Lattice gauge theory with quantum computers

RBRC
RIKEN BNL Research Center

Akio Tomiya (RIKEN-BNL)
akio.tomiya@riken.jp

T. Izubuchi, Y. Kikuchi (RBRC)
M. Honda (YITP)
B. Chakraborty (Cambridge)
arXiv: 2001.00485

Who am I?

Riken/BNL, particle physics, Lattice QCD, and ML

Publications

Detection of phase transition via convolutional neural networks
A Tanaka, A Tomiya
Journal of the Physical Society of Japan 86 (6), 063001

Evidence of effective axial $U(1)$ symmetry restoration at high temperature QCD
A Tomiya, G Cossu, S Aoki, H Fukaya, S Hashimoto, T Kaneko, J Noaki, ...
Physical Review D 96 (3), 034509

Deep learning and the $\mathrm{AdS}/\mathrm{CFT}$ correspondence
K Hashimoto, S Sugishita, A Tanaka, A Tomiya
Physical Review D 98 (4), 046019

Violation of chirality of the Möbius domain-wall Dirac operator from the eigenmodes
G Cossu, H Fukaya, A Tomiya, S Hashimoto
Physical Review D 93 (3), 034507

Deep learning and holographic QCD
K Hashimoto, S Sugishita, A Tanaka, A Tomiya
Physical Review D 98 (10), 106014

Quantum quench and scaling of entanglement entropy
P Caputa, SR Das, M Nozaki, A Tomiya
Physics Letters B 772, 53-57

Biography

2018 - : Postdoc in RIKEN-BNL (NY, US)
2015 - 2018 : Postdoc in CCNU (Wuhan, China)
2015 : PhD at Osaka

Outline

Outline

Motivation, Big goal

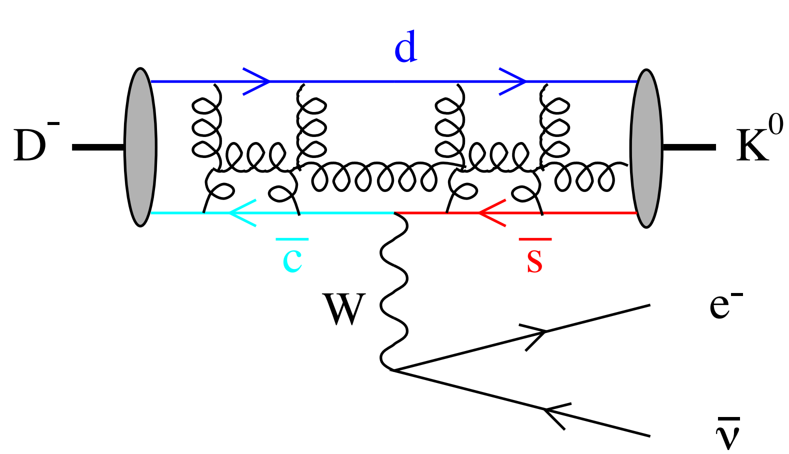
Akio Tomiya

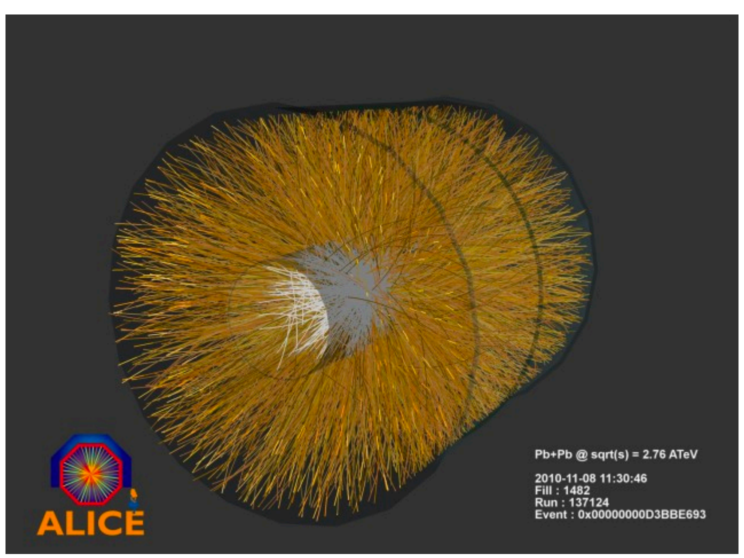
Non-perturbative calculation of QCD is important

QCD in 3 + 1 dimension

$$S = \int d^4x \left[-\frac{1}{4} \mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - gA\!\!\!/ - m\right)\psi \right]$$

$$Z = \int \mathscr{D}A\mathscr{D}\bar{\psi}\mathscr{D}\psi e^{\mathrm{i}S} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g[A_\mu, A_\nu]$$





- This describes…
    - inside of hadrons (bound state of quarks), mass of them
    - scattering of gluons, quarks
    - Equation of state of neutron stars, Heavy ion collisions, etc
- **Non-perturbative effects are essential**. How can we deal with?
    - Confinement
    - Chiral symmetry breaking

Motivation, Big goal

LQCD = Non-perturbative calculation of QCD

QCD in 3 + 1 dimension

$$S = \int d^4x \left[-\frac{1}{4} \mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - gA\!\!\!/ - m\right)\psi \right]$$

$$Z = \int \mathscr{D}A\mathscr{D}\bar{\psi}\mathscr{D}\psi e^{\mathrm{i}S} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g[A_\mu, A_\nu]$$

QCD in Euclidean 4 dimension

$$S = \int d^4x \left[+\frac{1}{4} \mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(\partial\!\!\!/ - gA\!\!\!/ - m\right)\psi \right]$$

$$Z = \int \mathscr{D}A\mathscr{D}\bar{\psi}\mathscr{D}\psi e^{-S}$$

← This can be regarded as a statistical system

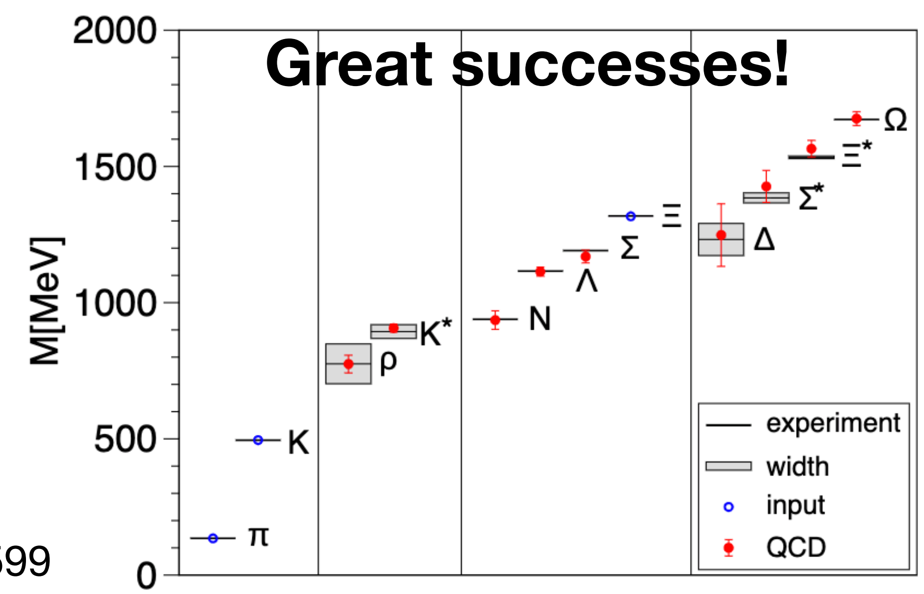
- Standard approach: Lattice QCD with Imaginary time and Monte-Carlo
  - LQCD = QCD + cutoff + irrelevant ops. = "Statistical mechanics"
  - Mathematically well-defined quantum field theory
  - **Quantitative** results are available = Systematic errors are controlled

Motivation, Big goal

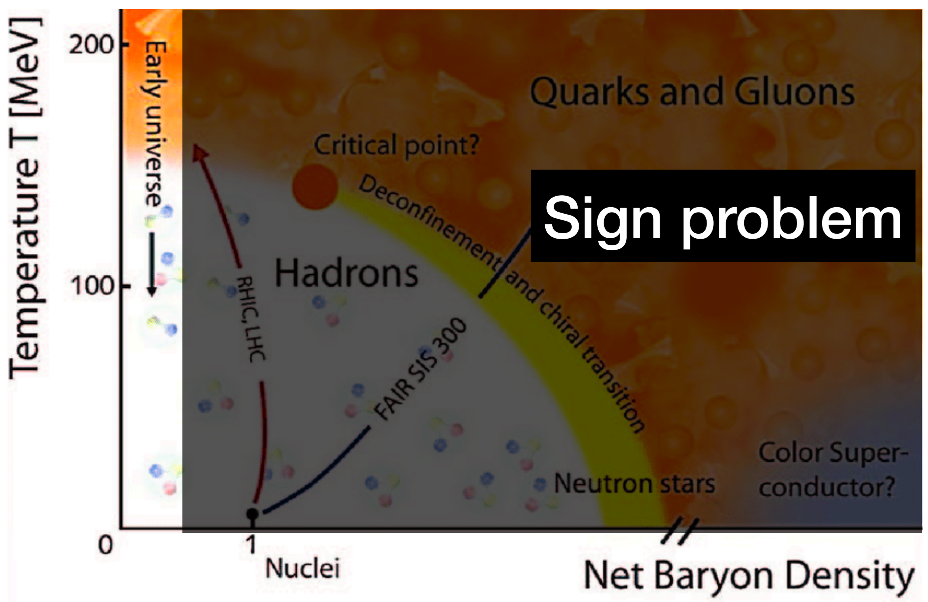
Sign problem prevents using Monte-Carlo

- Monte-Carlo is very powerful method to evaluate expectation values for "statistical system", like lattice QCD in imaginary time

$$\langle O[U] \rangle = \frac{1}{N_{\mathrm{conf}}} \sum_{c}^{N_{\mathrm{conf}}} O[U_c] + \mathcal{O}\left(\frac{1}{\sqrt{N_{\mathrm{conf}}}}\right) \qquad U_c \leftarrow P(U) = \frac{1}{Z} e^{-S[U]} \in \mathbb{R}_+$$



arXiv:0906.3599



- However, if we have, real time, finite theta, finite baryon density case, we cannot we use Monte-Carlo technique because e^{-S} becomes complex. This is no more probability.
- Hamiltonian formalism does not have such problem! But it requires huge memory to store quantum states, which cannot realized even on supercomputer.
- Quantum states should not be realized on classical computer but on quantum computer (Feynman 1982)

