Study of high temperature QCD with chiral fermions

OSAKA UNIVERSITY
Live Locally, Grow Globally

Hidenori Fukaya (Osaka U.)

for JLQCD collaboration

S. Aoki, Y. Aoki, G. Cossu, HF,

S. Hashimoto, T. Kaneko, C. Rohrhofer,

K. Suzuki, in preparation.

JLQCD's finite T project (2012~)

1. Nf=2 QCD with overlap fermions at fixed topology [G. Cossu] 2012-2013, on IBM BG/L, Hitachi SR11000
2. Nf=2 QCD with Mobius domain-wall fermions [G. Cossu, A.Tomiya] 2013-2015, on IBM BG/Q.
3. Nf=2 QCD with MDW, finer and larger lattices [Y. Aoki, K. Suzuki, C. Rohrhofer] 2016-2020, on IBM BG/Q and Oakforest-PACS [today's topic]
4. Nf=2+1 QCD with MDW started ! [I. Kanamori, Y. Nakamura joined.] 2020- Oakforest-PACS, Fugaku?

Results 2016-2019 (phase 3)

Symanzik gauge action

Nf=2 Mobius domain-wall fermion action

m = [1-10] m_{phys}

1/a = 0.075 fm (0.1fm in phase 2)

Lt = 8,10,12,14 [T=190-330MeV]

L=24,32,40,48 [1.8-3.6fm]

15000-30000trj.

Checking overlap/domain-wall consistency with reweighting.

Results 2016-2019 (phase 3)

Target observables are

- Dirac spectrum
- Topological charge,
- axial U(1) susceptibility,
- meson/baryon correlators,
- chiral susceptibility.

Special focus = axial U(1) anomaly

Anomalous WTI looks non-zero:

$$\langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion}$$

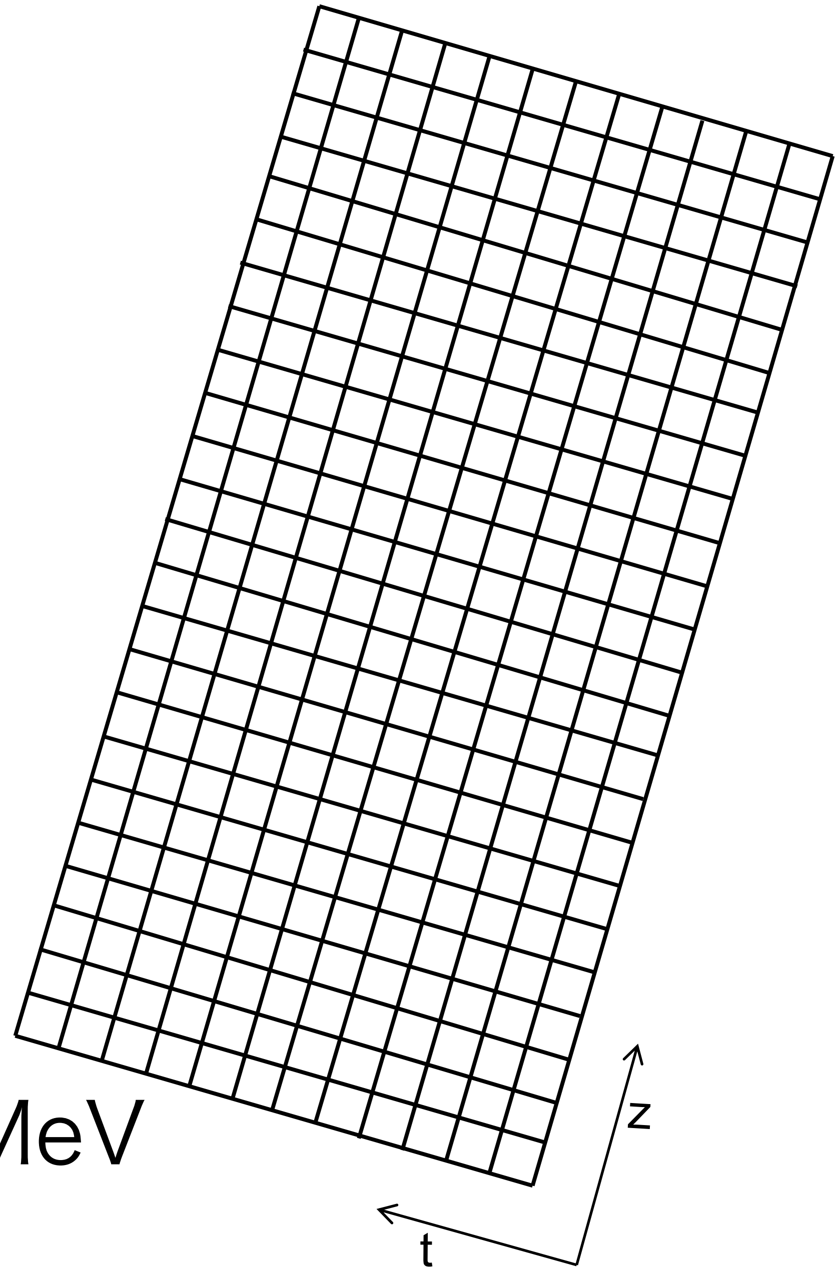
but the real question is if

$$\langle \langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') \rangle_{gluons} = \left\langle \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion} \right\rangle_{gluons} = 0???$$

to which only lattice QCD can answer.

Contents

Simulation setup



Nf=2 flavor QCD

1/a = 2.6 GeV (0.075fm)

Symanzik gauge action

L=24,32,40,48 [1.8-3.6fm]

Mobius domain-wall fermions with m_{res}<1MeV

Quark mass from 3MeV

(< phys. pt. ~4MeV) to 30MeV.

T=190, 220, 260, 330 MeV and higher.

(Lt=8,10,12,14)

Tc is estimated to be around 175MeV.

Overlap vs. Mobius domain-wall

$$D_{\rm ov}(m) = \left[\frac{1+m}{2} + \frac{1-m}{2}\gamma_5\,{\rm sgn}(H_M)\right]$$

perfect chiral sym.

numerically m_{res} ~ 1keV

good chiral sym.

$$D^{4D}_{\rm DW}(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5\frac{1-(T(H_M))^{L_s}}{1+(T(H_M))^{L_s}}$$

with L_5=16.

$$H_M = \gamma_5\frac{2D_W}{2+D_W}$$

numerically m_{res} ~ 1MeV

OV is obtained by exactly computing the sgn function for low-modes of H_M.

Violation of chiral symmetry enhanced at finite T

Checking chiral sym. for EACH eigenmode

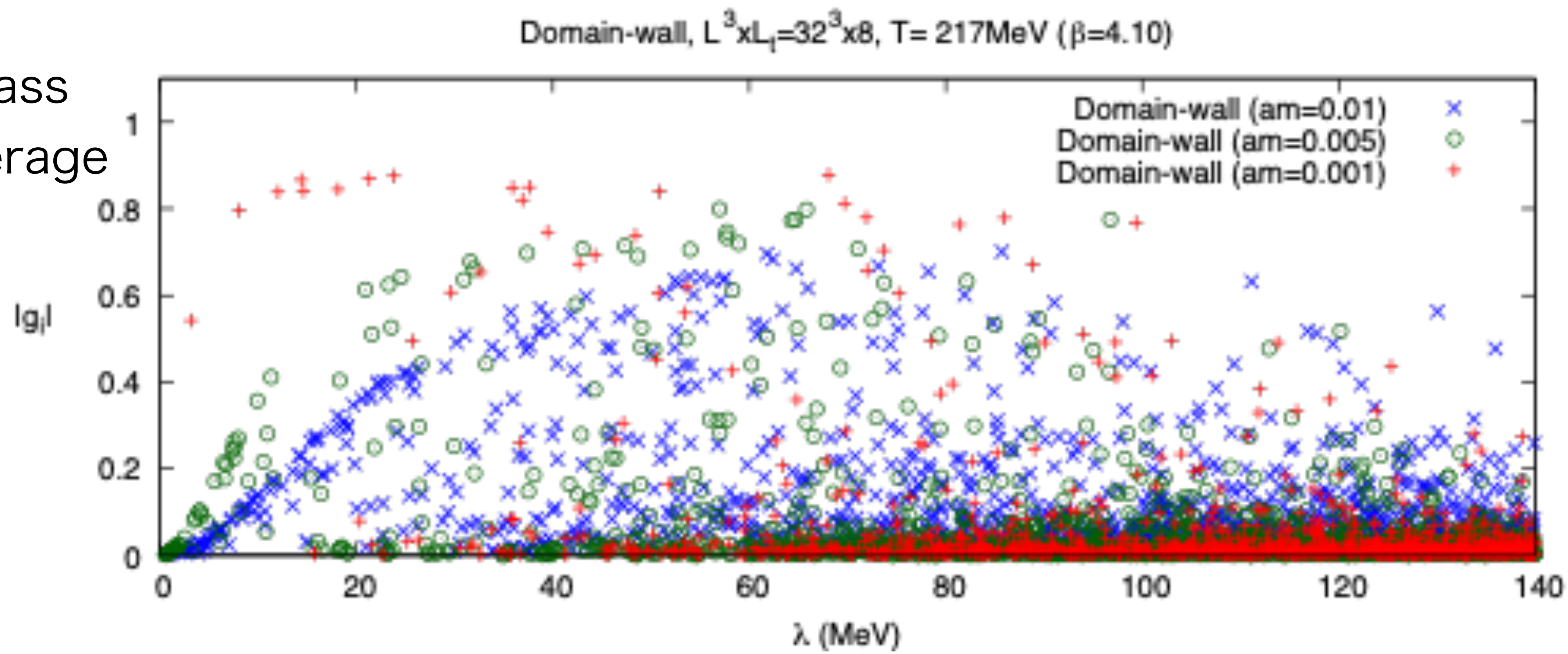
$$g_i = \left(v_i^\dagger, \frac{D\gamma_5 + \gamma_5 D - aRD\gamma_5 D}{\lambda_i} v_i \right)$$

Bad modes appear above Tc for a~0.1fm.

[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

Note: residual mass is (weighted) average of them.

For T=0, gi are consistent with residual mass.



Overlap/domain-wall reweighting

$$\langle O\rangle_{overlap} = \frac{\int dAO[\det D_{\rm ov}(m)]^2 e^{-S_G}}{\int dA[\det D_{\rm ov}(m)]^2 e^{-S_G}} = \frac{\int dAOR[\det D^{\rm 4D}_{\rm DW}(m)]^2 e^{-S_G}}{\int dAR[\det D^{\rm 4D}_{\rm DW}(m)]^2 e^{-S_G}} = \frac{\langle OR\rangle_{domain-wall}}{\langle R\rangle_{domain-wall}}$$

$$R \equiv \frac{\det[D_{\rm ov}(m)]^2}{\det[D^{\rm 4D}_{\rm DW}(m)]^2}$$

Essential for a > 0.1fm. [our previous work]

DW and OV are consistent for a~0.08fm.

