Ψ
1.0 GeV	2.0 GeV
1.6 GeV	2.6 GeV
2.5 GeV	3.3 GeV

- $\mathbf{p} = (1,0,0)$ $\mathbf{q} = (1/2,0,1)$ in units of $2\pi/L \sim 0.64$GeV
- $U^{\mu\nu}$ is $O(a)$ improved without improving the axial current

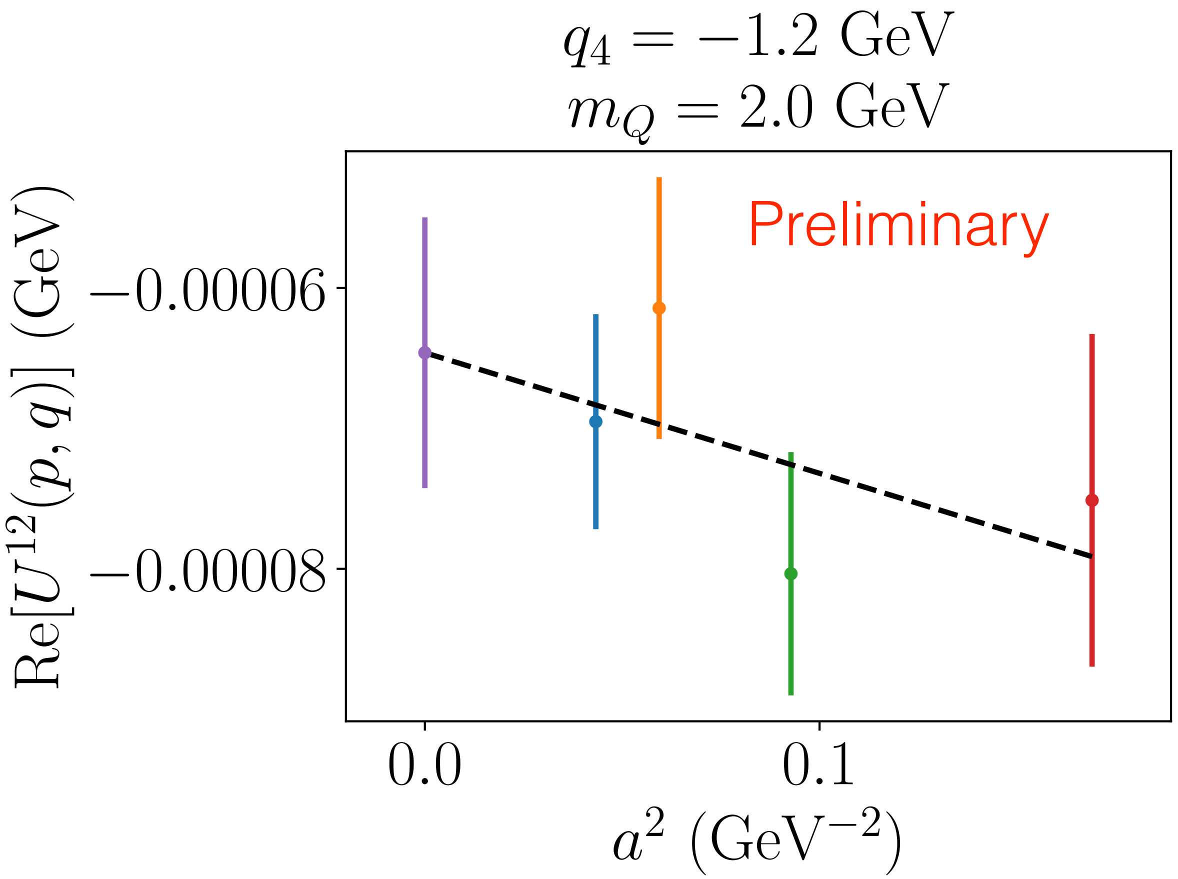
Excited-state contamination