Short summary

Sign problem prevent to use conventional method

- QCD describes perturbative and non-perturbative phenomena
- Lattice QCD with imaginary time is non-perturbative and quantitive method, which is evaluated by Monte-Carlo
- Sign problem, which is occurred in real time/finite theta/finite baryon density cace, prevents us to use the Monte-Carlo
- Hamiltonian formalism is one solution but we cannot construct the Hilbert space because of the dimensionality
- Quantum simulation/computer is natural realization the Hamiltonian formalism

Question?

Outline

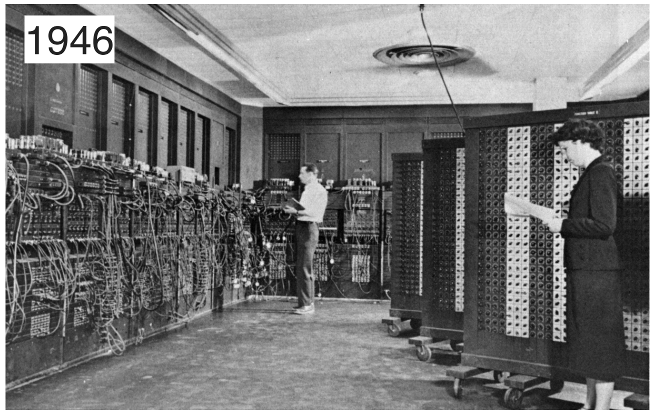
1. The sign problem in Quantum field theory 4P
2. Quantum computer 7P
3. Schwinger model with lattice-Hamiltonian formalism 10P
4. Adiabatic preparation of vacuum 3P
5. Results 6P

Quantum computer?
Towards beyond classical computers

Classical

$0,1$

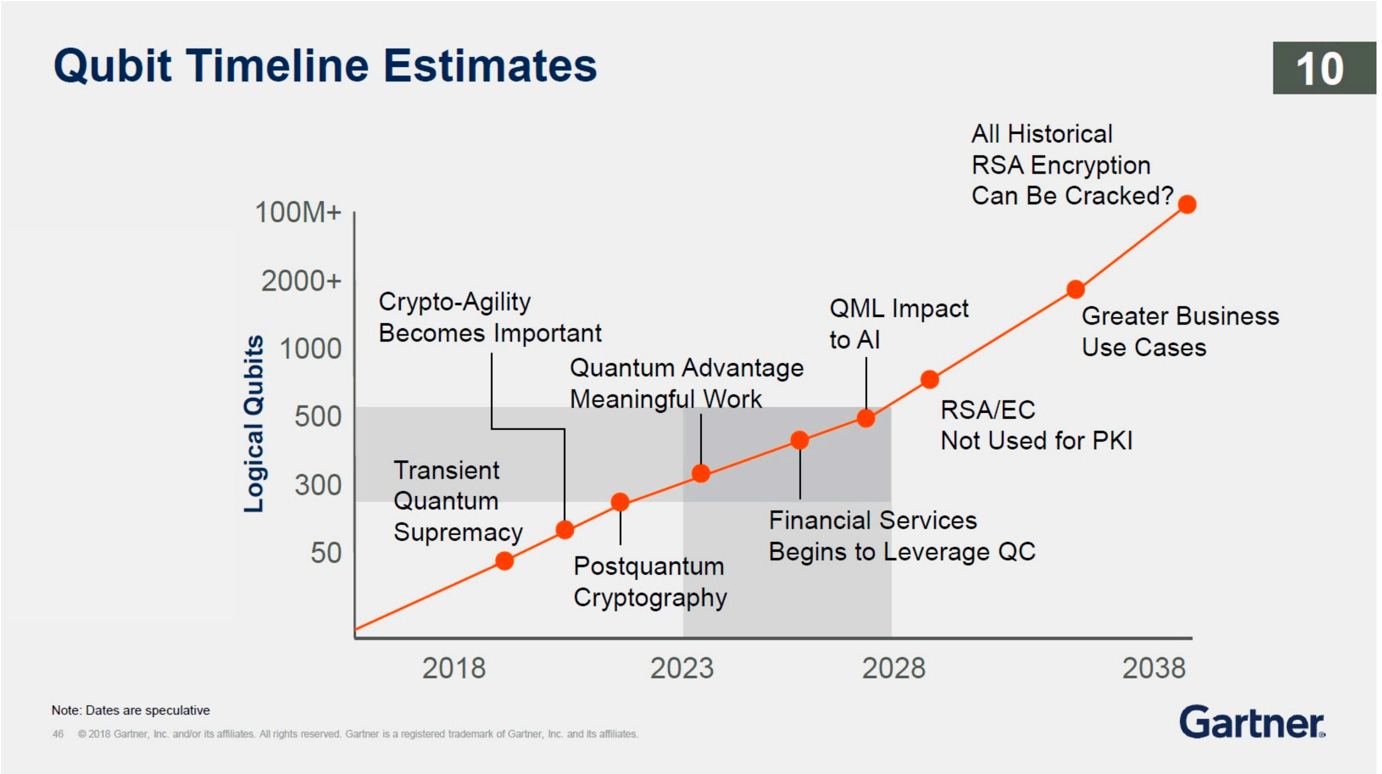
Data →Machine→Data



Quantum

$|0\rangle, |1\rangle$

State →Machine→State



Lattice gauge theory with quantum computer could be a future "common tool"

https://uk.pcmag.com/forward-thinking/117979/gartners-top-10-strategic-technology-trends

Quantum computer?

We can perform bit operation + α

Classical

$$011 \xrightarrow{\text{“NOT”}} 100$$

Quantum

$$|011\rangle = |0\rangle \otimes |1\rangle \otimes |1\rangle \xrightarrow{\text{“NOT”}} |100\rangle = |1\rangle \otimes |0\rangle \otimes |0\rangle$$

In addition,

$$|0\rangle \otimes |0\rangle \xrightarrow{\text{“Entangling”}} CN_{\overrightarrow{10}} H_0 |0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}} |0\rangle \otimes (|0\rangle + |1\rangle)$$

$$= \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

But, what is benefit for physicist?

Quantum computer?

For physicists : Circuit ~ time evolution of quantum spins

Example1.

Transverse Ising model on 3 sites (Open boundary)

$$H = -\sum_{<j,k>} Z_j Z_k - h \sum_j X_j = -Z_0 Z_1 - Z_1 Z_2 - hX_0 - hX_1 - hX_2$$

X_j: Pauli matrix of x on site j

Z_j: Pauli matrix of z on site j

h: size of external field

Time evolution for infinitesimal (real) time ε:

$$\mathrm{e}^{-\mathrm{i}H\epsilon} = \mathrm{e}^{-\mathrm{i}(-Z_0Z_1 - Z_1Z_2 - hX_0 - hX_1 - hX_2)\epsilon}$$

$$\approx \underbrace{\mathrm{e}^{-\mathrm{i}(-Z_0Z_1 - Z_1Z_2)\epsilon}}_{\equiv U_{ZZ}(\epsilon)} \underbrace{\mathrm{e}^{-\mathrm{i}(-hX_0 - hX_1 - hX_2)\epsilon}}_{\equiv U_X(\epsilon)} + O(\epsilon^2)$$ (Suzuki-Trotter expansion)

Qubit = spin

$|0\rangle = |\uparrow\rangle$

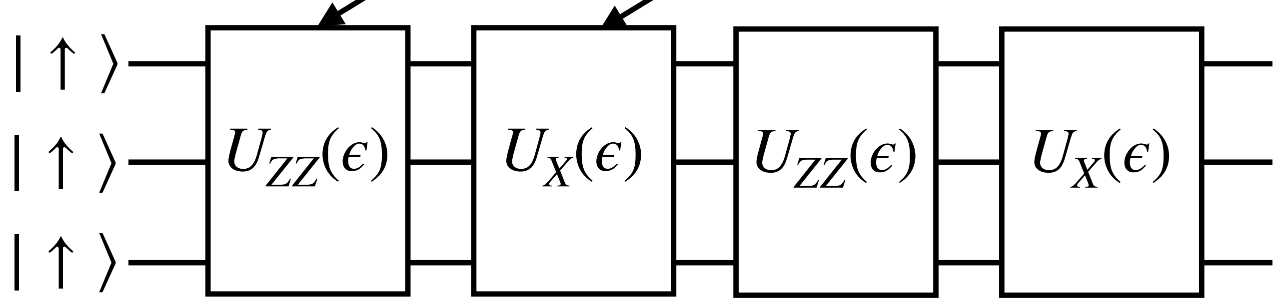
$|1\rangle = |\downarrow\rangle$

Unitary trasformation on a qubit = gate

$R_Z(\theta)|\psi\rangle = \mathrm{e}^{-\mathrm{i}\frac{1}{2}\theta Z}|\psi\rangle$ ~ Hamiltonian evol.