(for Meson/Baryon study, we use DW)

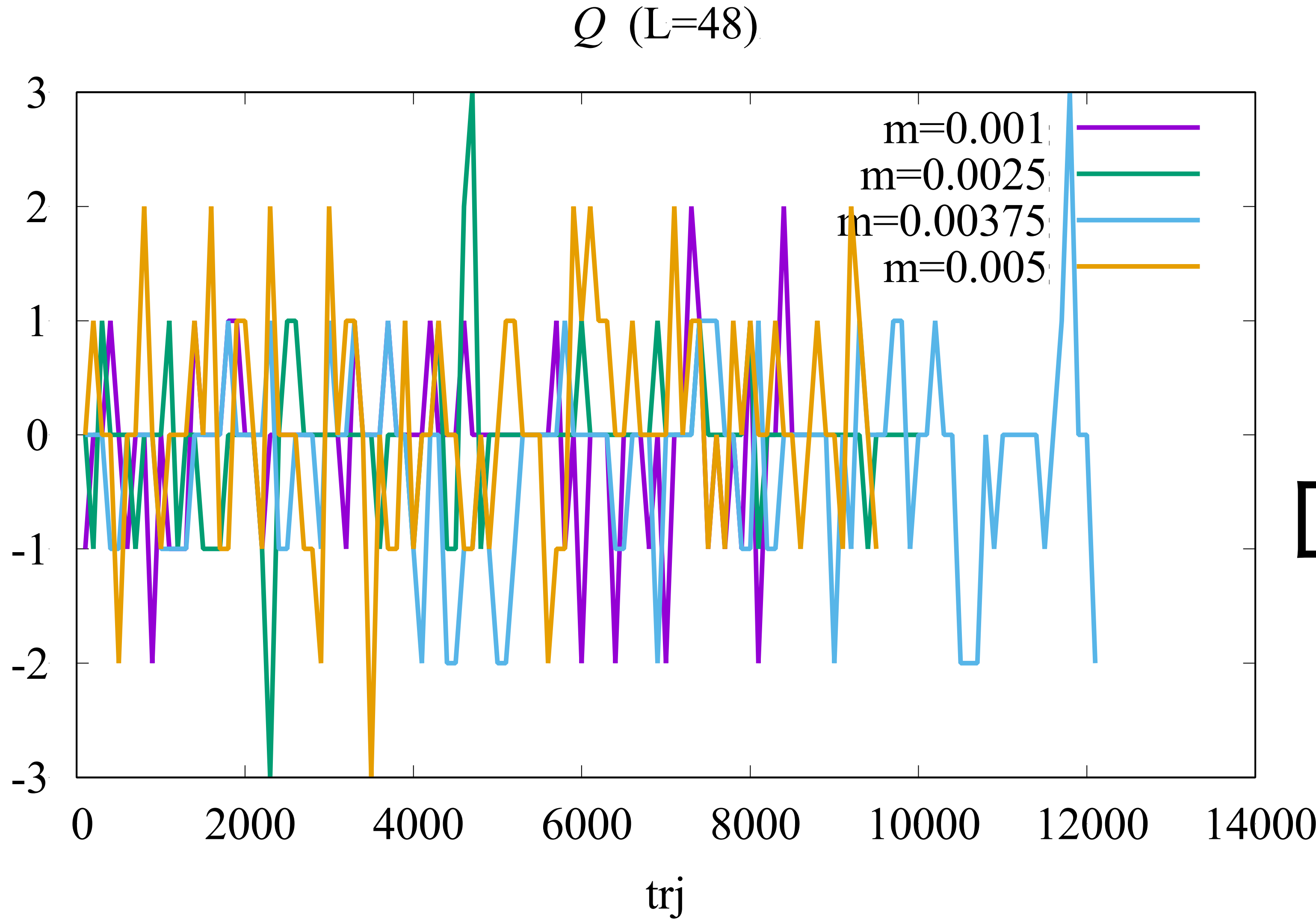
[this work].

Bonus = topology tunnelings

For dynamical overlap fermion,

we needed to fix the topology.

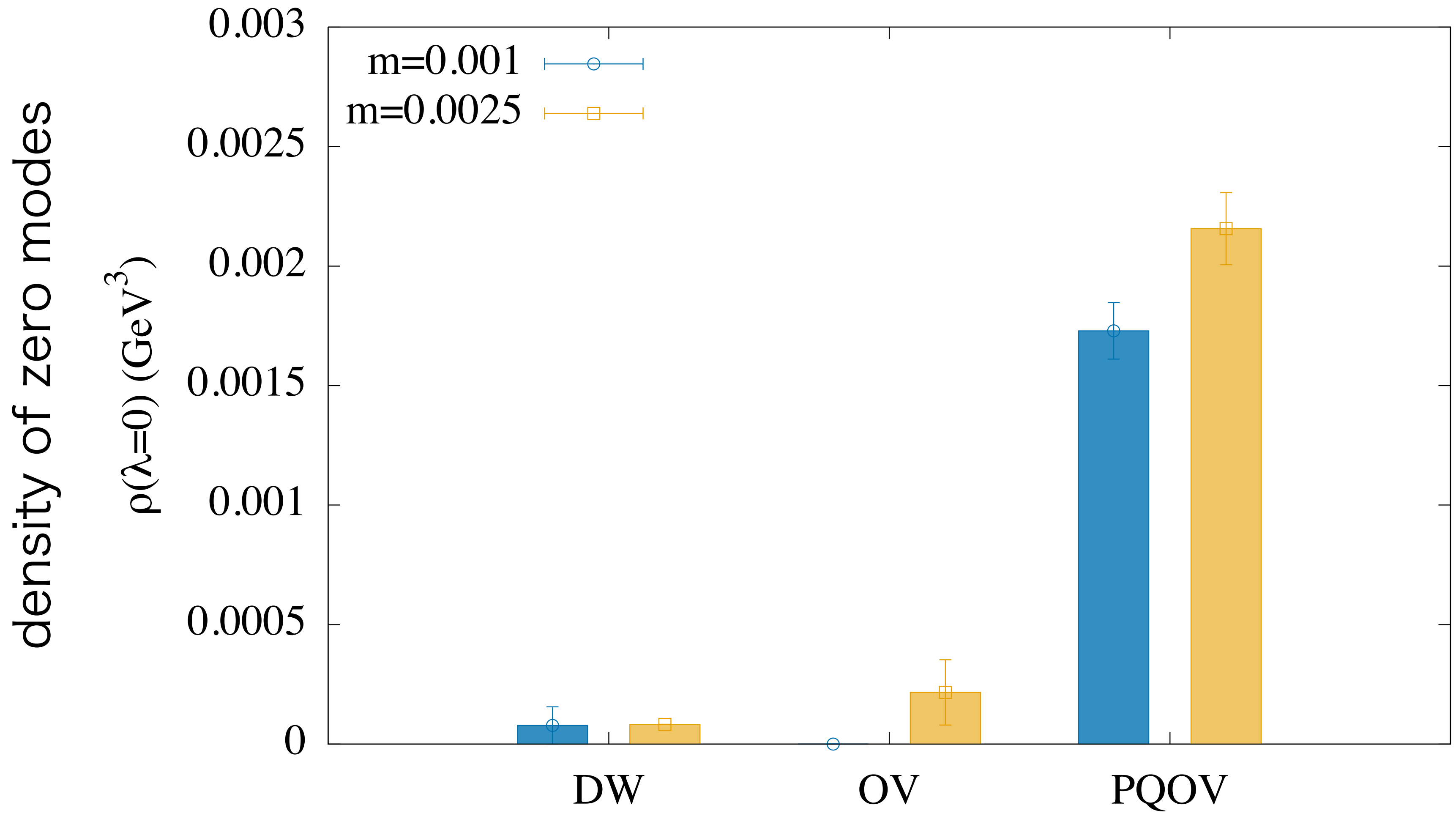
But DW + OV reweighting, we do not.



Data at T=220MeV

The use of overlap only in valence sector is dangerous !

In our work, reweighted OV and DW are consistent. But partially quenched OV is NOT. Fake chiral zero modes appear.



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Dirac spectrum

Zero eigenvalues are related to SU(2)xSU(2) breaking, through the Banks-Casher relation,

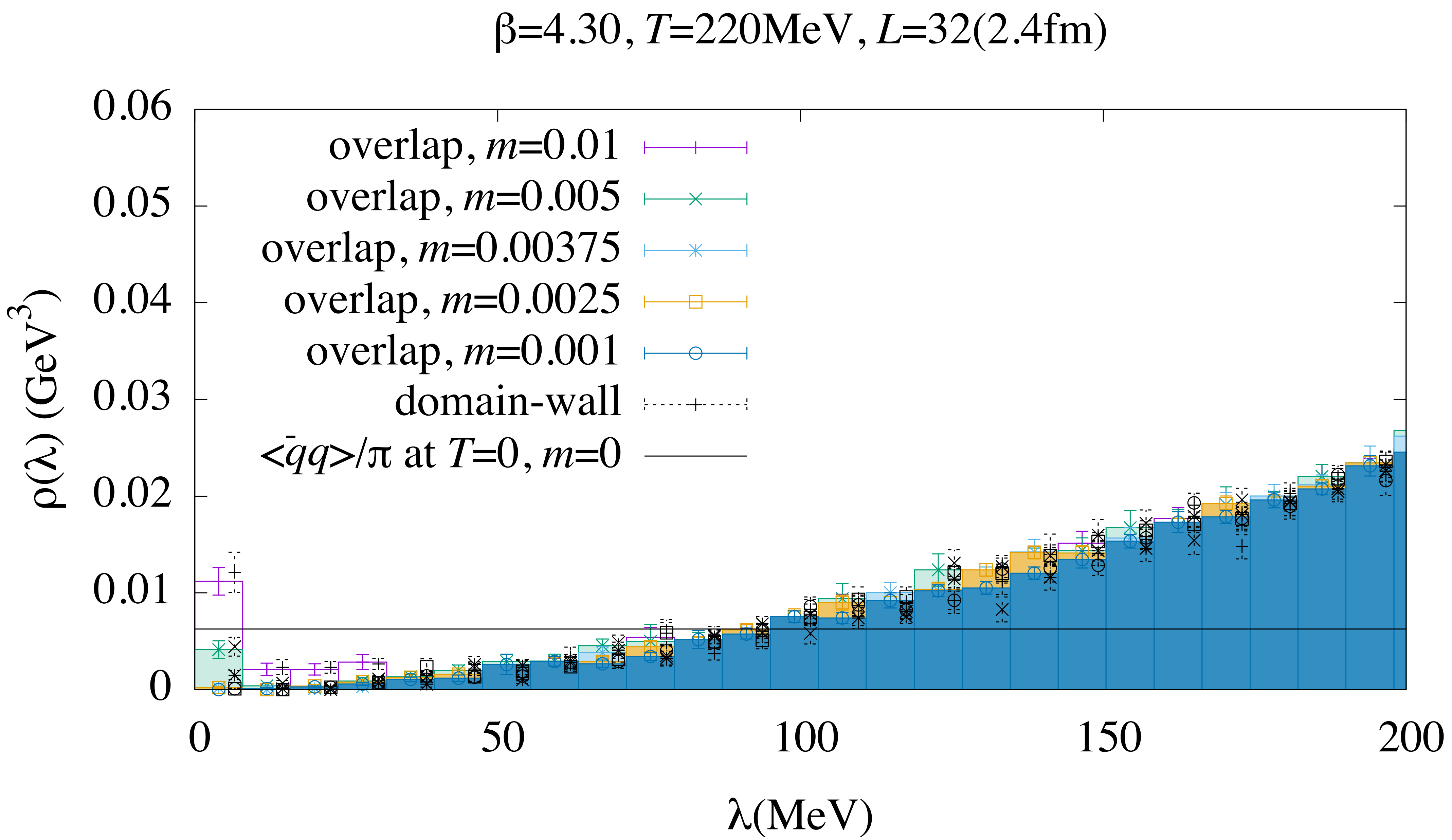
$$\lim_{m\to 0}\lim_{V\to\infty}\langle\bar{q}q\rangle = \pi\rho(0) \quad \rho(\lambda) = \frac{1}{V}\sum_i \langle\delta(\lambda-\lambda_i(A))\rangle$$

$\lambda_i(A)$: i-th eigenvalue of Dirac op. with gauge background A.