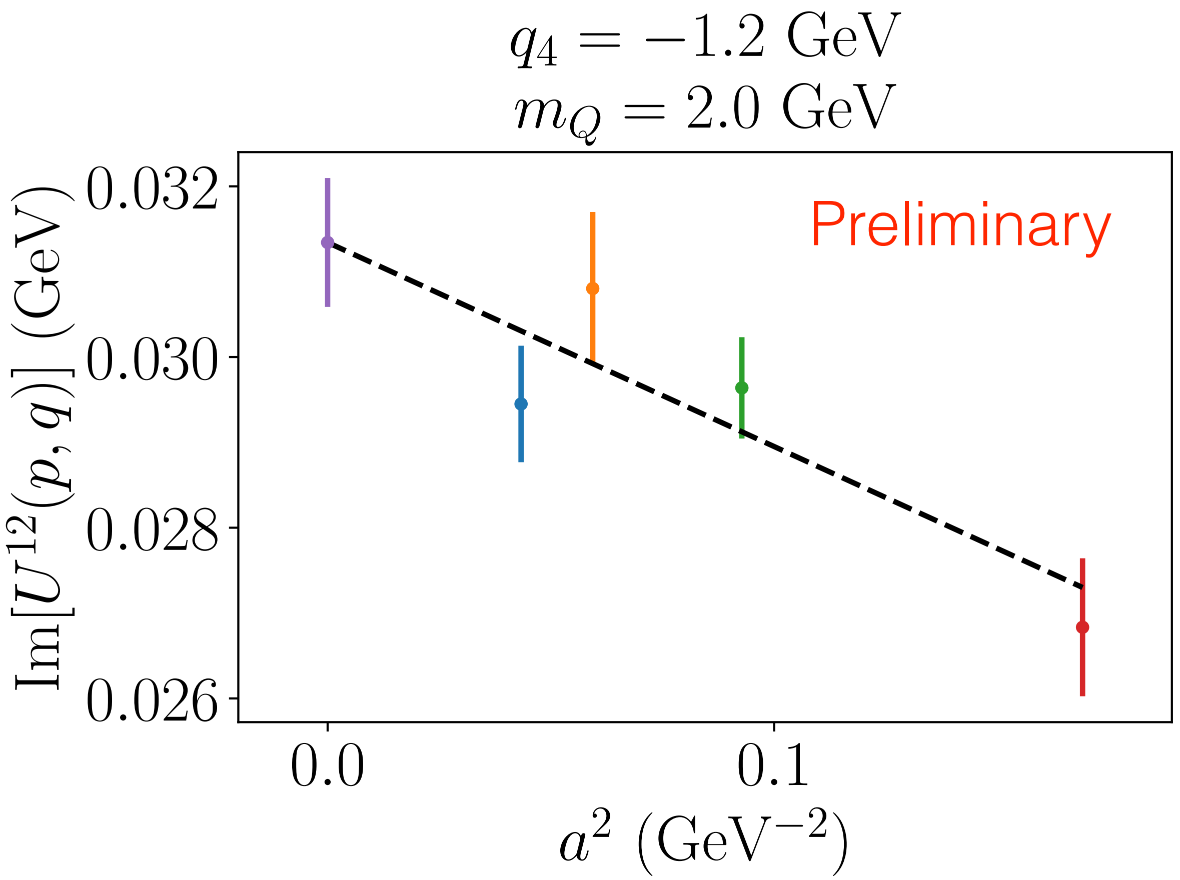






Continuum extrapolation of U^{12}





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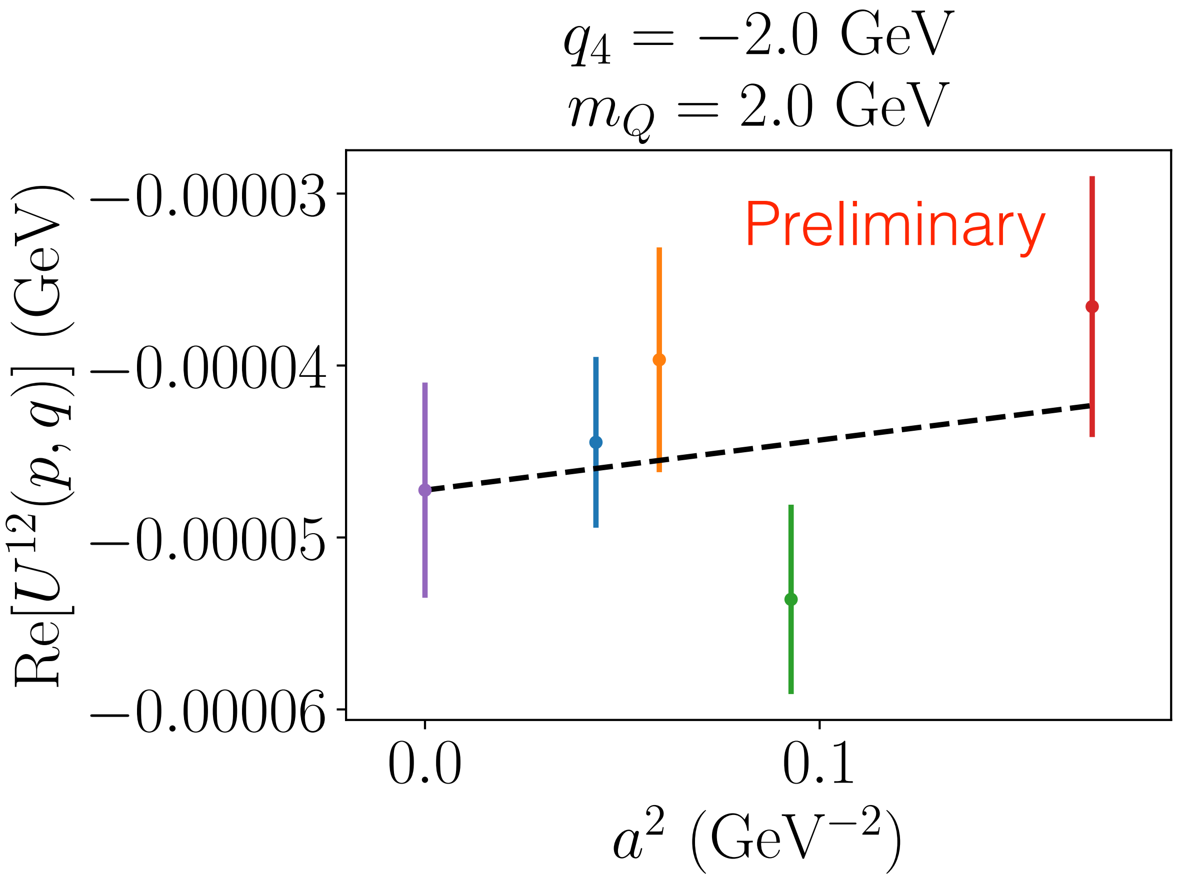