We can make these boxes from **gates** (ask me later)

$$\mathrm{e}^{-\mathrm{i}Ht} \underbrace{|\uparrow\rangle \otimes |\uparrow\rangle \otimes |\uparrow\rangle}_{= |\Omega(0)\rangle} =$$



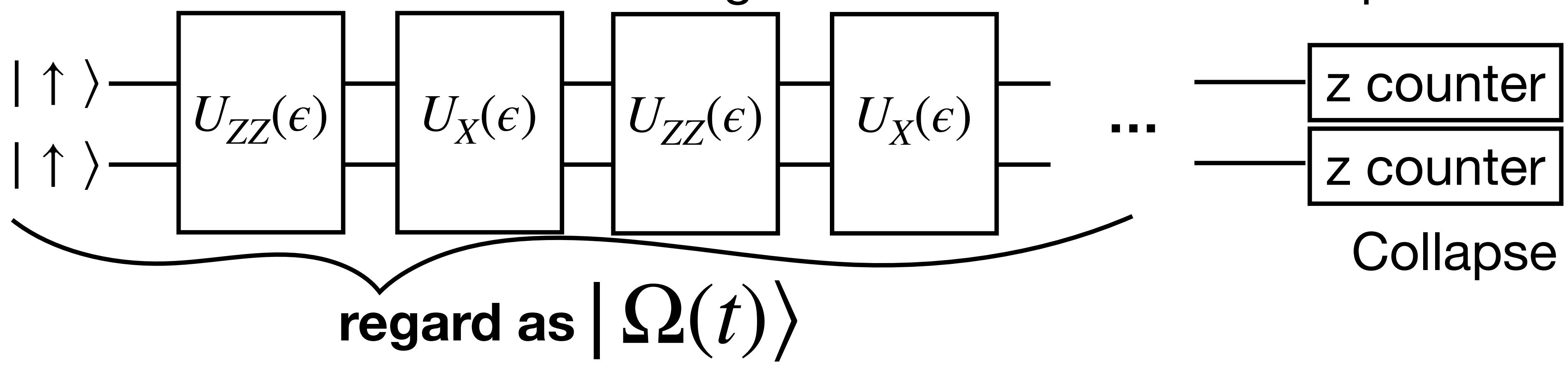
$$\cdots = |\Omega(t)\rangle$$

In this way, we can (re)produce, Hamiltonian time evolution using a quantum circuit.
Here we can evaluate the systematic error from the expansion and
reduce it by using higher order decomposition (leapfrog etc)
Quantum computer actually can realize any unitary transformation (skipping proof)

Quantum computer?

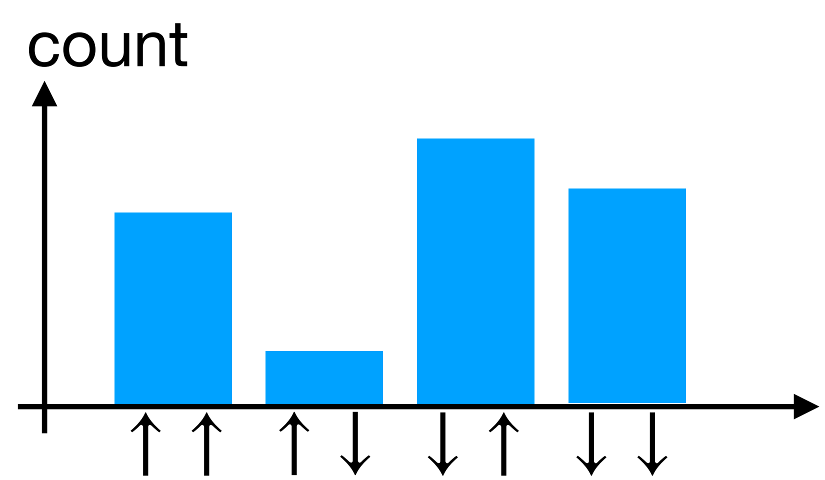
For physicists : Circuit ~ time evolution of quantum spins

Example2 We can make wave functional for a given Hamiltonian for 2nd quantized system



z counter measures spins up/down in probability (Born rule), many trial gives histogram:

$$\#\text{ of } \uparrow\uparrow \propto |\langle\uparrow\uparrow|\Omega(t)\rangle|^2$$
$$\#\text{ of } \uparrow\downarrow \propto |\langle\uparrow\downarrow|\Omega(t)\rangle|^2$$
$$\#\text{ of } \downarrow\uparrow \propto |\langle\downarrow\uparrow|\Omega(t)\rangle|^2$$
$$\#\text{ of } \downarrow\downarrow \propto |\langle\downarrow\downarrow|\Omega(t)\rangle|^2$$



On the other hand, the magnetization is,

$$\langle\Omega(t)|\sum_{k=1}^{2} Z_k|\Omega(t)\rangle = \sum_{k=1}^{2}\sum_{\Psi=\uparrow\uparrow,\cdots}\langle\Omega(t)|Z_k|\Psi\rangle\langle\Psi|\Omega(t)\rangle$$

(insert complete set)

$$= \sum_{k=1}^{2}\sum_{\Psi=\uparrow\uparrow,\cdots}(-1)^{\Psi_k}|\langle\Psi|\Omega(t)\rangle|^2$$

$\Psi = \uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow$

(can be constructed from data)

$\Psi_k = $ spin on site k

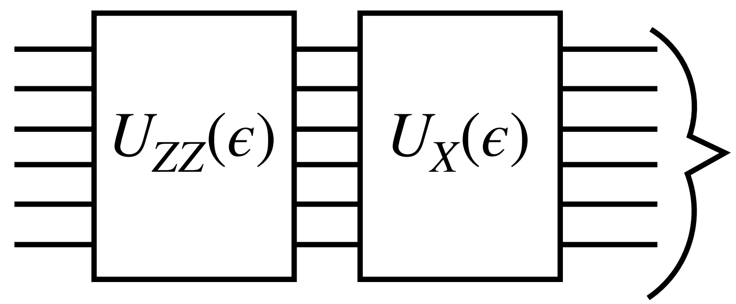
We can calculate expectation values!

Quantum computer?

Quantum computer is under developing

Quantum computer is theoretically universal, namely it can mimic any unitary transformation, but practically …

1. The number of qubits are not many.



up to 53 (world record)

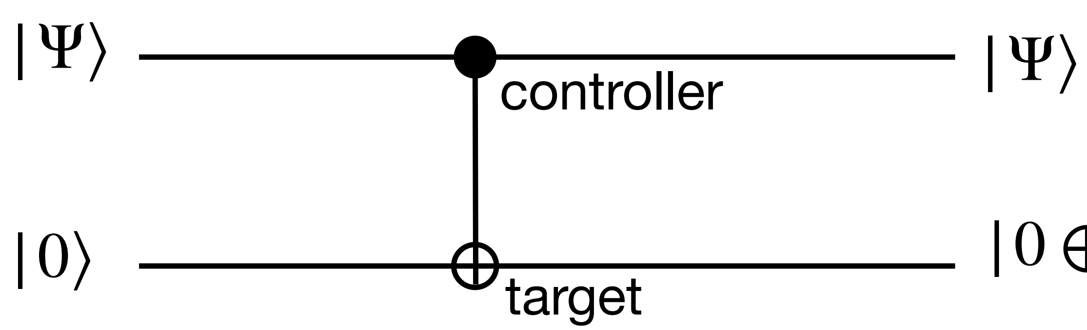
2. Gate operations are inaccurate.

= We cannot make quantum circuit deeper.

e.g.) Control-not (CNOT) gate

If • side is 0, gate does nothing on the target ⊕

If • side is 1, gate flips the target ⊕ side.



$$|0 \oplus \Psi\rangle = \begin{cases} |0\rangle & (\text{if } \Psi = 0) \\ |1\rangle & (\text{if } \Psi = 1) \end{cases}$$

$$|\,{}_{\text{actual}}\langle 0 \oplus \Psi | 0 \oplus \Psi \rangle_{\text{ideal}}\,| \sim 0.97$$ (machine dependent, 1903.10963)

Operations are sometimes wrongly performed.