and axial U(1) anomaly through the index theorem,

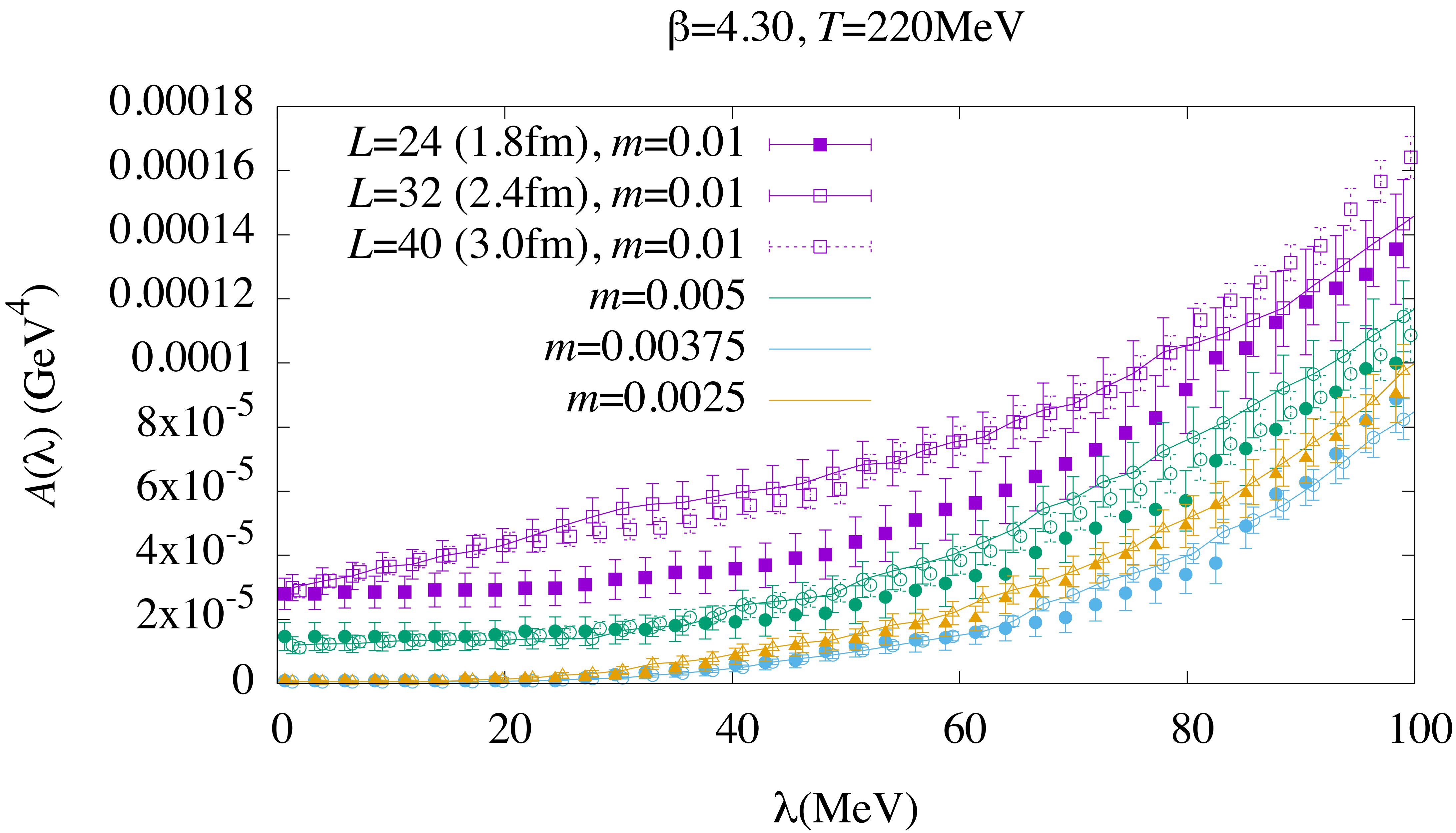
$$n_+ - n_- = \frac{1}{32\pi^2}\int d^4x\, \epsilon_{\mu\nu\rho\sigma}\mathrm{tr}_c F^{\mu\nu}F^{\rho\sigma}$$

Dirac spectrum at T=220MeV



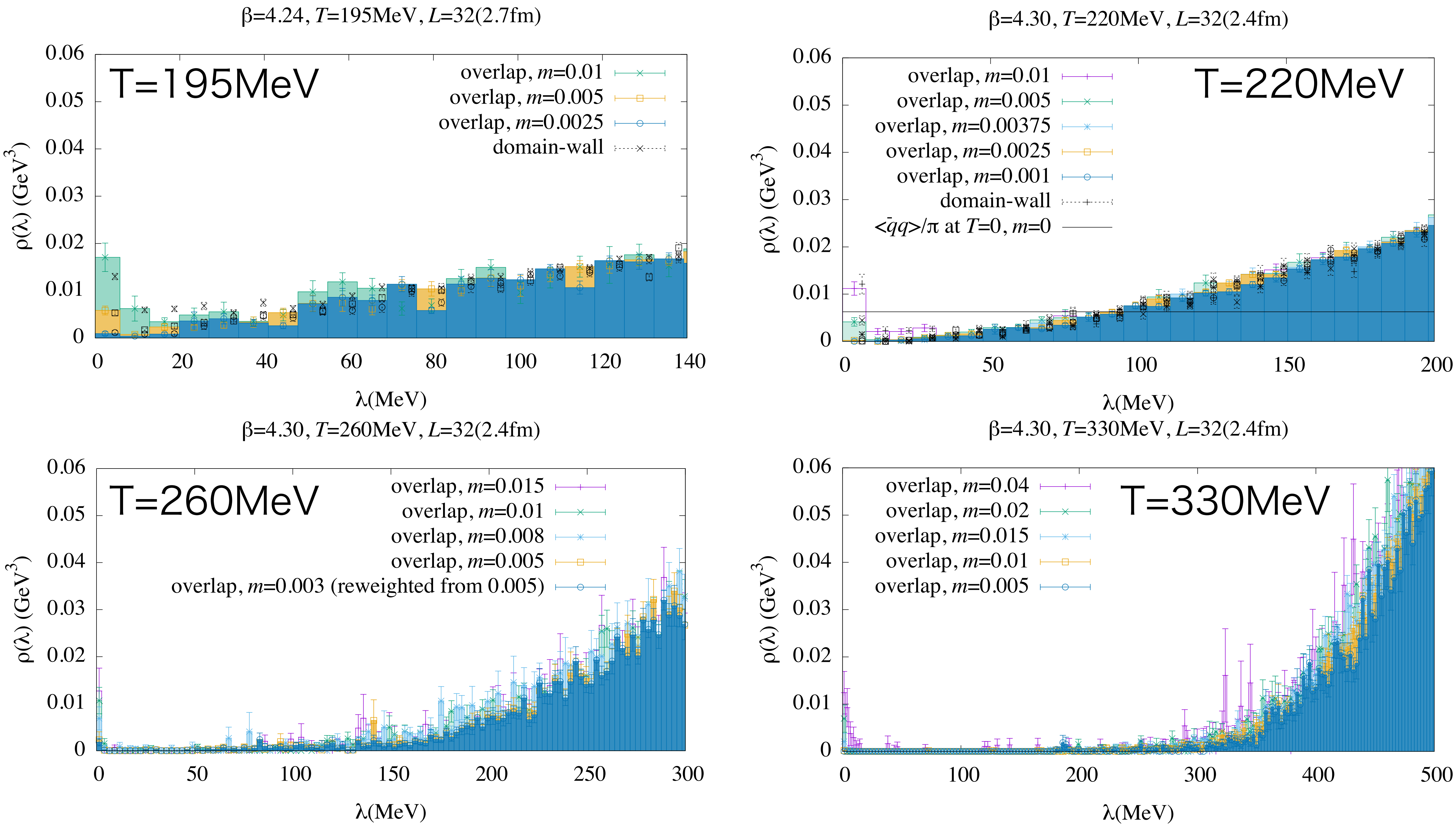
* A remarkable peak at zero but disappears as m→0. U(1)A?
* Strong supression of non-zero near zero modes. SU(2)?
* DW and OV are consistent.

Different volumes

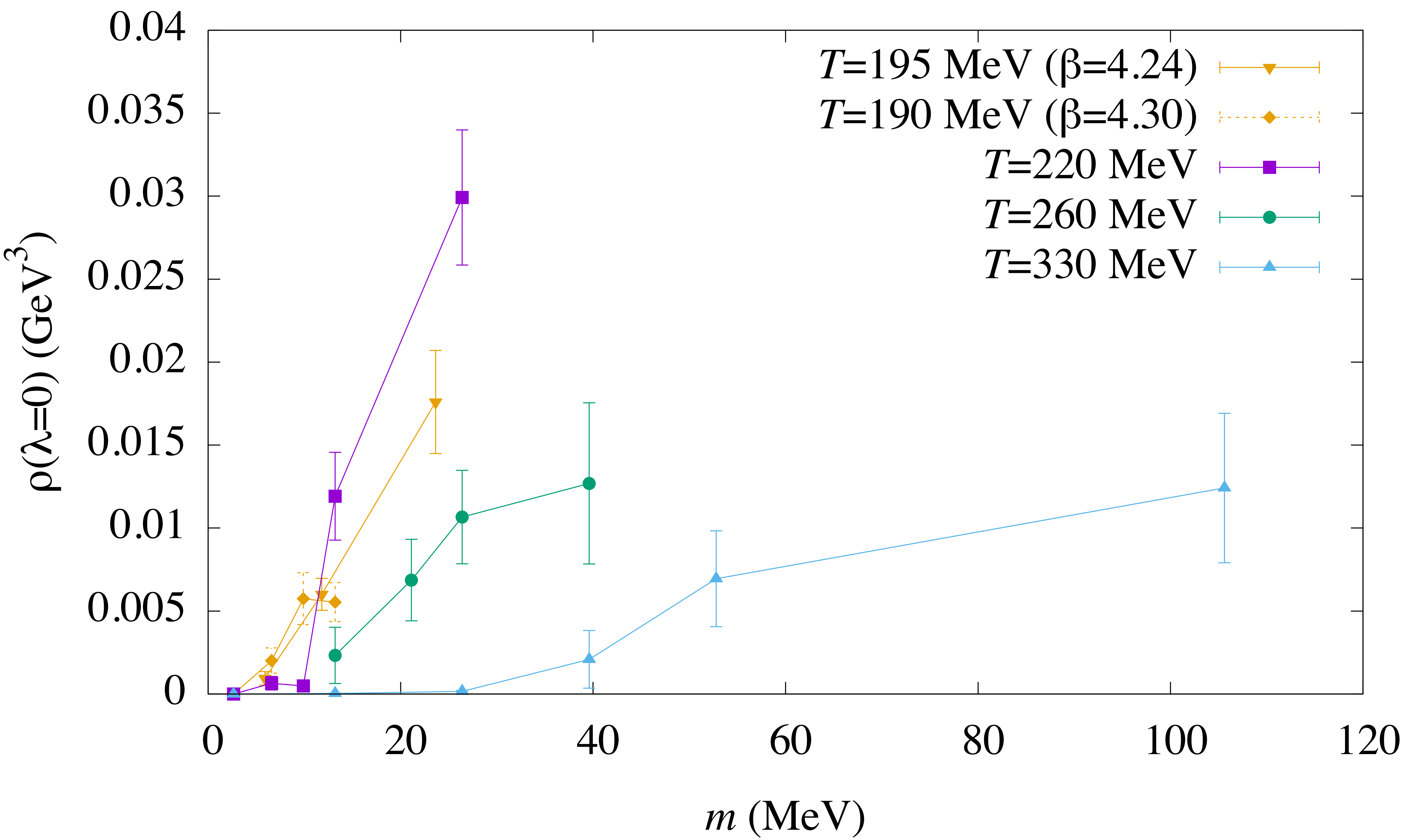


* 3 different volumes show consistent results
* except for L=24 m=0.01 (heaviest data, L/Lt=2)

The larger T, the larger the pseudo-gap.