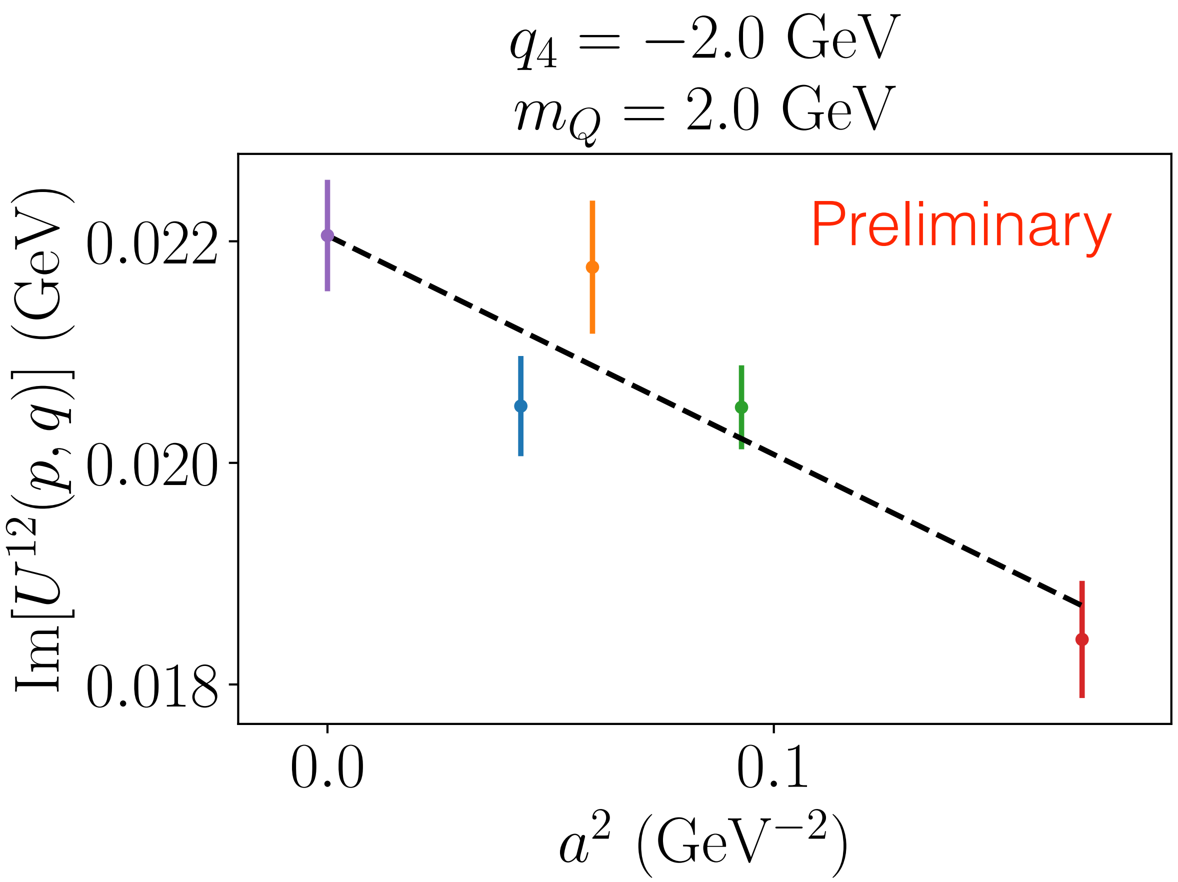
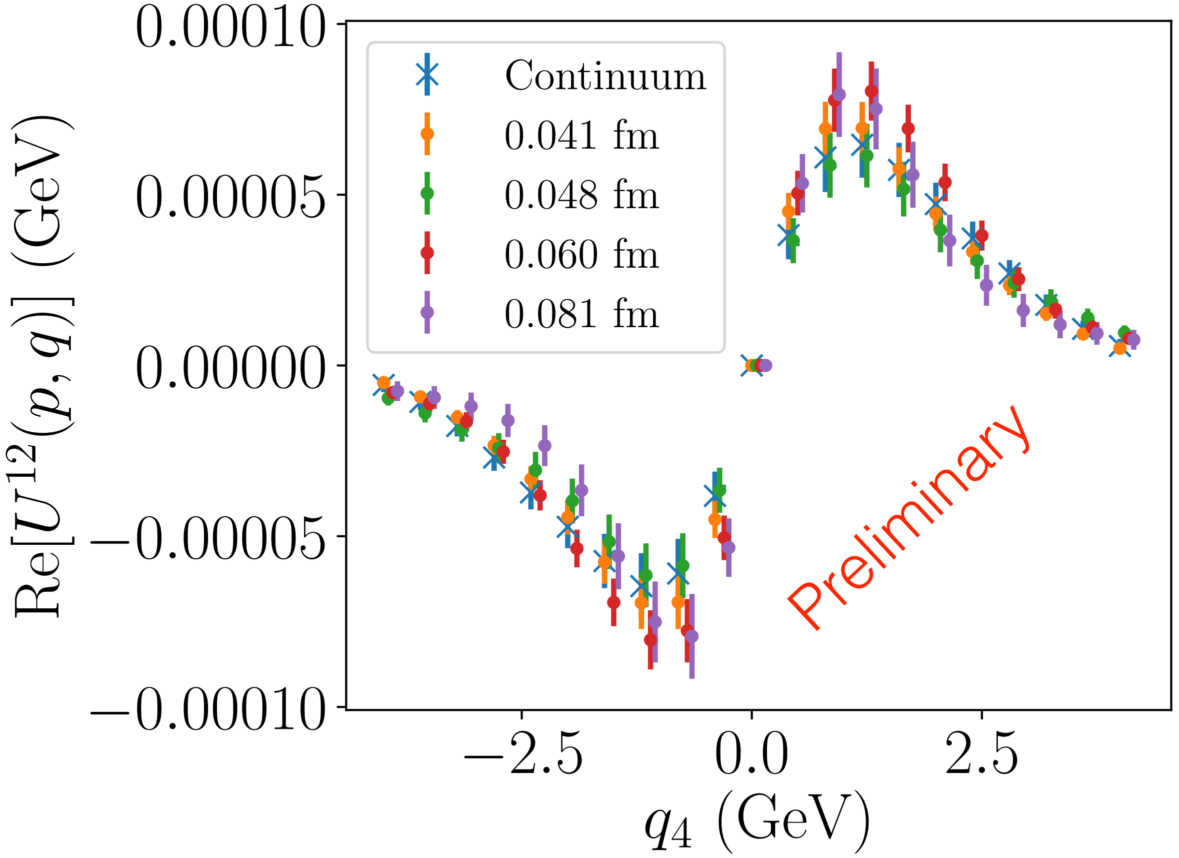
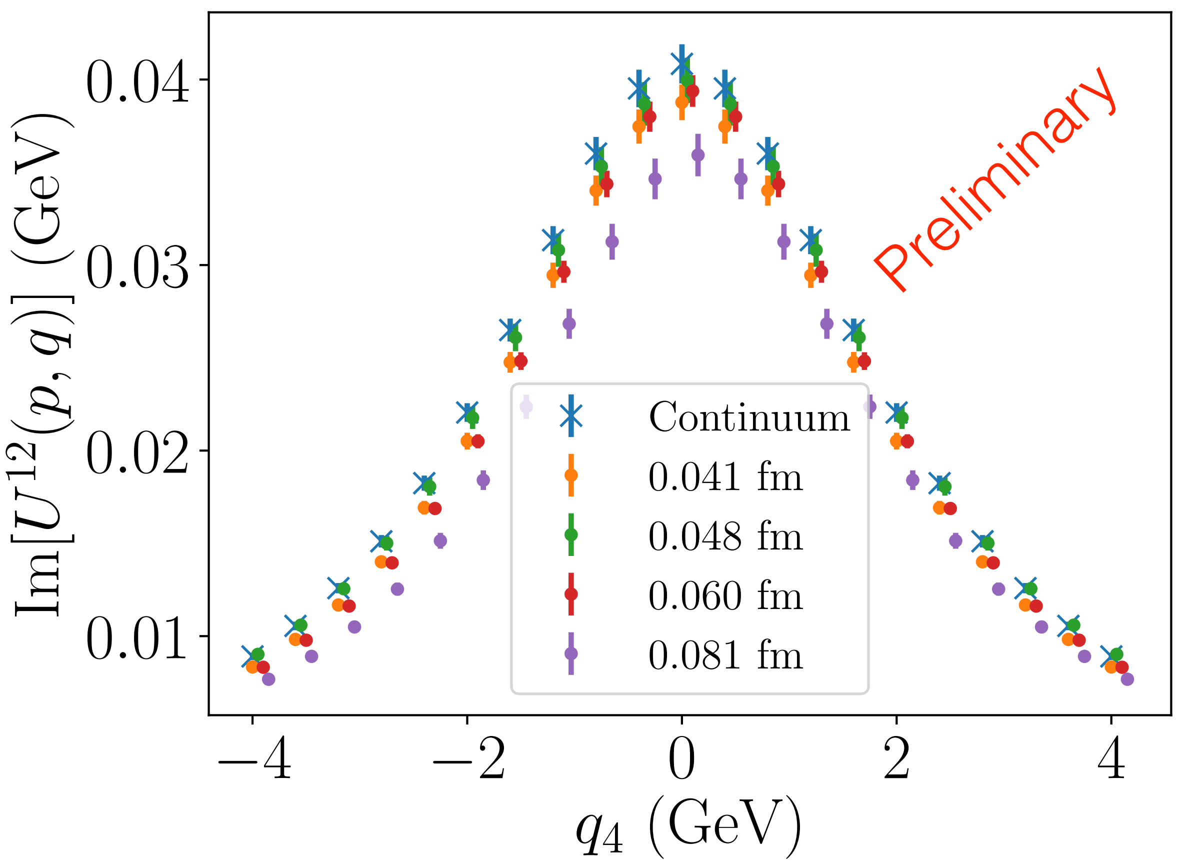