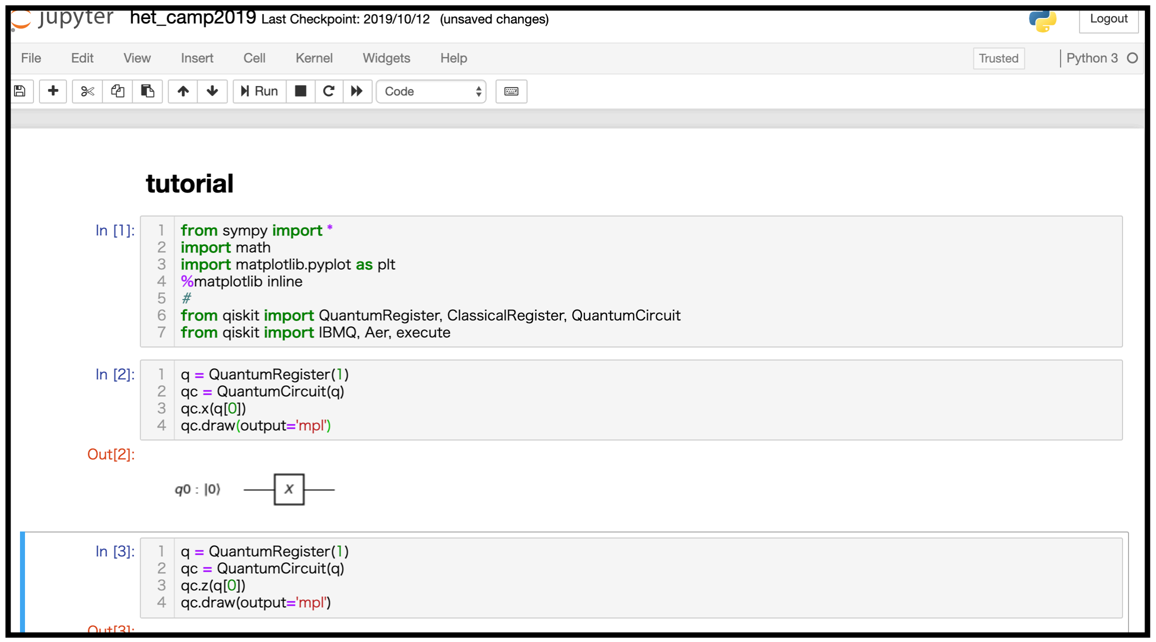
In order to study machine independent parts, we use a simulator instead of real one.

Quantum computer?

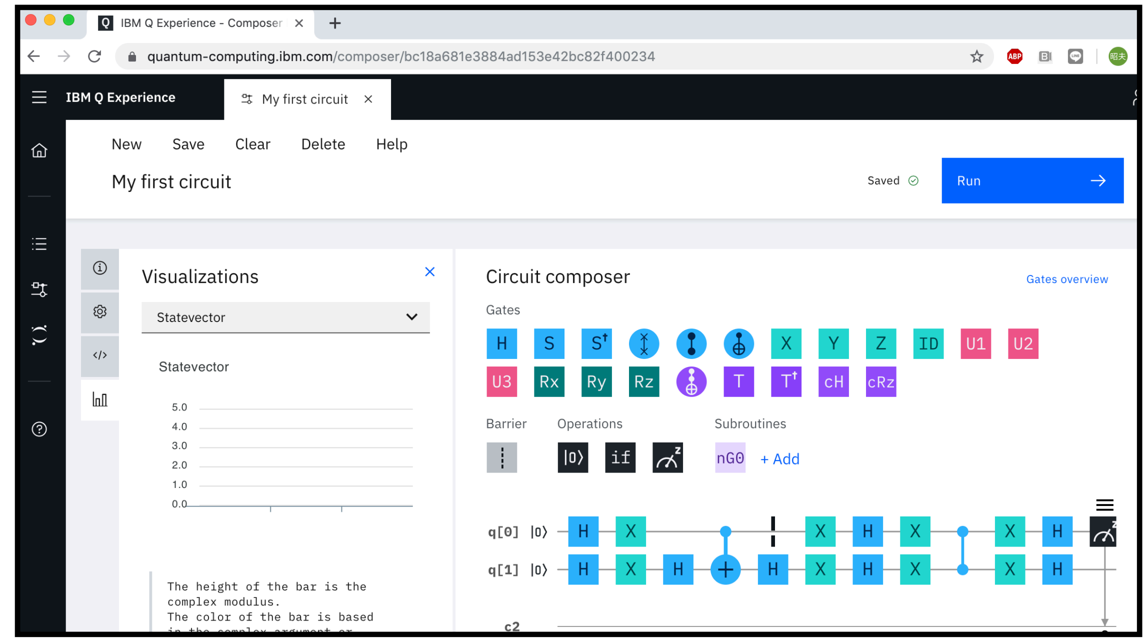
IBM Q is available and free

Akio Tomiya

From Jupyter/python



From Browser to real machine



Several frameworks are available;

- Qiskit : *de facto* standard (IBM)
- Qulacs : Fastest simulator (QunaSys, Japan)
- Blueqat : I think this is easiest (MDR, Japan)

etc…

Short summary

Quantum computer?

- Quantum computer is developing technology. Current one is noisy so far
- Once hamiltonian is constructed, we can perform time evolution using quantum circuit in principle
- Comment1: We use simulator but our technology can be used in future machines with error-correction. Time resolves this problem.
- Comment2: Simulation of quantum computer by classical machine is generally exponentially hard. To calculate large problem, we need real device.

Question?

Outline

Schwinger model

＝2D QED: Solvable at m=0, similar to QCD in 4D.

Schwinger model = QED in 1+1 dimension

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\left(i\partial\!\!\!/ - gA\!\!\!/ - m \right)\psi + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} \right]$$

Similarities to QCD in 3+1

- Confinement
- Chiral symmetry breaking (different mechanism), gapped even m=0

$$\langle \overline{\psi}\psi \rangle = -\frac{e^{\gamma} g}{\pi^{3/2}} = -g0.16\cdots$$

[Y. Hosotani,…]

- Theta term is essential for CP violation and causes the sign problem but in this talk we omit this one (please refer our paper for θ≠0)
- Vacuum decay by external electric field (Schwinger effect)

Hamiltonian of Schwinger model

＝2D QED: Solvable at m=0, similar to QCD in 4D.

Schwinger model = QED in 1+1 dimension

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left(i\partial\!\!\!/ - gA\!\!\!/ - m \right) \psi \right]$$

- Strategy

  1. Derive Hamiltonian with gauge fixing

  2. Rewrite gauge field to fermions using Gauss' law

  3. Use Jordan-Wigner transformation → Spin system

Akio Tomiya

Hamiltonian of Schwinger model

What I want explain in this section

Schwinger model = QED in 1+1 dimension

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - gA\!\!\!/ - m\right)\psi \right]$$

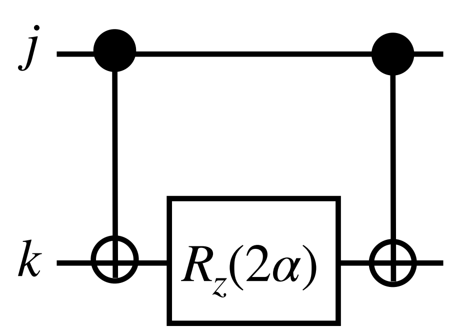
- Strategy(1gauge fix, 2Gauss' law, 3Jordan-Wigner trf)

Schwinger model on the lattice (staggered fermion, OBC, Spin rep.)

$$H = \frac{1}{4a}\sum_n \left[X_n X_{n+1} + Y_n Y_{n+1}\right] + \frac{m}{2}\sum_n (-1)^n Z_n + \frac{g^2 a}{2}\sum_n \left[\sum_{j=1}^{n}\left(\frac{Z_j + (-1)^j}{2}\right) + \epsilon_0\right]^2$$

$$\mathrm{e}^{-\mathrm{i}H\epsilon} \approx \mathrm{e}^{-\mathrm{i}H_Z\epsilon}\mathrm{e}^{-\mathrm{i}H_{XX}\epsilon}\mathrm{e}^{-\mathrm{i}H_{YY}\epsilon}\mathrm{e}^{-\mathrm{i}H_{ZZ}\epsilon}$$

$$U_{Z_jZ_k}(\alpha) = \mathrm{e}^{\alpha \mathrm{i} Z_j Z_k} =$$



$$R_z(\theta) = \exp(\mathrm{i}\frac{1}{2}\theta\sigma_z)$$

Hamiltonian of Schwinger model

＝2D QED: Solvable at m=0, similar to QCD in 4D.