(Near)zero mode peaks



Consistent with zero BEFORE the chiral limit.

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✔ 2. Lattice setup

Nf=2 QCD w/ MDWF and rewegihteg overlap. at T=190-330MeV near physical m~4MeV.

3. Numerical results
   - Dirac spectrum has a peak but vanishes in the m→0 limit.
   - Topology
   - U(1) susceptibility
   - Meson correlators
   - Chiral susceptibility
4. Summary

Topological susceptibility

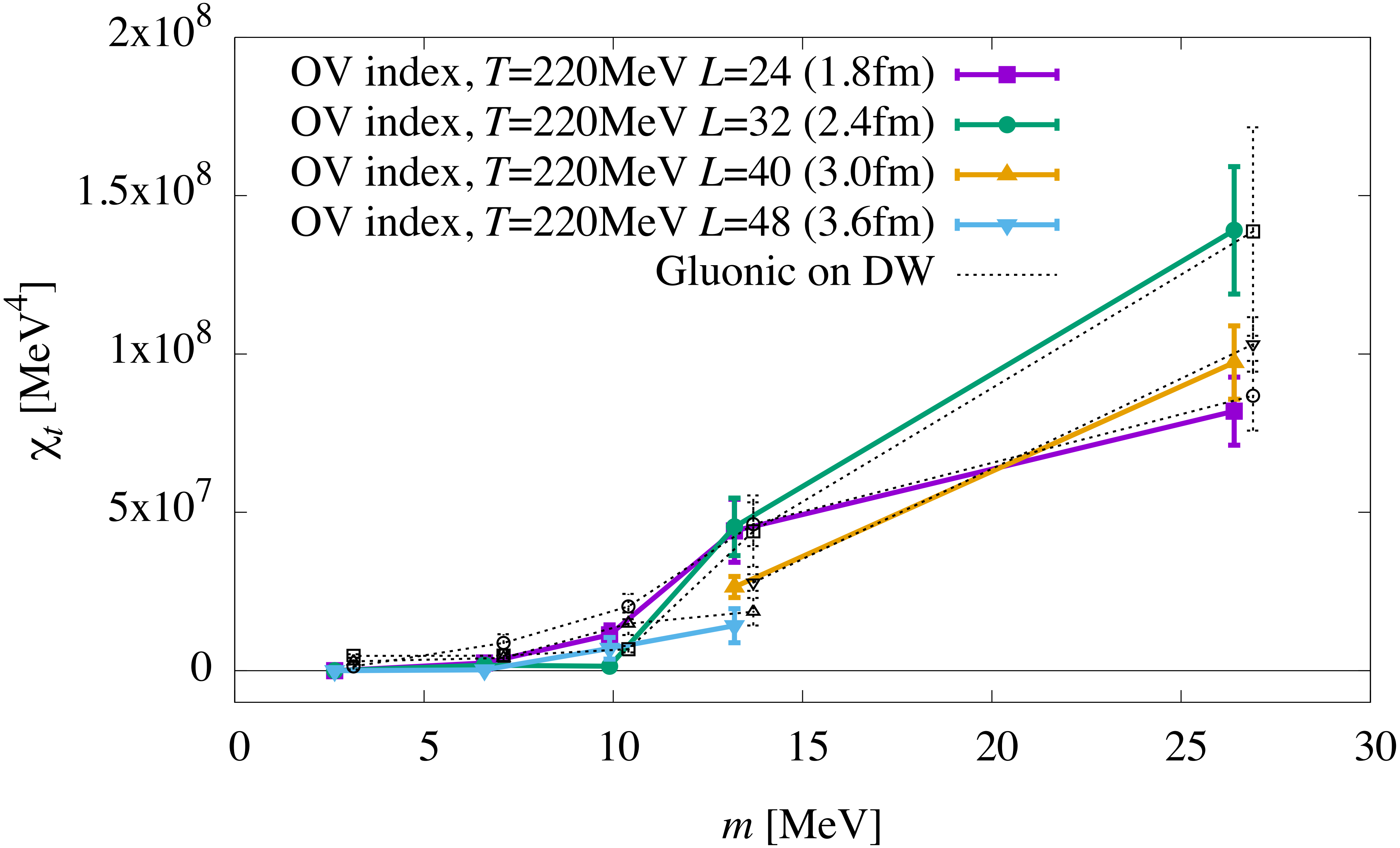
$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$

$$Q = n_+ - n_- = \frac{1}{32\pi^2} \int d^4x \mathrm{Tr} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

Overlap Dirac index (w/ OV/DW reweghting)	Gluonic definition (using clover term)

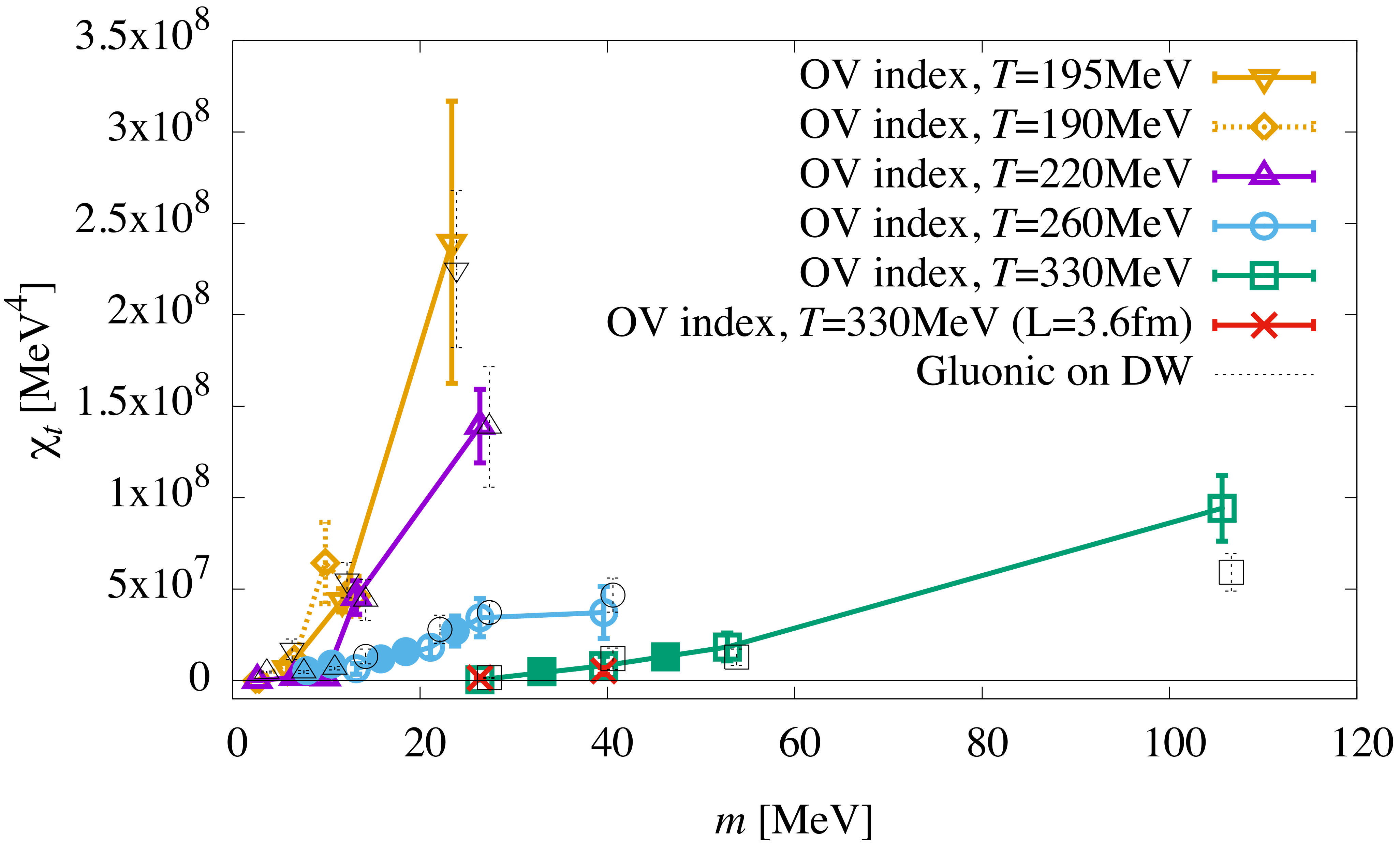
We try both definitions.

Topological susceptibility at T=220MeV



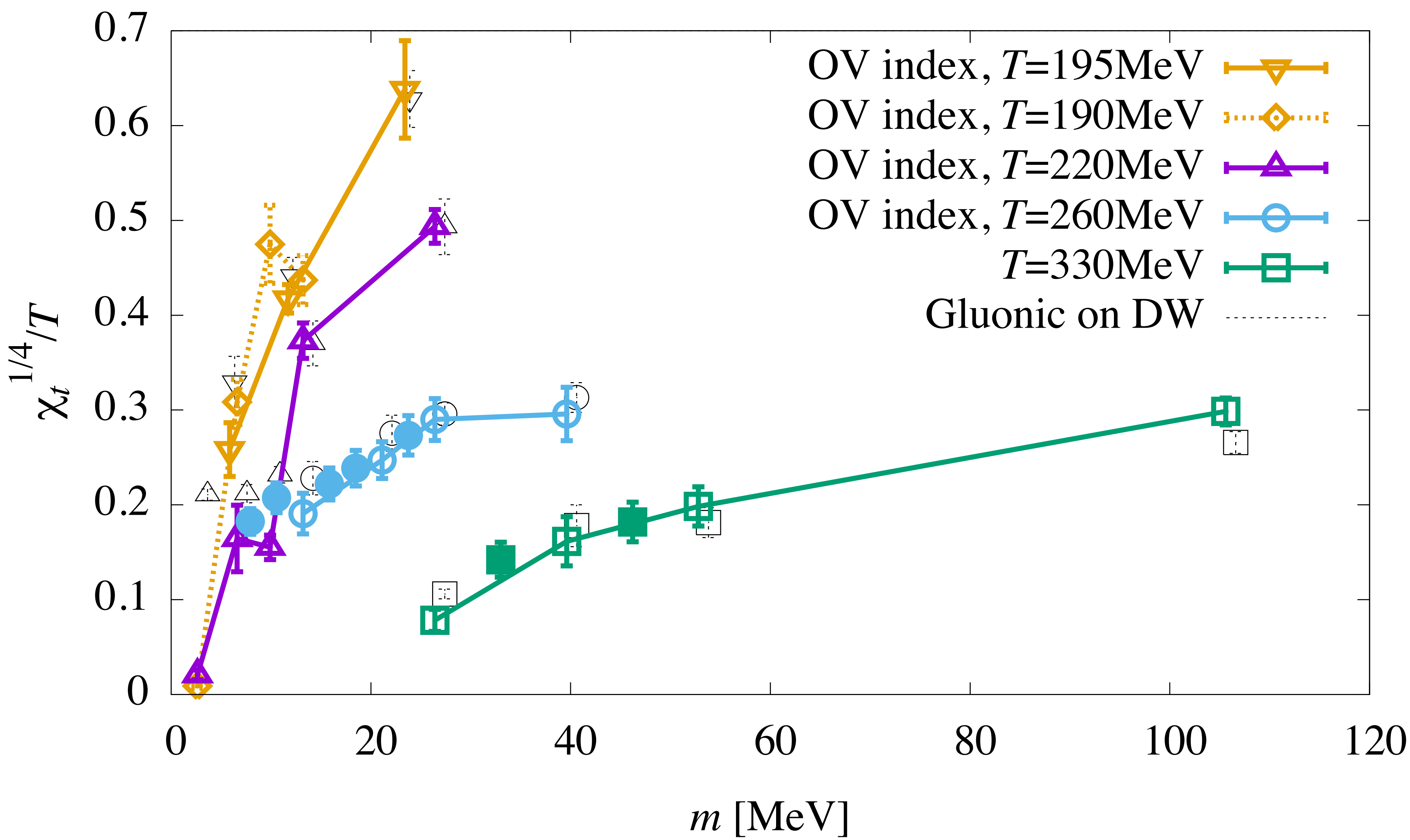
* Strong supression around m=10MeV.
* Data for L=1.8-3.6 fm are consistent.
* Gluonic def. on DW and reweighted OV index agree.