Results of U^{12}





OPE fits in momentum and position spaces

- Momentum space: fit the continuum-limit U^{12} to

$$U^{12}(p,q) = \frac{2iq_3 E_\pi}{\tilde{Q}^2}\left[C_W^{(0)}(\tilde{Q}^2) f_\pi + \frac{6(p\cdot q)^2 - p^2 q^2}{6(\tilde{Q}^2)^2} C_W^{(2)}(\tilde{Q}^2) f_\pi \langle \xi^2 \rangle + \ldots\right] + \mathcal{O}(1/\tilde{Q}^3)$$

- Position space: Fourier transform ✔

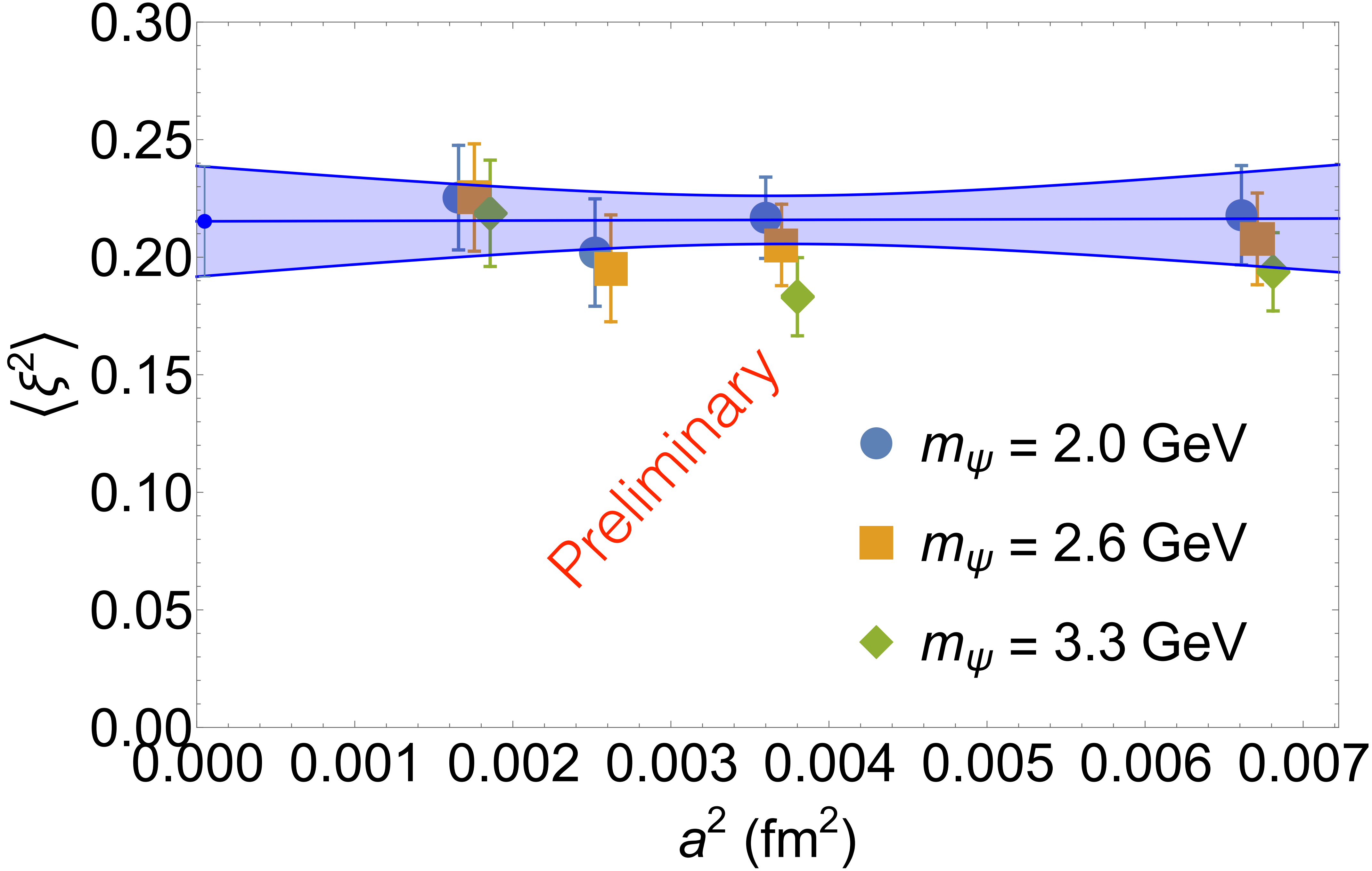
$$\tilde{U}^{\mu\nu}(p,\mathbf{q},\tau) = \int d\tau\ e^{-iq_4\tau} U^{\mu\nu}(p,q)$$