(detail)

Schwinger model = QED in 1+1 dimension

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left(i\partial\!\!\!/ - gA\!\!\!/ - m \right) \psi \right]$$

$$\Pi(x) = \frac{\partial \mathscr{L}}{\partial \dot{A}^1(x)} = \dot{A}(x) = E(x)$$

$A_0 = 0$

$$\begin{cases} H = \int dx \left[-i\bar{\psi}\gamma^1(\partial_1 + igA_1)\psi + m\bar{\psi}\psi + \frac{1}{2}\Pi^2 \right] \\ \partial_x E = g\bar{\psi}\gamma^0\psi \end{cases}$$

(Gauss' law constraint)

This constrains time evolution to be gauge invariant

Lattice Hamiltonian formalism

Hamiltonian on a discrete space

(detail)

Schwinger model in continuum

$$H = \int dx \left[-\mathrm{i}\bar{\psi}\gamma^1(\partial_1 + \mathrm{i}gA_1)\psi + m\bar{\psi}\psi + \frac{1}{2}\Pi^2 \right]$$

Gauss' law $\partial_x E = g\bar{\psi}\gamma^0\psi$

$$-\frac{1}{g}\Pi(x) \to L_n \qquad \text{upper componentof } \psi \to \chi_{\text{even}-\text{site}}$$

$$-agA_1(x) \to \phi_n \qquad \text{lower componentof } \psi \to \chi_{\text{odd}-\text{site}}$$

Schwinger model on the lattice (staggered fermion)

$$H = -\frac{\mathrm{i}}{2a}\sum_{n=1}^{N-1}\left[\chi_{n+1}^\dagger \mathrm{e}^{-\mathrm{i}\phi_n}\chi_n - \chi_n^\dagger \mathrm{e}^{\mathrm{i}\phi_n}\chi_{n+1}\right] + m\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n + \frac{g^2a}{2}\sum_{n=1}^{N-1}L_n^2$$

Gauss' law $L_n - L_{n-1} = \chi^\dagger\chi_n - \frac{1}{2}\left(1-(-1)^n\right)$

Lattice Schwinger model = spin system

Gauge trf, open bc, Gauss law -> pure fermionic system

(detail)

Schwinger model on the lattice (staggered fermion)

$$H = -\frac{\mathrm{i}}{2a}\sum_{n=1}^{N-1}\left[\chi_{n+1}^{\dagger}\mathrm{e}^{-\mathrm{i}\phi_n}\chi_n - \chi_n^{\dagger}\mathrm{e}^{\mathrm{i}\phi_n}\chi_{n+1}\right] + m\sum_{n=1}^{N}(-1)^n\chi_n^{\dagger}\chi_n + \frac{g^2 a}{2}\sum_{n=1}^{N-1}L_n^2$$

Gauss' law $L_n - L_{n-1} = \chi^{\dagger}\chi_n - \frac{1}{2}\left(1-(-1)^n\right)$

$L_0 = \epsilon_0 \in \mathbb{R}$ **(open B.C.), and insert "Gauss' law"**

$$\begin{cases} U_n = \prod_{j=1}^{n-1}\mathrm{e}^{-\mathrm{i}\phi_j} \\ \chi_n \to U_n\chi_n \\ \mathrm{e}^{-\mathrm{i}\phi_{n-1}} \to U_{n-1}\mathrm{e}^{-\mathrm{i}\phi_{n-1}}U_n^{\dagger} \end{cases}$$

remnant gauge transformation

Schwinger model on the lattice (staggered fermion, OBC)

$$H = -\frac{\mathrm{i}}{2a}\sum_{n}\left[\chi_{n+1}^{\dagger}\chi_n - \chi_n^{\dagger}\chi_{n+1}\right] + m\sum_{n}(-1)^n\chi_n^{\dagger}\chi_n + \frac{g^2 a}{2}\sum_{n}\left[\sum_{j}^{n}\left(\chi_j^{\dagger}\chi_j - \frac{1-(-1)^j}{2}\right) + \epsilon_0\right]^2$$

Lattice Schwinger model

We requires anticommutations to fermions

(detail)

Schwinger model on the lattice (staggered fermion, OBC)

$$H = -\frac{\mathrm{i}}{2a}\sum_{n}\left[\chi_{n+1}^{\dagger}\chi_{n} - \chi_{n}^{\dagger}\chi_{n+1}\right] + m\sum_{n}(-1)^{n}\chi_{n}^{\dagger}\chi_{n} + \frac{g^{2}a}{2}\sum_{n}\left[\sum_{j}^{n}\left(\chi_{j}^{\dagger}\chi_{j} - \frac{1-(-1)^{j}}{2}\right) + \epsilon_{0}\right]^{2}$$

System is quantized by assuming the canonical anti-commutation relation

$$\{\chi_{j}^{\dagger}, \chi_{k}\} = \mathrm{i}\delta_{jk} \qquad j, k = \text{site index}$$

On the other hand, Pauli matrices satisfy anti-commutation as well

$$\{\sigma^{\mu}, \sigma^{\nu}\} = 2\delta_{\mu\nu}\mathbf{1} \qquad \mu, \nu = 1,2,3$$

Quantum spin-chain case, each site has Pauli matrix, but they are “commute”.
We can absorb difference of statistical property using Jordan Wigner transformation

Jordan-Wigner transformation: $$\chi_{n} = \frac{X_{n} - \mathrm{i}Y_{n}}{2}\prod_{j<n}(\mathrm{i}Z_{j})$$

X_j: Pauli matrix of x on site j
Y_j: Pauli matrix of y on site j
Z_j: Pauli matrix of z on site j

This guarantees the statistical property

This reproduce correct Fock space.

We can rewrite the Hamiltonian in terms of spin-chain

Akio Tomiya

Lattice Schwinger model = spin system

Jordan-Wigner transformation: Fermions ~ Spins

(detail)

Schwinger model on the lattice (staggered fermion, OBC)

$$H = -\frac{\mathrm{i}}{2a}\sum_n \left[\chi_{n+1}^\dagger \chi_n - \chi_n^\dagger \chi_{n+1}\right] + m\sum_n (-1)^n \chi_n^\dagger \chi_n + \frac{g^2 a}{2}\sum_n \left[\sum_j^n \left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right) + \epsilon_0\right]^2$$

$$\begin{cases} \chi_n = \dfrac{X_n - \mathrm{i}Y_n}{2}\displaystyle\prod_{j<n}(\mathrm{i}Z_j) \\ \chi_n^\dagger = \dfrac{X_n + \mathrm{i}Y_n}{2}\displaystyle\prod_{j<n}(-\mathrm{i}Z_j) \end{cases}$$

Jordan-Wigner transformation

X_j: Pauli matrix of x on site j

Y_j: Pauli matrix of y on site j

Z_j: Pauli matrix of z on site j

Schwinger model on the lattice (staggered fermion, OBC, Spin rep.)

$$H = \frac{1}{4a}\sum_n \left[X_n X_{n+1} + Y_n Y_{n+1}\right] + \frac{m}{2}\sum_n (-1)^n Z_n + \frac{g^2 a}{2}\sum_n \left[\sum_{j=1}^n \left(\frac{Z_j + (-1)^j}{2}\right) + \epsilon_0\right]^2$$

[Y. Hosotani 9707129]

Lattice Schwinger model = spin system

Jordan-Wigner transformation: Fermions ~ Spins

(detail)

Schwinger model on the lattice (staggered fermion, OBC, Spin rep.)

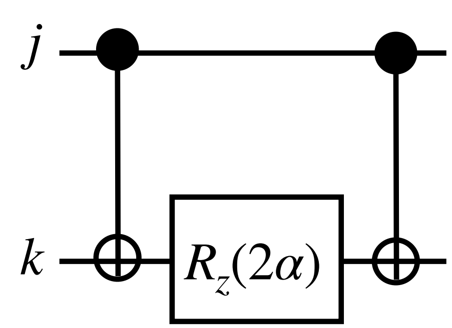
$$H = \frac{1}{4a}\sum_n \left[X_n X_{n+1} + Y_n Y_{n+1}\right] + \frac{m}{2}\sum_n (-1)^n Z_n + \frac{g^2 a}{2}\sum_n \left[\sum_{j=1}^{n}\left(\frac{Z_j + (-1)^j}{2}\right) + \epsilon_0\right]^2$$

Evolution by each term can be represented by gates (with Suzuki-Trotter expansion):

$$|0\rangle_{\text{circuit}} = |\uparrow\rangle_{\text{spin}}$$
$$|1\rangle_{\text{circuit}} = |\downarrow\rangle_{\text{spin}}$$

e.g.)

$$U_{Z_jZ_k}(\alpha) = \mathrm{e}^{\alpha \mathrm{i} Z_j Z_k} =$$



$$R_z(\theta) = \exp(\mathrm{i}\frac{1}{2}\theta\sigma_z)$$

Skipping detailed calculation but, this realizes correct unitary evolution

$$U_{Z_0Z_1}(\alpha)|\uparrow\rangle_0|\uparrow\rangle_1 = \mathrm{e}^{\alpha \mathrm{i} Z_j Z_k}|\uparrow\rangle_0|\uparrow\rangle_1 = \mathrm{e}^{+\alpha}|\uparrow\rangle_0|\uparrow\rangle_1$$
$$U_{Z_0Z_1}(\alpha)|\downarrow\rangle_0|\downarrow\rangle_1 = \mathrm{e}^{\alpha \mathrm{i} Z_j Z_k}|\downarrow\rangle_0|\downarrow\rangle_1 = \mathrm{e}^{+\alpha}|\downarrow\rangle_0|\downarrow\rangle_1$$
$$U_{Z_0Z_1}(\alpha)|\downarrow\rangle_0|\uparrow\rangle_1 = \mathrm{e}^{\alpha \mathrm{i} Z_j Z_k}|\downarrow\rangle_0|\uparrow\rangle_1 = \mathrm{e}^{-\alpha}|\downarrow\rangle_0|\uparrow\rangle_1$$
$$U_{Z_0Z_1}(\alpha)|\uparrow\rangle_0|\downarrow\rangle_1 = \mathrm{e}^{\alpha \mathrm{i} Z_j Z_k}|\uparrow\rangle_0|\downarrow\rangle_1 = \mathrm{e}^{-\alpha}|\uparrow\rangle_0|\downarrow\rangle_1$$