Different temperatures



* Sharp drop at FINITE quark mass
* Gluonic def. on DW and reweighted OV index agree.

Taking 4th root



- * Topology fluctuation is suppressed to ~ m^4.
- * goes down to a few MeV, at most.

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Axial U(1) susceptibility

Definition: Difference between S and PS triplet correlator

We try 2ways:

$$\Delta(m) = \sum_x \left[\langle \pi(x)\pi(0)\rangle - \langle \delta(x)\delta(0)\rangle\right],$$

→ DW w/ noise method

Spectral decomposition (with overlap D)

$$\Delta(m) = \frac{1}{V(1-m^2)^2}\left\langle \sum_{\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

→ OV w/ reweighting

λ_m : eigenvalues of $H_{ov}(m) = \gamma_5[(1-m)D_{ov} + m]$

Chiral zero-mode subtraction

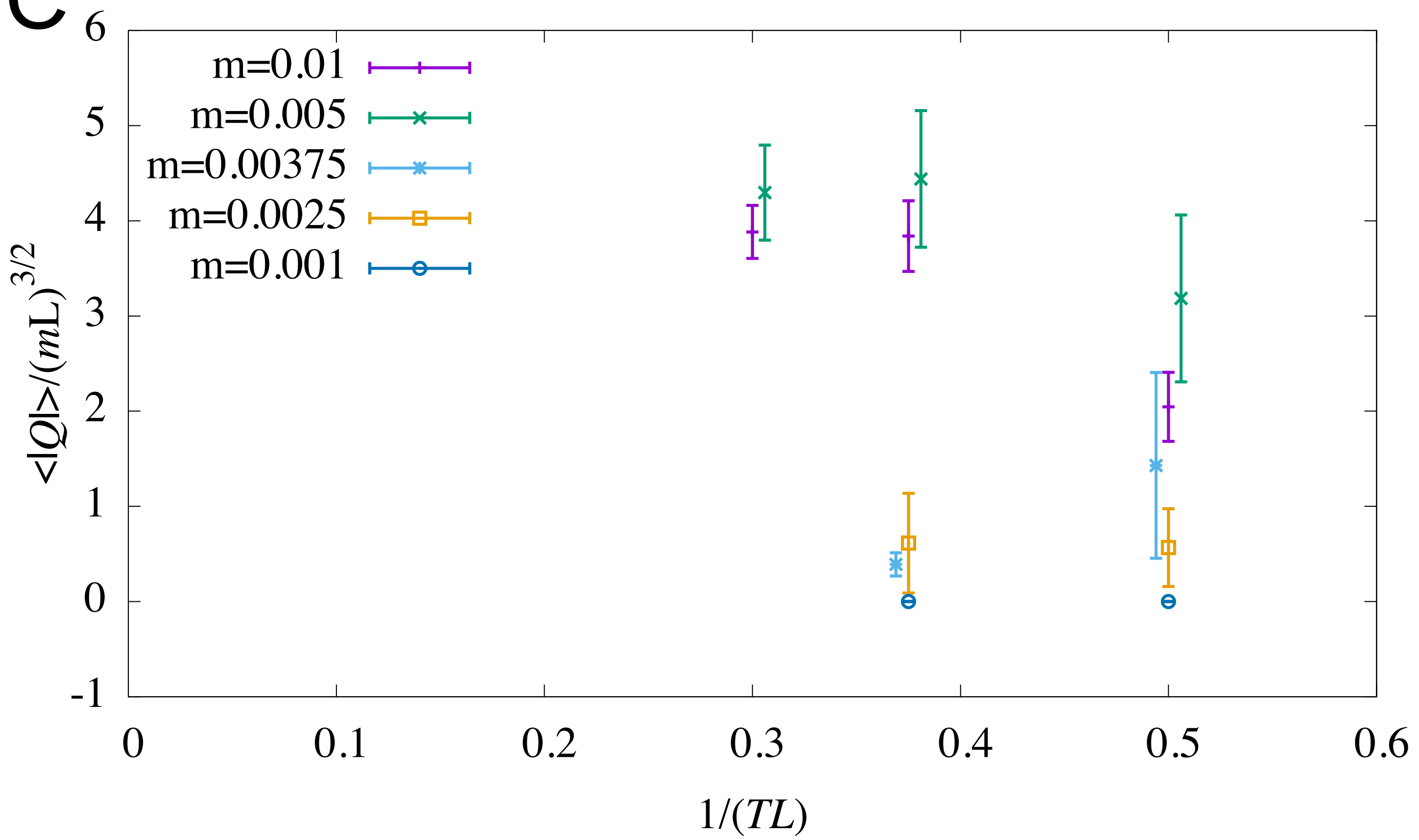
$$\Delta(m) = \frac{\langle|Q|\rangle}{m^2V(1-m^2)^2} + \frac{1}{V(1-m^2)^2}\left\langle \sum_{\lambda_m\neq 0} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

We find chiral zero modes are noisy.

-> subtract by hand: $\bar{\Delta}(m) \equiv \Delta(m) - \frac{2\langle|Q|\rangle}{m^2(1-m^2)^2V}$

This is justified since

$$\langle|Q|\rangle \propto V^{1/2} \; (\propto L^{3/2})$$



UV subtraction

In the expansion in valence quark mass,

$$\bar{\Delta}(m_v) = \frac{a}{m_v^2} + b + m_v^2 c + O(m_v^4)$$

c has a UV divergence, while we are interested in the IR part, a and b.

-> Let us remove c, so that

$$\bar{\Delta}^{UV subt.}(m) = \frac{a}{m^2} + b + O(m^4)$$

by linear combinations w/ 3 m_v's.

UV subtraction

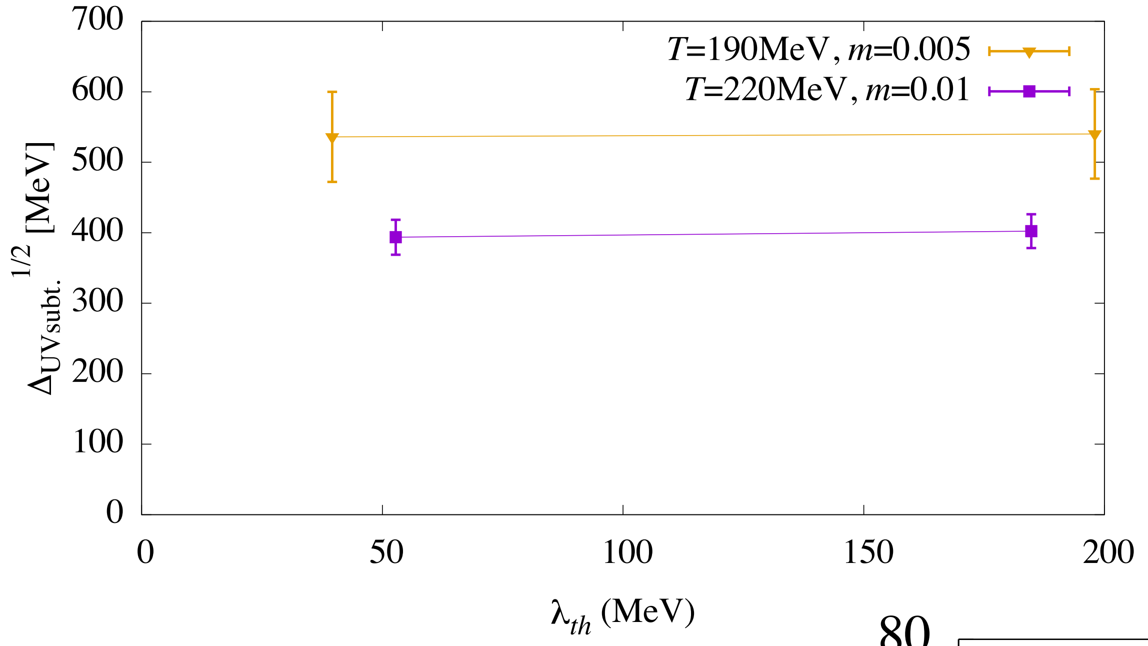
More concretely,

$$\bar{\Delta}^{UV\,subt.}(m) = \frac{m_2^2 m_3^2}{m_2^2 - m_3^2}\left[\frac{\bar{\Delta}(m_1) - \bar{\Delta}(m_2)}{m_1^2 - m_2^2} - \frac{\bar{\Delta}(m_1) - \bar{\Delta}(m_3)}{m_1^2 - m_3^2}\right] + \frac{(m_1^2 + m_2^2)(m_1^2 + m_3^2)}{m_3^2 - m_2^2}\left[\frac{m_1^2\bar{\Delta}(m_1) - m_2^2\bar{\Delta}(m_2)}{m_1^4 - m_2^4} - \frac{m_1^2\bar{\Delta}(m_1) - m_3^2\bar{\Delta}(m_3)}{m_1^4 - m_3^4}\right],$$

where we choose

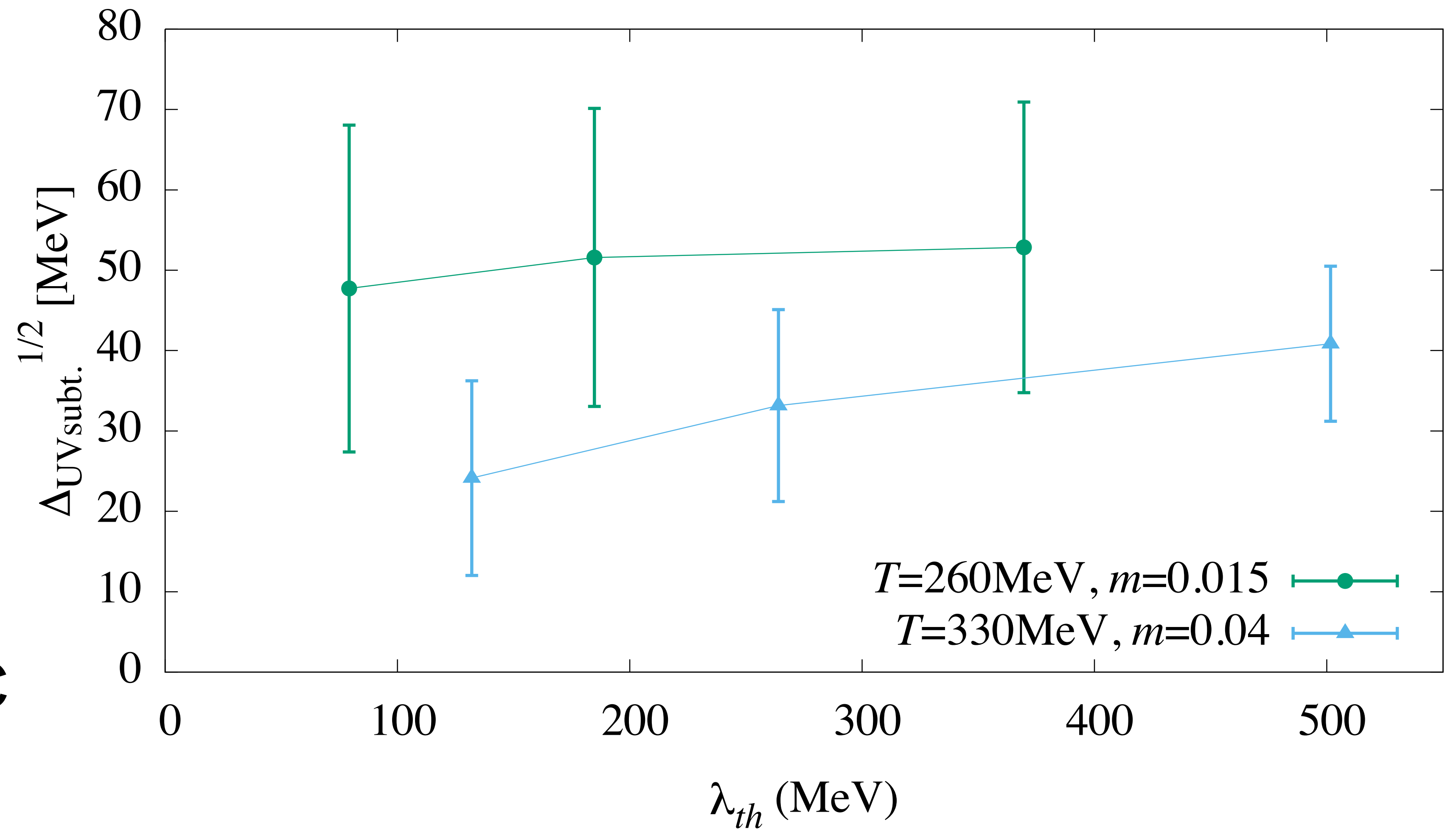
$$m_1 = m, \;\; m_2 = 0.95m \;\; m_3 = 1.05m.$$