➞ Allows for determining $\langle \xi^2 \rangle$ at finite lattice spacing

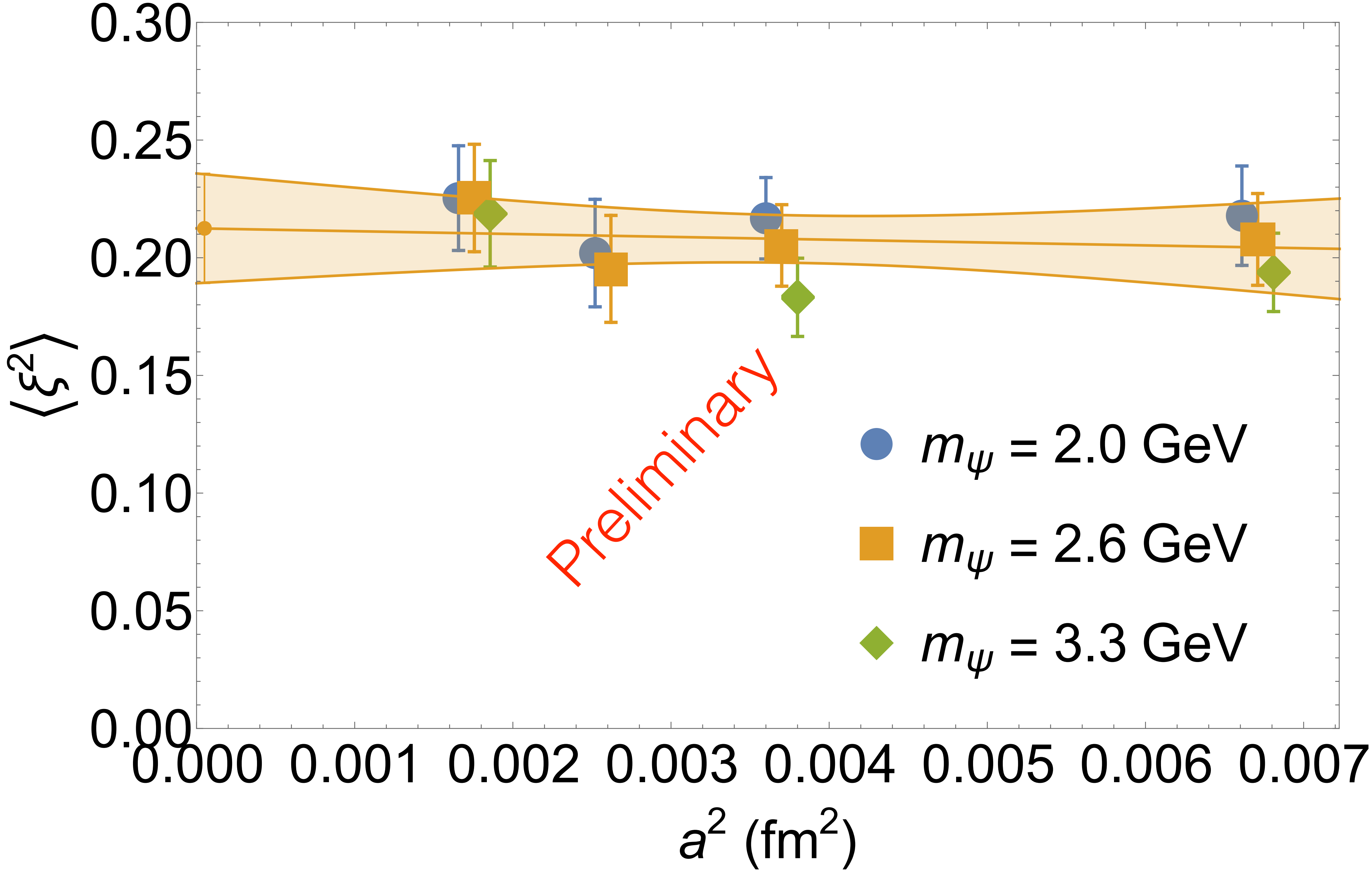
➞ Offers a different analysis procedure

➞ Less sensitive to Z_A and b_A

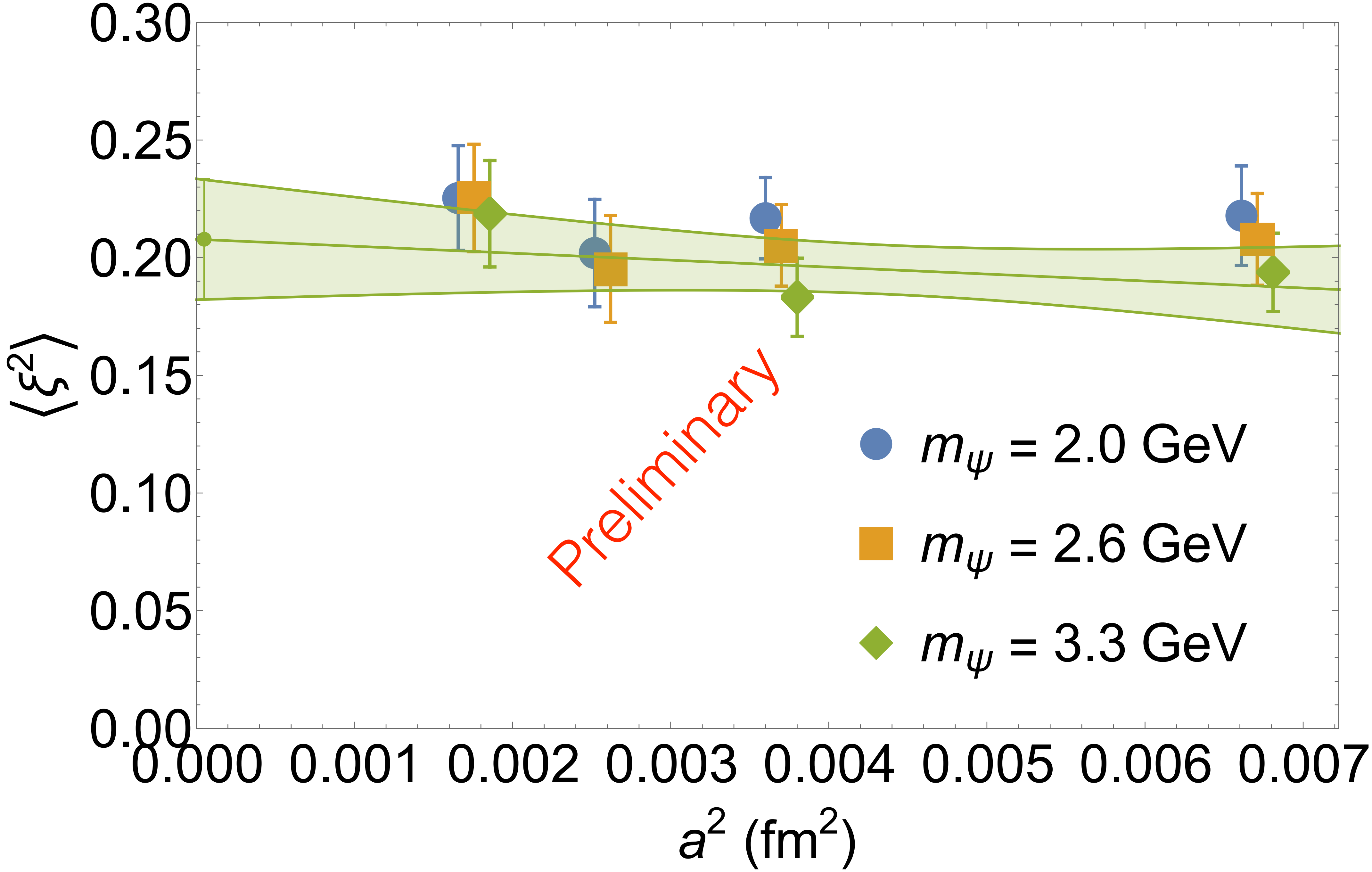
Continuum extrapolation for $\langle \xi^2 \rangle$