Lattice Schwinger model = spin system

Akio Tomiya

Jordan-Wigner transformation: Fermions ~ Spins

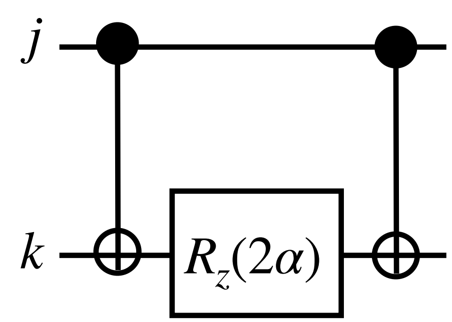
Schwinger model on the lattice (staggered fermion, OBC, Spin rep.)

$$H = \frac{1}{4a}\sum_n \left[X_n X_{n+1} + Y_n Y_{n+1}\right] + \frac{m}{2}\sum_n (-1)^n Z_n + \frac{g^2 a}{2}\sum_n \left[\sum_{j=1}^{n}\left(\frac{Z_j + (-1)^j}{2}\right) + \epsilon_0\right]^2$$

Evolution by each term can be represented by gates (with Suzuki-Trotter expansion):

e.g.)

$$U_{Z_j Z_k}(\alpha) = \mathrm{e}^{\alpha \mathrm{i} Z_j Z_k} =$$



$$R_z(\theta) = \exp(\mathrm{i}\frac{1}{2}\theta\sigma_z)$$

Then, we can evaluate,

$$\mathrm{e}^{-\mathrm{i}Ht}\,|0\rangle \otimes |1\rangle \otimes \cdots \otimes |0\rangle \otimes |1\rangle$$

(trivial ground state for m, g->∞)

To calculate chiral condensate, we have to prepare the vacuum for full Hamiltonian.

$$|\Omega\rangle_{\mathrm{exact}} \neq |0\rangle \otimes |1\rangle \otimes \cdots \otimes |0\rangle \otimes |1\rangle$$

Next section, we discuss state preparation.

Short summary

Lattice Schwinger model = spin system

- Schwinger model, 1+1 dimensional QED, is a toy model for QCD in 3+1 dim.
- Lattice Schwinger model + open boundary = Spin model
- We can realize time evolution of lattice Schwinger model using circuit.
- We want to reproduce analytic value for the chiral condensate at m=0 in the continuum,

$$\langle \overline{\psi}\psi \rangle = -\frac{\mathrm{e}^{\gamma} g}{\pi^{3/2}} = -g 0.16\cdots$$

to study usability of quantum computer/circuit

Question?

Outline

Adiabatic preparation of vacuum

To calculate VEV, vacuum is needed

(Following is slightly simplified from our paper, but essentially same)

$$H_0 = \frac{m}{2}\sum_n (-1)^n Z_n + \frac{g^2 a}{2}\sum_n \left[\frac{1}{2}\sum_{j=1}^{n}\left(Z_j + (-1)^j\right)\right]^2$$

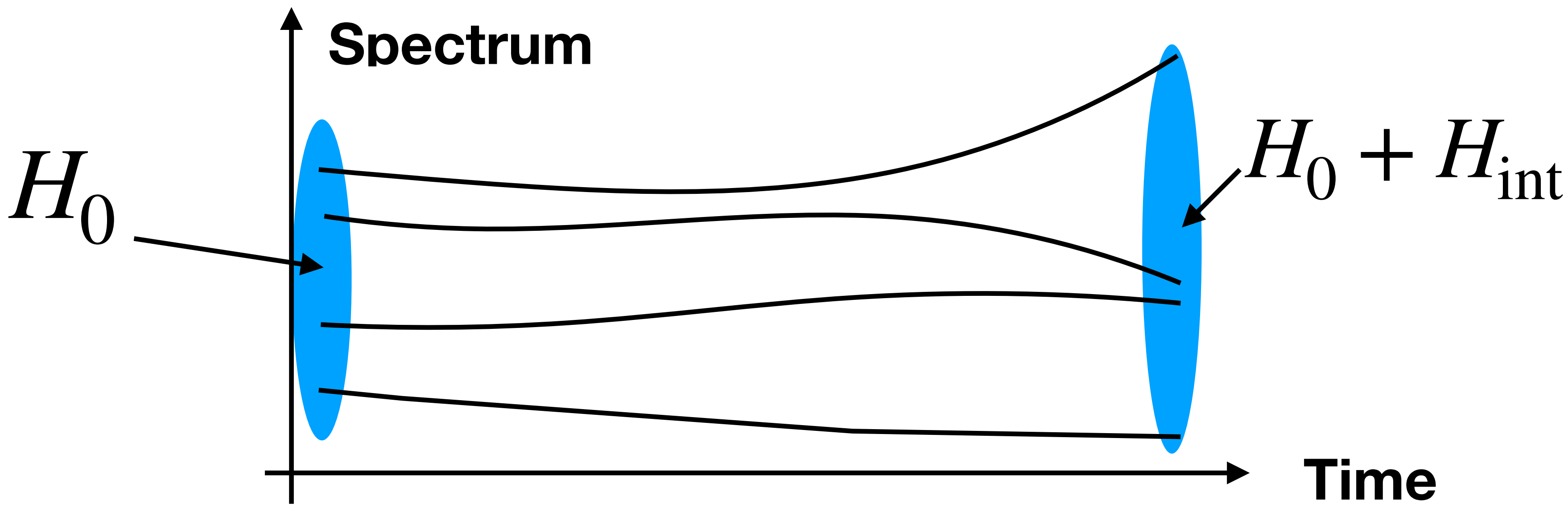
: This has a trivial vacuum (Neel ordered)

$$H_{\rm int} = \frac{1}{4a}\sum_n \left[X_n X_{n+1} + Y_n Y_{n+1}\right]$$

: Kinetic term in original QFT

$$H(t) = H_0 + \frac{t}{T}H_{\rm int} \qquad 0 < t < T$$

We can use adiabatic theorem!

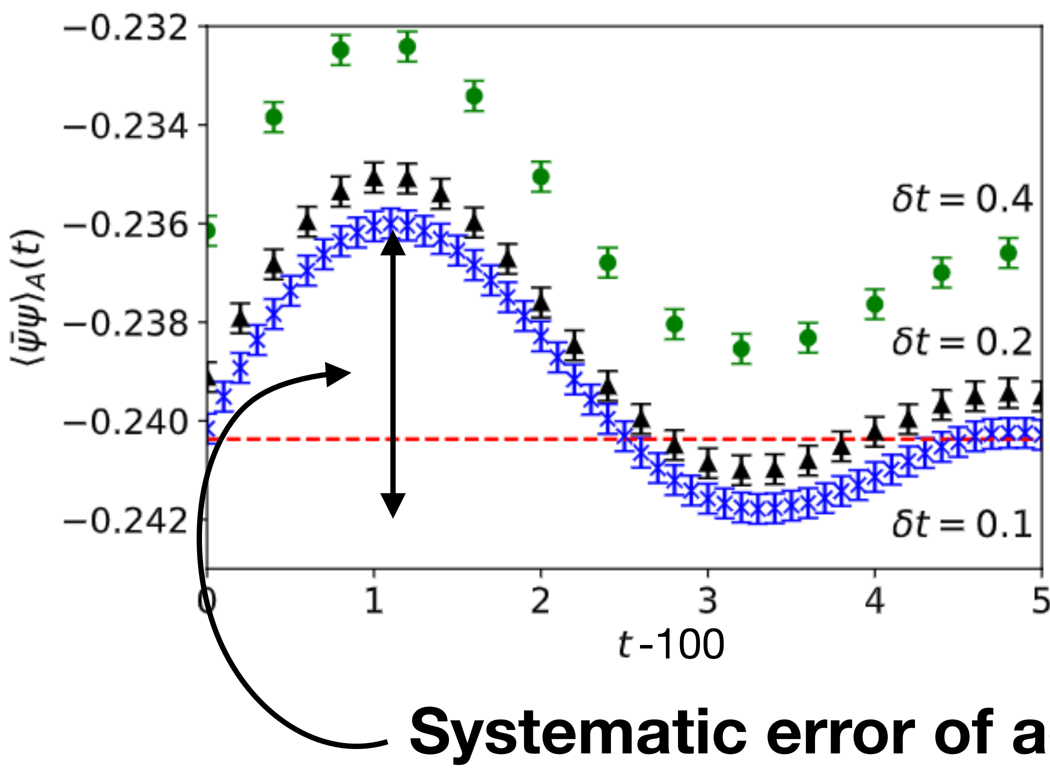


$$|\Omega\rangle_{\rm exact} = \lim_{T\to\infty} \hat{\rm T} e^{-i\int^T dt H(t)} |\Omega\rangle_{\rm trivial}$$

Adiabatic state preparation

We can control systematic error from adiabatic st. prep.