Low-mode saturation

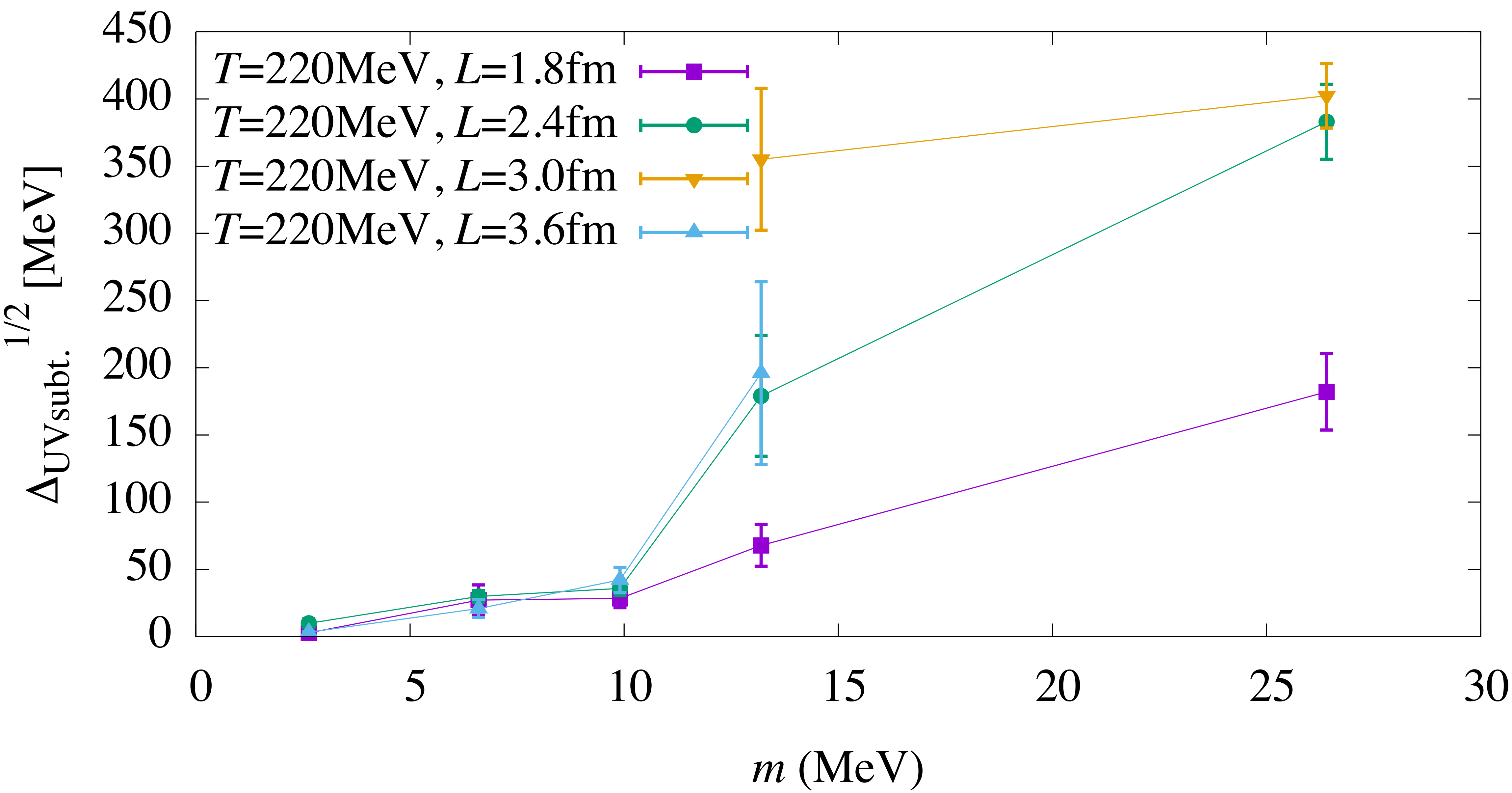


It looks O.K. for lower T

But for higher T, we need higher modes contribution using stochastic method.

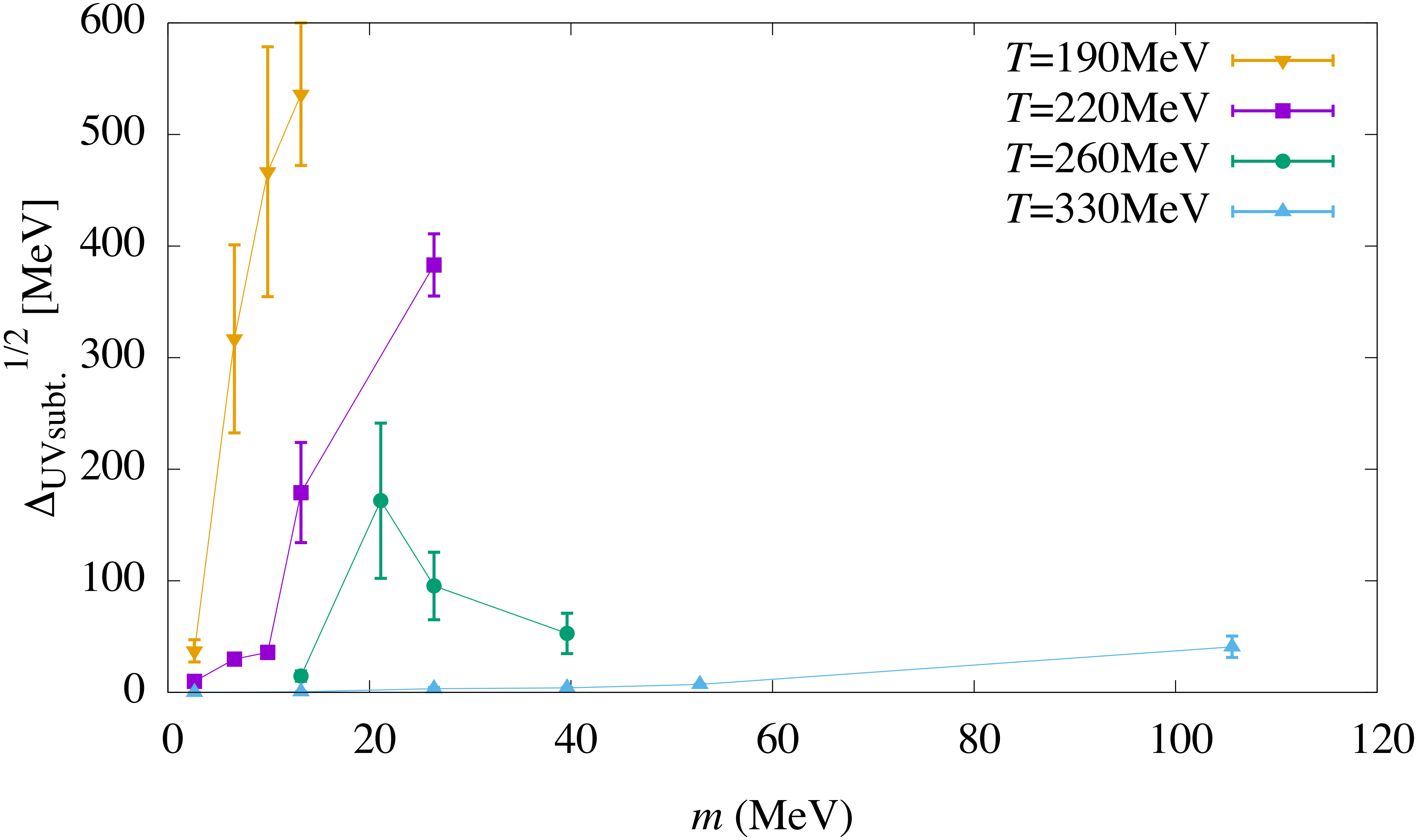


Axial U(1) susceptibility at T=220MeV



* Different volumes show consistent results, except for L=24 at heavier masses (L/Lt=2)
* anomaly goes down to a few MeV, at most.

Different temperatures



* Axial U(1) anomaly goes down to a few MeV, at most.
* Results here are with low-modes only. K. Suzuki is (re)analyzing the stochastic measurements.

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   - Meson correlators
   - Chiral susceptibility
4. Summary

“Meson” correlator

We consider spacial correlator in z direction,

$$C_\Gamma(z) = -\sum_{x,y,t} \langle \bar{u}\Gamma d(x,y,z,t)\bar{d}\Gamma u(0,0,0,0)\rangle,$$

where

$$\Gamma = \gamma_5(PS),\ 1(S),\ \gamma_{1,2}(V),\ \gamma_5\gamma_{1,2}(A),\ \gamma_4\gamma_3(T_t) \text{ and } \gamma_5\gamma_4\gamma_3(X_t).$$

* We find that the chiral symmetry is good enough with MDW.
* Rotationally symmetric average taken.
* Low-mode averaging is performed for noisy ensembles.

Tensor channels

We find that comparison

$$\bar{q}\tau^a q(x)\bar{q}\tau^a q(0) \leftrightarrow \bar{q}\tau^a\gamma_5 q(x)\bar{q}\tau^a\gamma_5 q(0)$$

↘ too noisy.

is difficult. Instead, we investigate

$$\bar{q}\tau^a\gamma_4\gamma_3 q(x)\bar{q}\tau^a\gamma_4\gamma_3 q(0) \leftrightarrow \bar{q}\tau^a\gamma_5\gamma_4\gamma_3 q(x)\bar{q}\tau^a\gamma_5\gamma_4\gamma_3 q(0)$$

For the reference, we also study $SU(2)_L \times SU(2)_R$ pair in vector channel,

$$\bar{q}\tau^a\gamma_1 q(x)\bar{q}\tau^a\gamma_1 q(0) \leftrightarrow \bar{q}\tau^a\gamma_1\gamma_5\gamma_1 q(x)\bar{q}\tau^a\gamma_5\gamma_1 q(0)$$

Is it really "meson"?