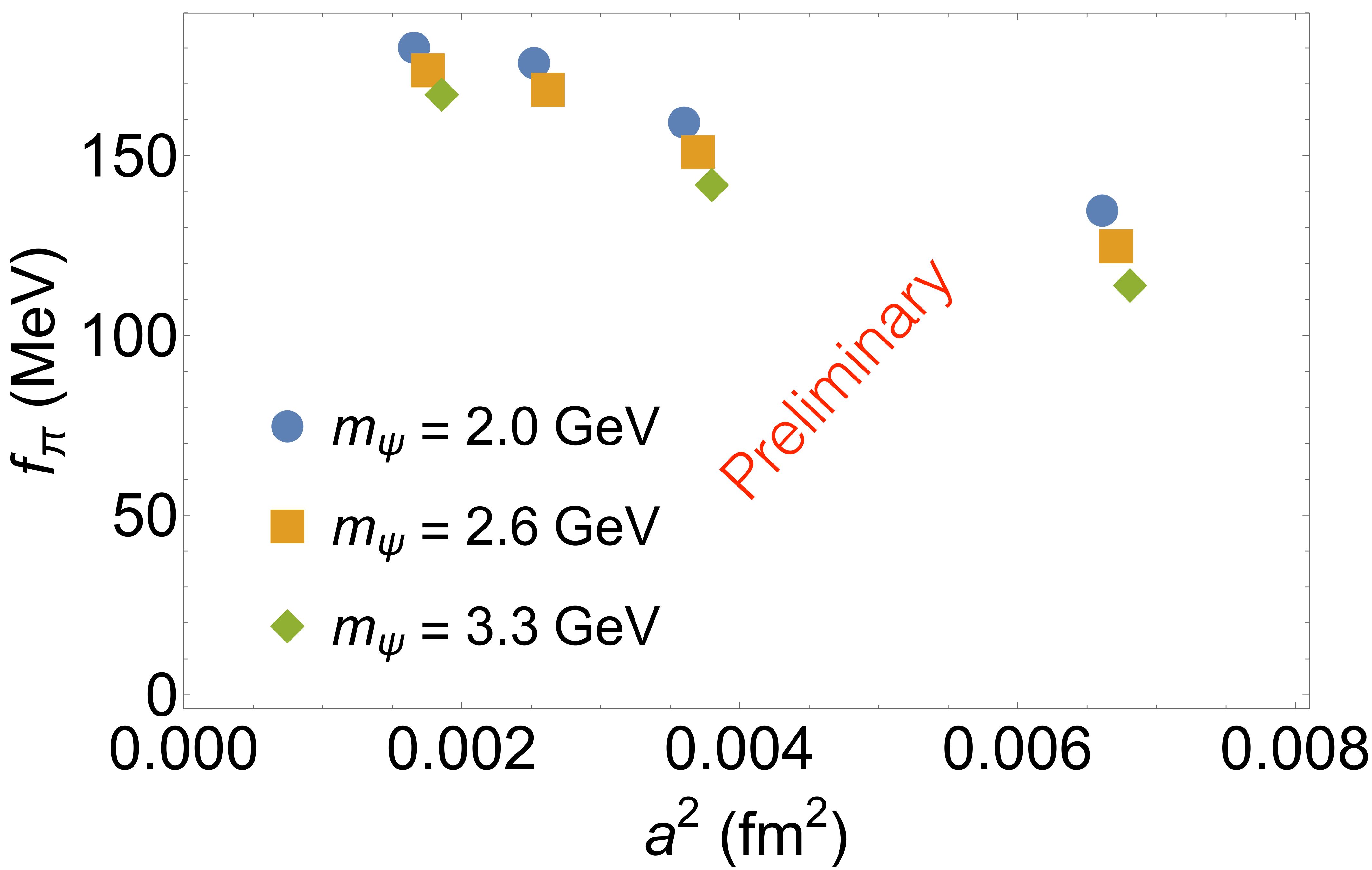
Continuum extrapolation for $\langle \xi^2 \rangle$