Adiabatic time T >> 1/gap, it looks converge



Systematic error of adiabatic state preparation

State prep.	Good	Bad
We use→ **Adiabatic**	Systematic error is under control. It can be eliminated by extrapolation	Huge cost (Depth is required)
Variational (commonly used in Quant. chemistry)	Economical (Magically good quality)	Depends on ansatz, in principle

Short summary

Adiabatic state preparation is systematically controlled

- To calculate vacuum expectation values, we need vacuum for full Hamiltonian
- Adiabatic state preparation is costly but sources of systematic errors are clear, safe to use.
- Note: Adiabatic state preparation becomes inefficient if the system approaches to gapless region ($\theta=\pi$). In the paper, we use improved time evolution operator

Question?

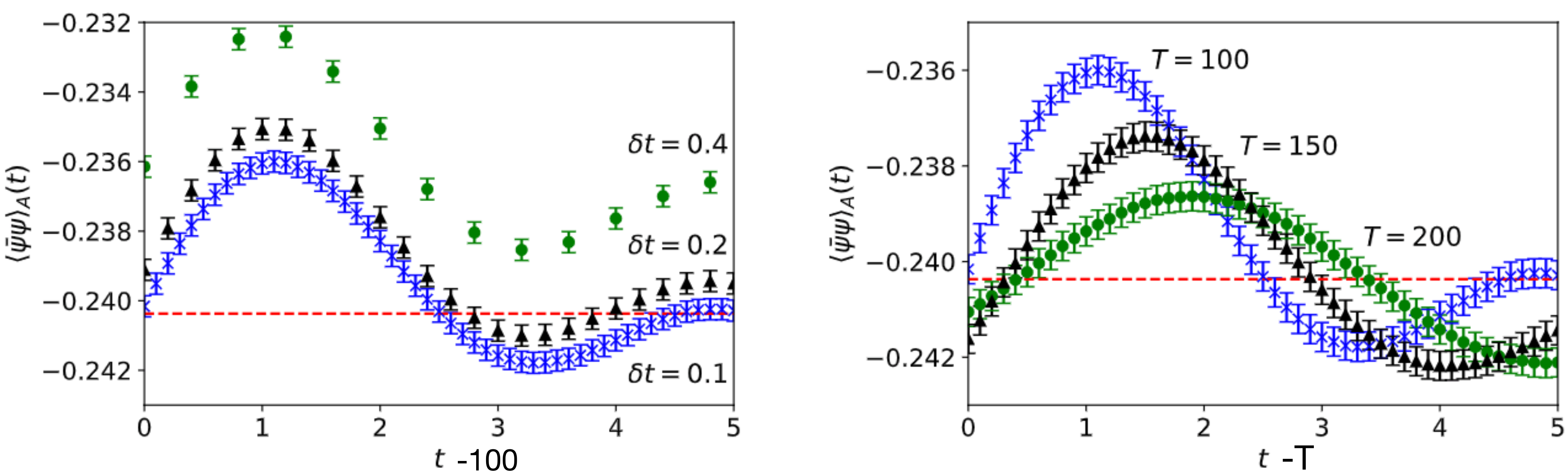
Outline

Results

Akio Tomiya

Chiral condensate with certain limits

- We calculate chiral condensate for m =0, m>0 in lattice Schwinger model
- We have taken limits,
    1. Large volume limit (Nx->0)
    2. Continuum limit (a->0)
- Limits for adiabatic state preparation are not taken yet but under control
    - Step size for Trotter decomposition (Left panel)
    - Large adiabatic time lime (Right panel)

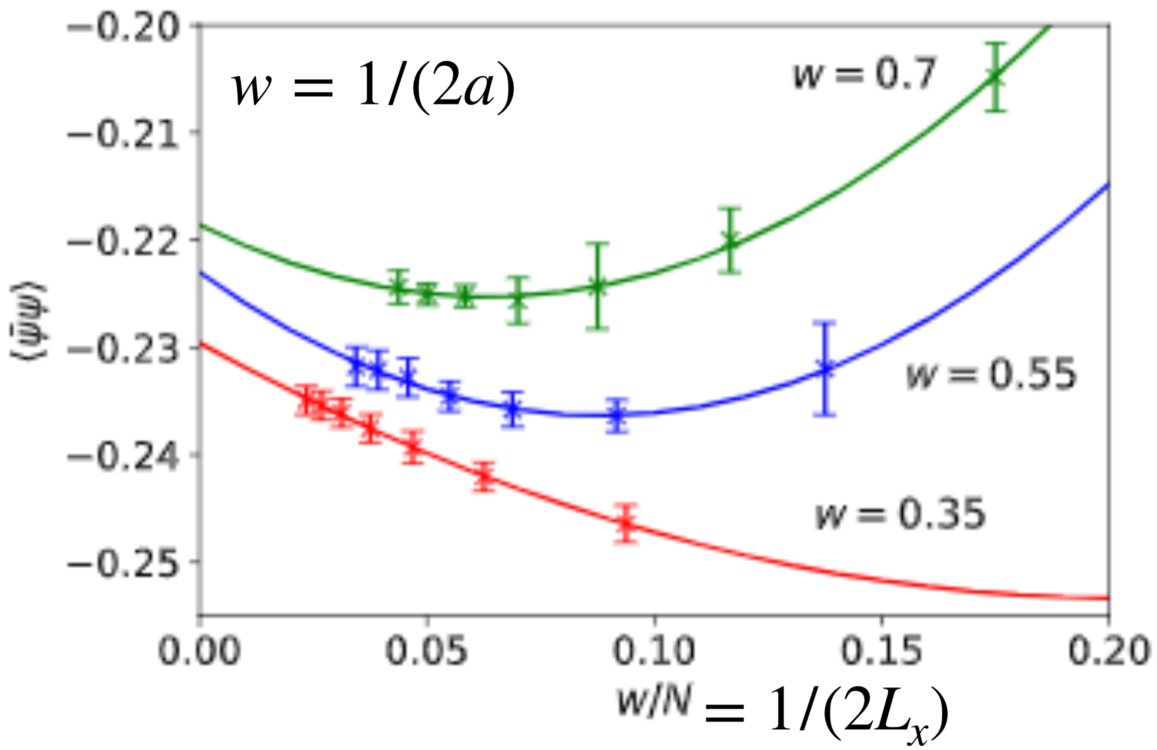


We take step size as 0.1 and adiabatic time as 100

Results: Large vol. & Cont. limit

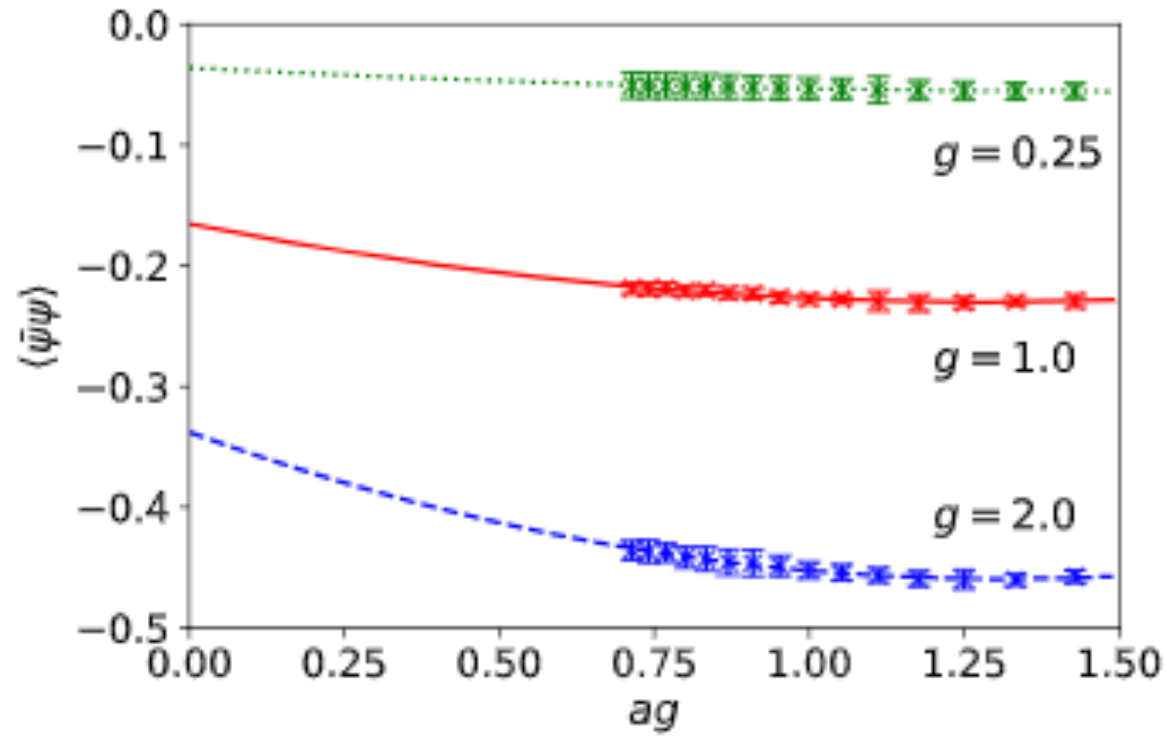
Systematic errors from theory are under control

Large volume limit via state pre.



Error bar includes systematic and statistical error. Statistics = 10^6 shots

Continuum limit via state pre.