In general form (of bosonic correlator),

$$C_\Gamma(z) = \int dM \rho_\Gamma(M) \int \frac{dp_z}{2\pi} \frac{2Me^{ip_z z}}{p_z^2 + M^2} = \int dM \rho_\Gamma(M) e^{-Mz}.$$

it depends on the details of spacial spectral function $\rho_\Gamma(M)$. If it has

1. an isolated pole $\propto \delta(M - m_g) \to$ $C_\Gamma(z) \sim e^{-m_g z}$
2. a cut (like 2-quark states) from m_{th},

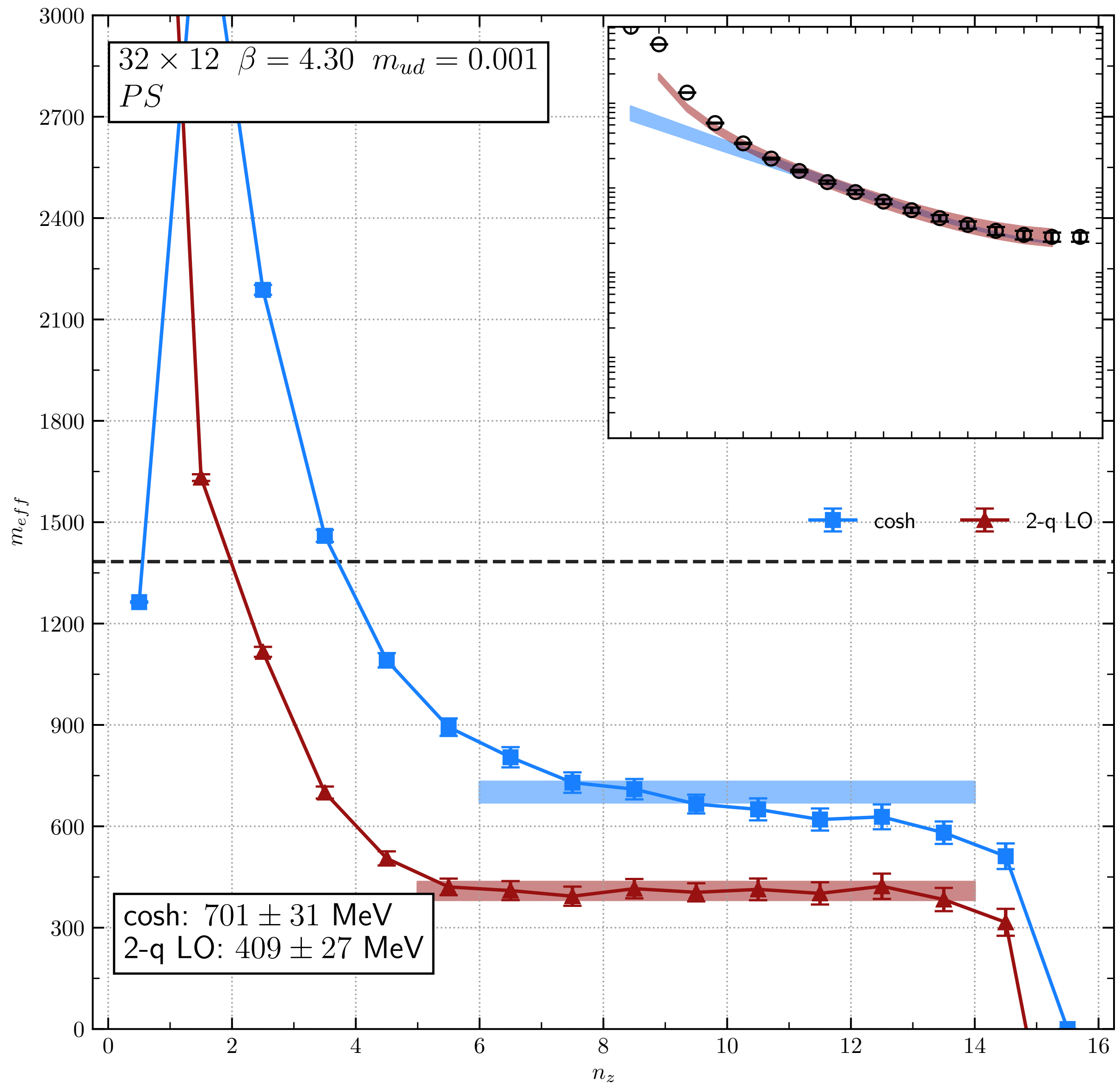
$$\rho_\Gamma(M) = \theta(M - m_{th})\,(c_0 + c_1 M + \cdots)$$

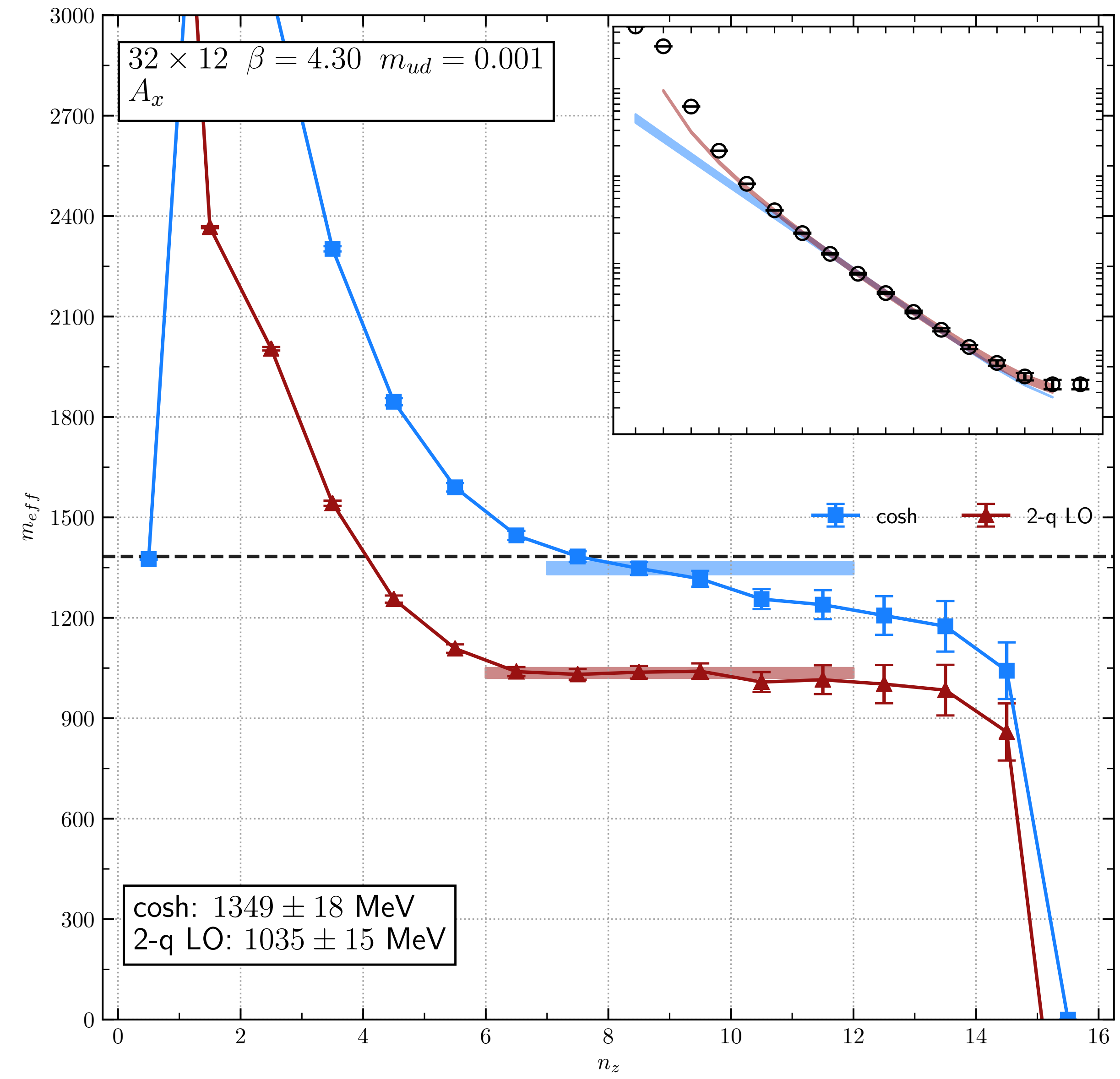
$$\to \quad C_\Gamma(z) \sim e^{-m_{th} z}\left(1/z + O(1/z^2)\right)$$

Pole vs. cut

Effective mass plots favor 2-quark picture.