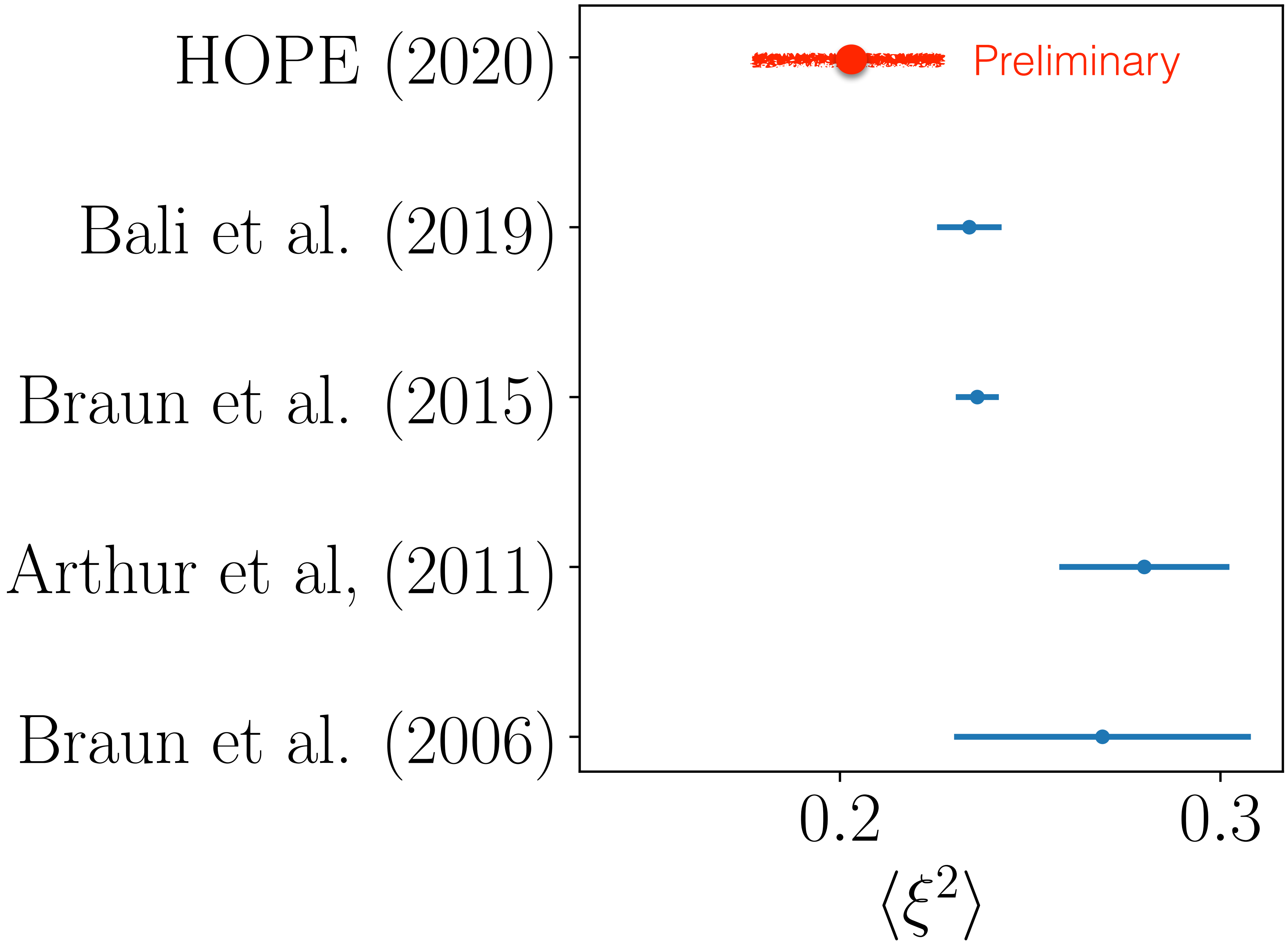
Continuum extrapolation for $\langle \xi^2 \rangle$



Continuum extrapolation for f_π from $\langle \xi^0 \rangle$



Comparing with other calculations



Conclusion and outlook

- The HOPE method is completely worked out for $\phi_\pi(x, \mu)$
- In general, need large $\mathbf{p}$ for accessing non-leading moments
- A strategy is found for computing $\langle \xi^2 \rangle$ at low $\mathbf{p}$
- Numerical result shows the validity of the HOPE method
- Future: higher $\langle \xi^n \rangle$ and other partonic quantities

Backup slides

Enhancing the signal: the need

We work with $|\omega| = \left|\frac{2p \cdot q^2}{\tilde{Q}}\right| < 1$

Leading contribution in $\mathrm{Im}[U^{12}]$ is $\sim\langle \xi^0 \rangle$

Leading contribution in $\mathrm{Re}[U^{12}]$ is $\sim\langle \xi^2 \rangle \omega^2$

→ Much noisier compared to $\mathrm{Im}[U^{12}]$

Enhancing the signal: the idea

We work with $|\omega| < 1$ where Minkowskian $U^{\mu\nu}$ is imaginary.

From $U^{\mu\nu}_{\text{Minkowski}}(p,q) = \int_{-\infty}^{\infty} d\tau\, e^{-q_0\tau} R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q})$.

→ $R^{\mu\nu}$ is imaginary.

Back to Euclidean space:

$$\text{Re}[U^{\mu\nu}(\mathbf{p},q)] = \text{Re}\left[\int_{-\infty}^{\infty} d\tau\, R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q}) e^{-iq_4\tau}\right]$$

$$\propto \int_0^{\infty} d\tau\, [R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q}) - R^{\mu\nu}(-\tau;\mathbf{p},\mathbf{q})] \sin(q_4\tau)$$

γ_5 hermiticity → $= R^{\mu\nu}(\tau;\mathbf{p},\mathbf{q}) + R^{\mu\nu}(\tau;-\mathbf{p},\mathbf{q})$

More correlated → reduced error

Enhancing the signal: the result