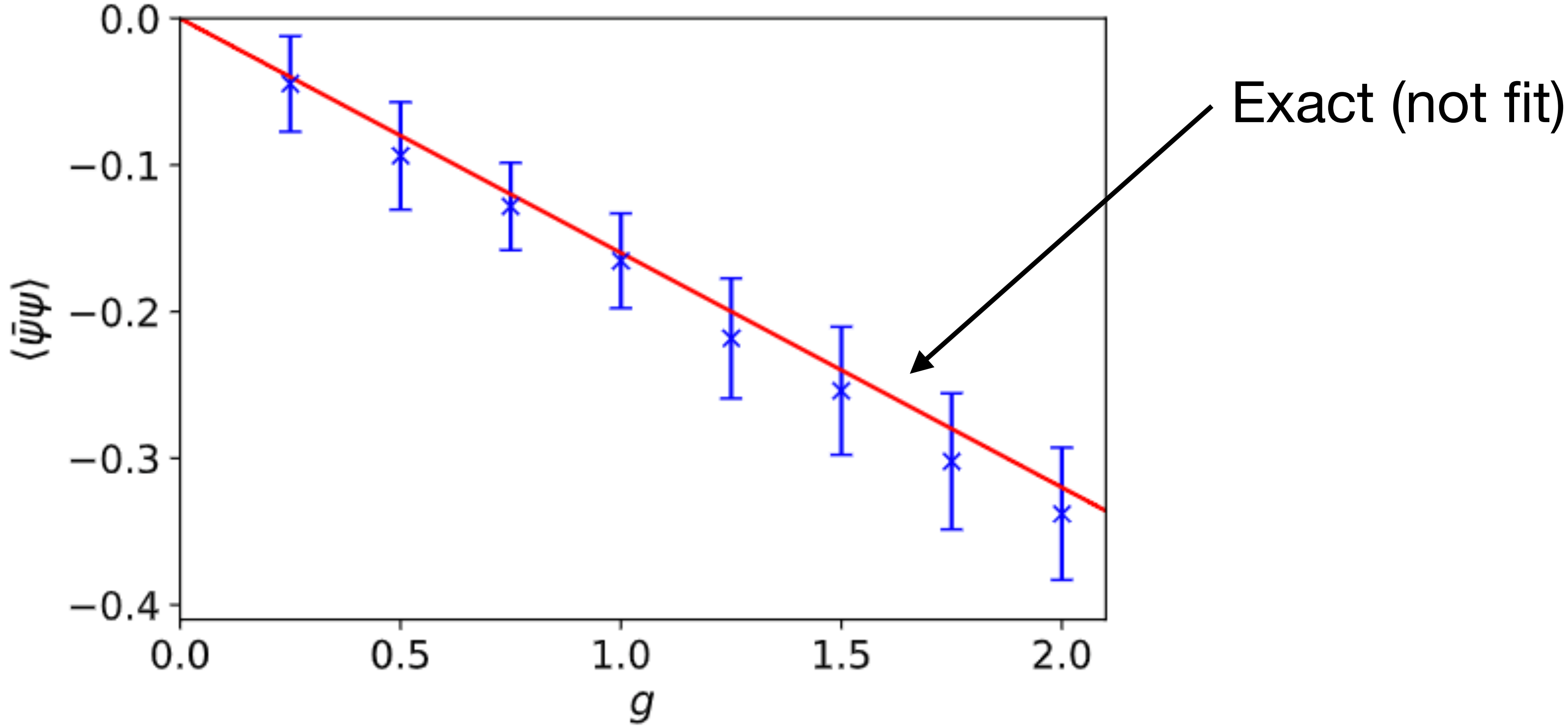


Error bar are asymptotic error for finite volume limit extrp.

Results: Large vol. & Cont. limit

Systematic errors from theory are under control

Results for massless Schwinger model are consistent with analytic value



Analytic value

$$\langle\overline{\psi}\psi\rangle = -\frac{e^{\gamma}g}{\pi^{3/2}} = -g0.160\cdots$$

Adiabatic preparation
V→∞, a→0

$$\langle\overline{\psi}\psi\rangle = -g0.160\cdots$$

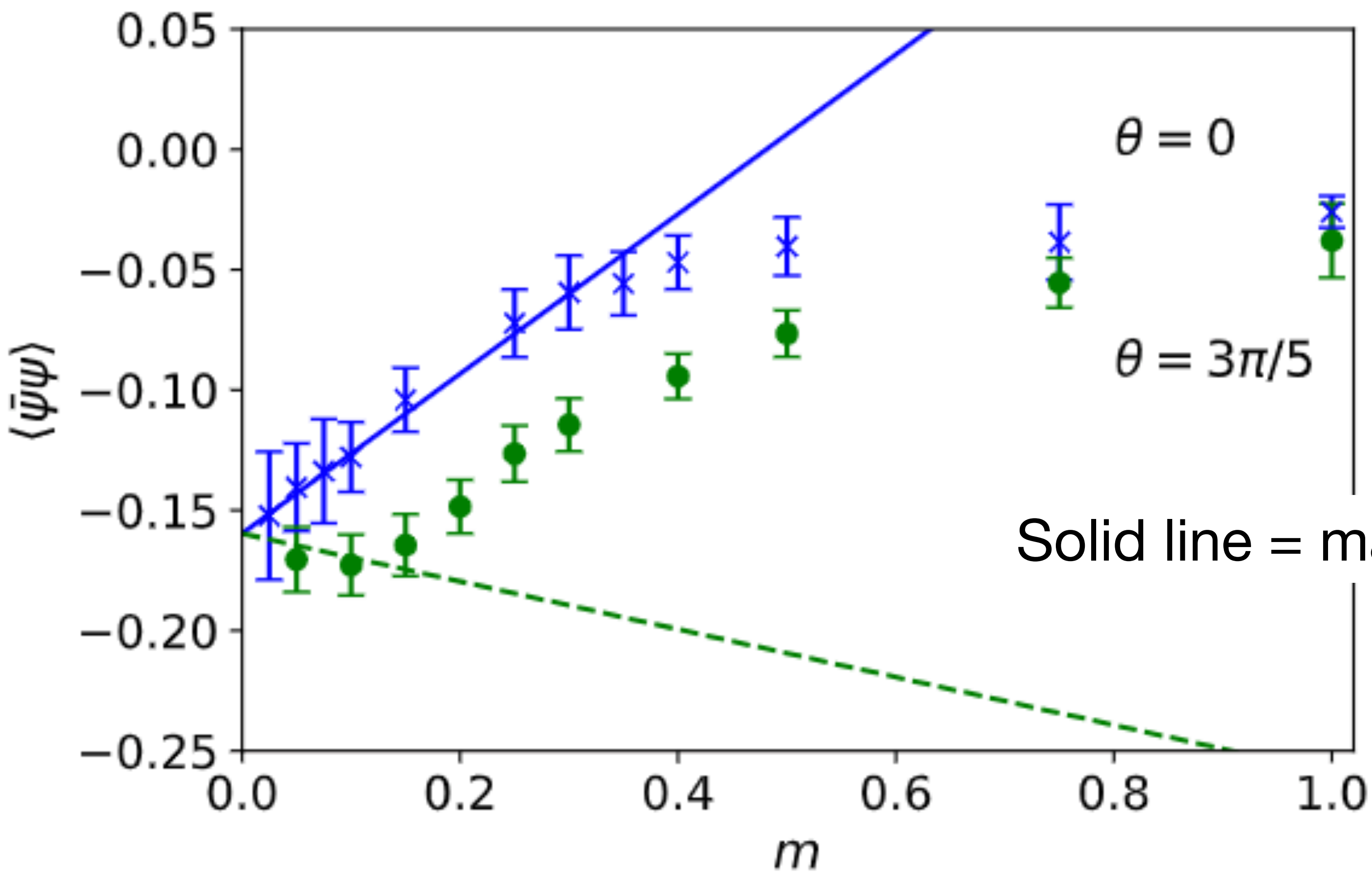
So far so good!

Results: Large vol. & Cont. limit

Systematic errors from theory are under control

Massive case and its time dependence (skipping all details)

For massive case, results via mass perturbation is known.
Result depends on θ as well as QCD



Solid line = mass perturbation

Our result for |m| < 1 reproduces mass perturbation as well as theta dependence. Large mass regime, we observe deviation

Towards on real machine

Real machine is noisy

- We need to care the fidelity: “accuracy” of operation of gates on qubits.
- Each time step = 250(# of 1-qubit gates)+270(# of 2-qubit gates)
- The number of time steps = T / δt = 1000
- Each gate operation has error, we need improvement.
    - Hardware side: Error correction, reliable qubits/operations
    - Theory side: improvement of decomposition & annealing process, this is discussed in our paper

- Towards to realize QCD, we need
    - Efficient higher dimensional version of “Jordan-Wigner” transf.
    - Development of treatment for continuous gauge d.o.f.
    - A number of (reliable) qubits
    - Efficient way of state preparation with controlling error

Summary

QFT calculation by Quantum computer

- We are investigating chiral condensate in the Schwinger model
- Errors from limits (Large volume, continuum) are under control
- Adiabatic state preparation works well
- We reproduce results both of massless and massive case
- Future work: Other observables, time depending process, etc

Thanks!