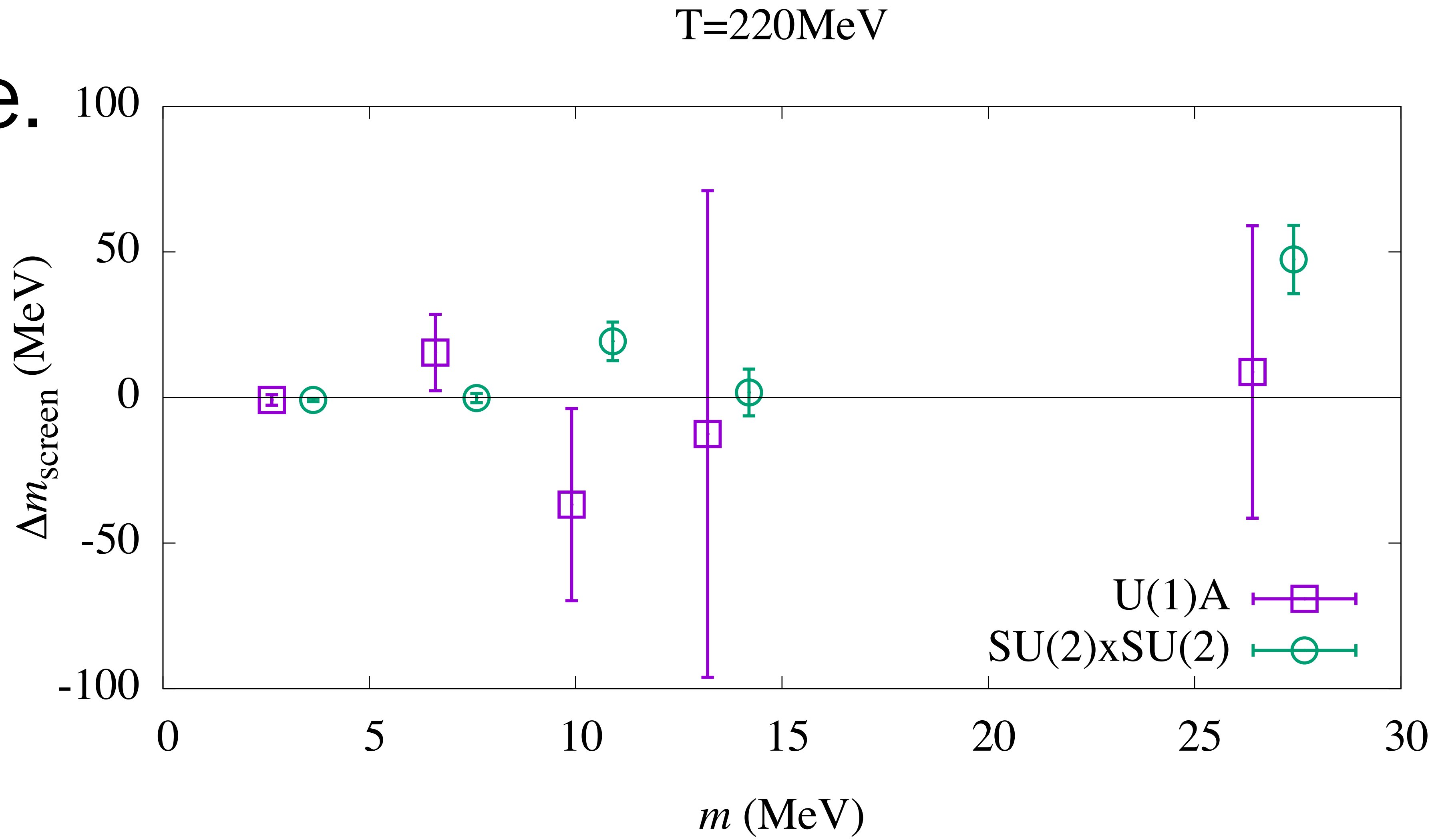
T=220MeV





Screening mass difference

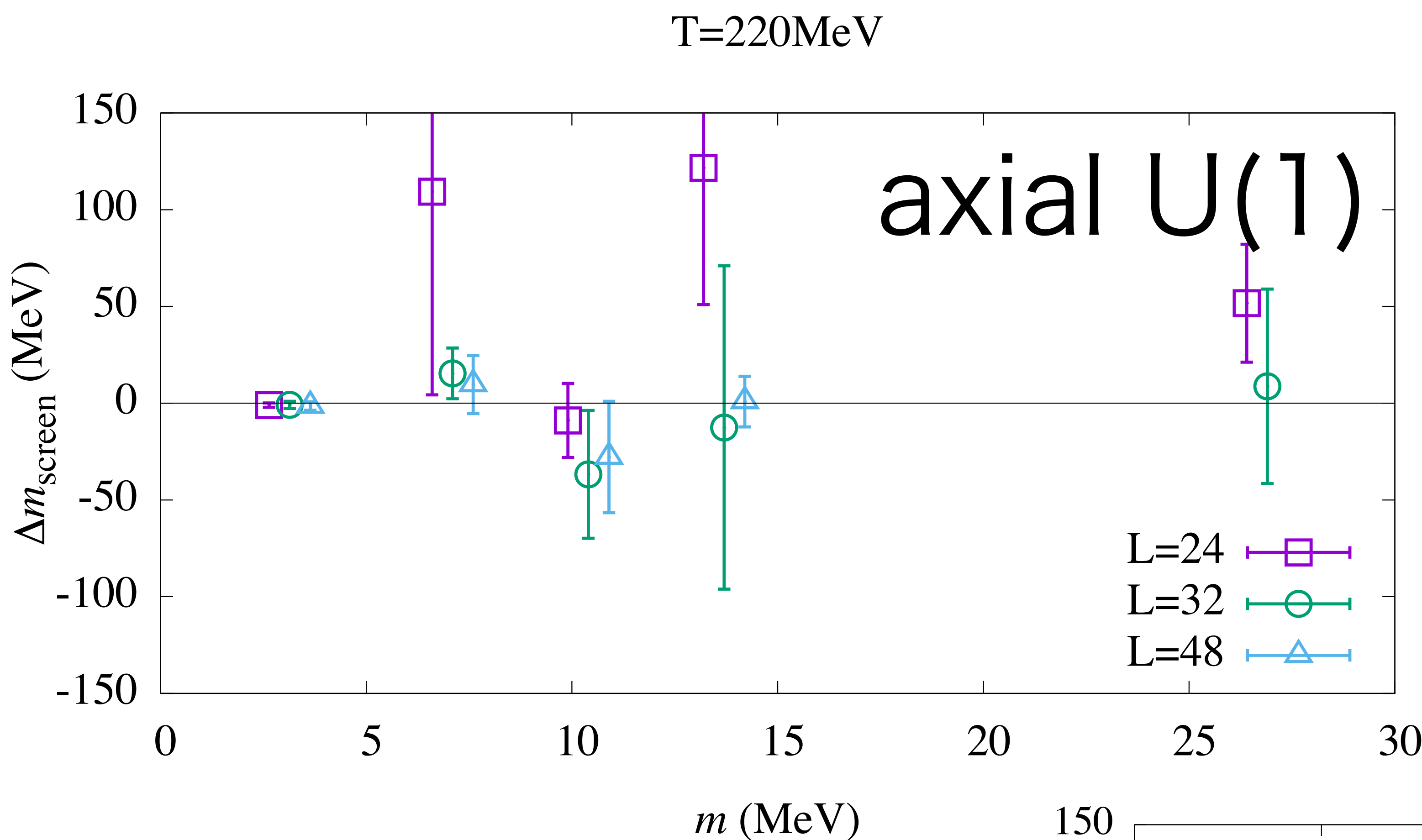
We fit our data to Ae^{-mz}/z and see their difference.



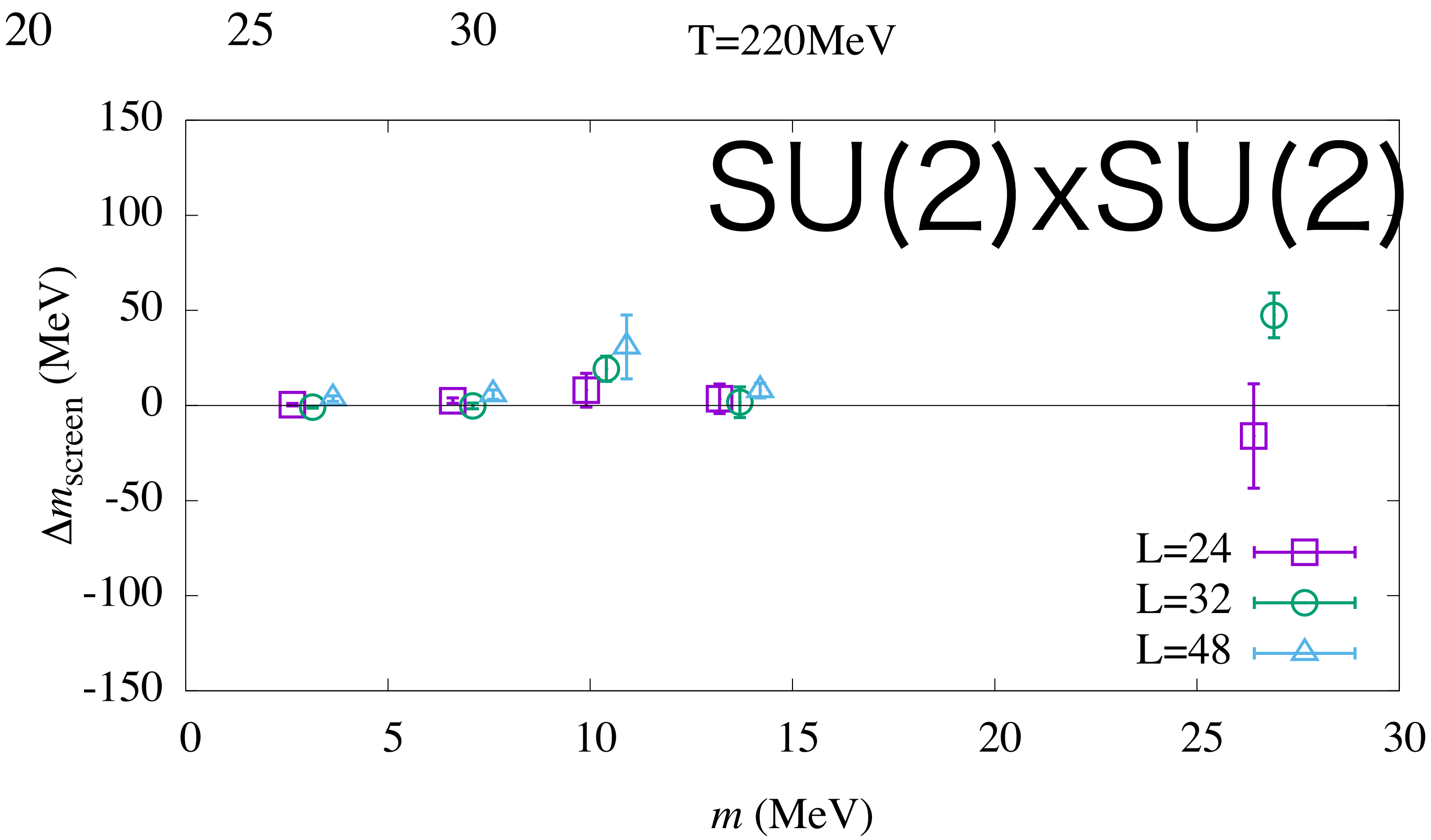
Anomaly effect disappears to a few MeV (~1% of temperature), at most.

Note: m_{screen} itself ~ 1GeV ～ $2\pi T$.

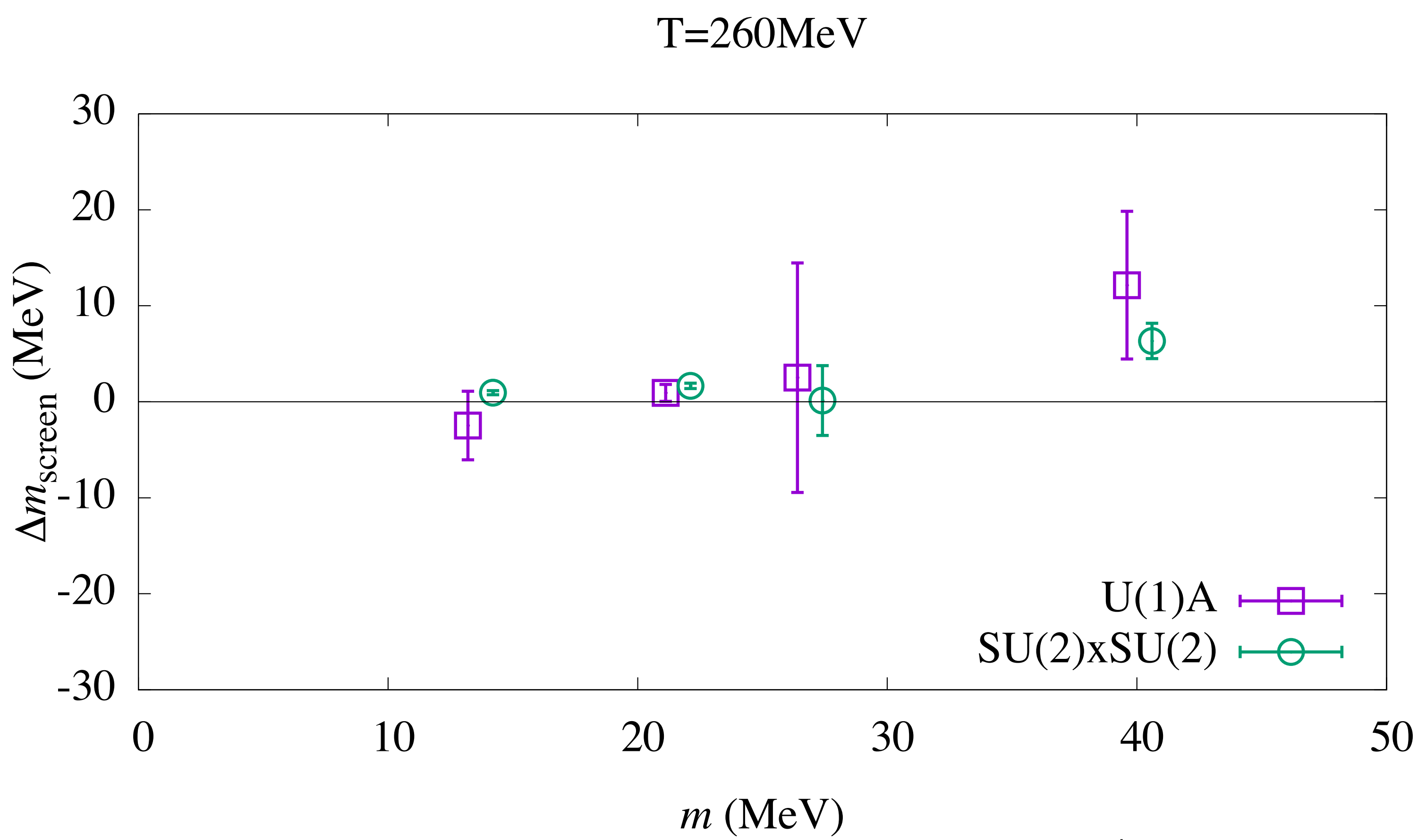
Volume dependence



The data are consistent with each other, except for noisy L=24.



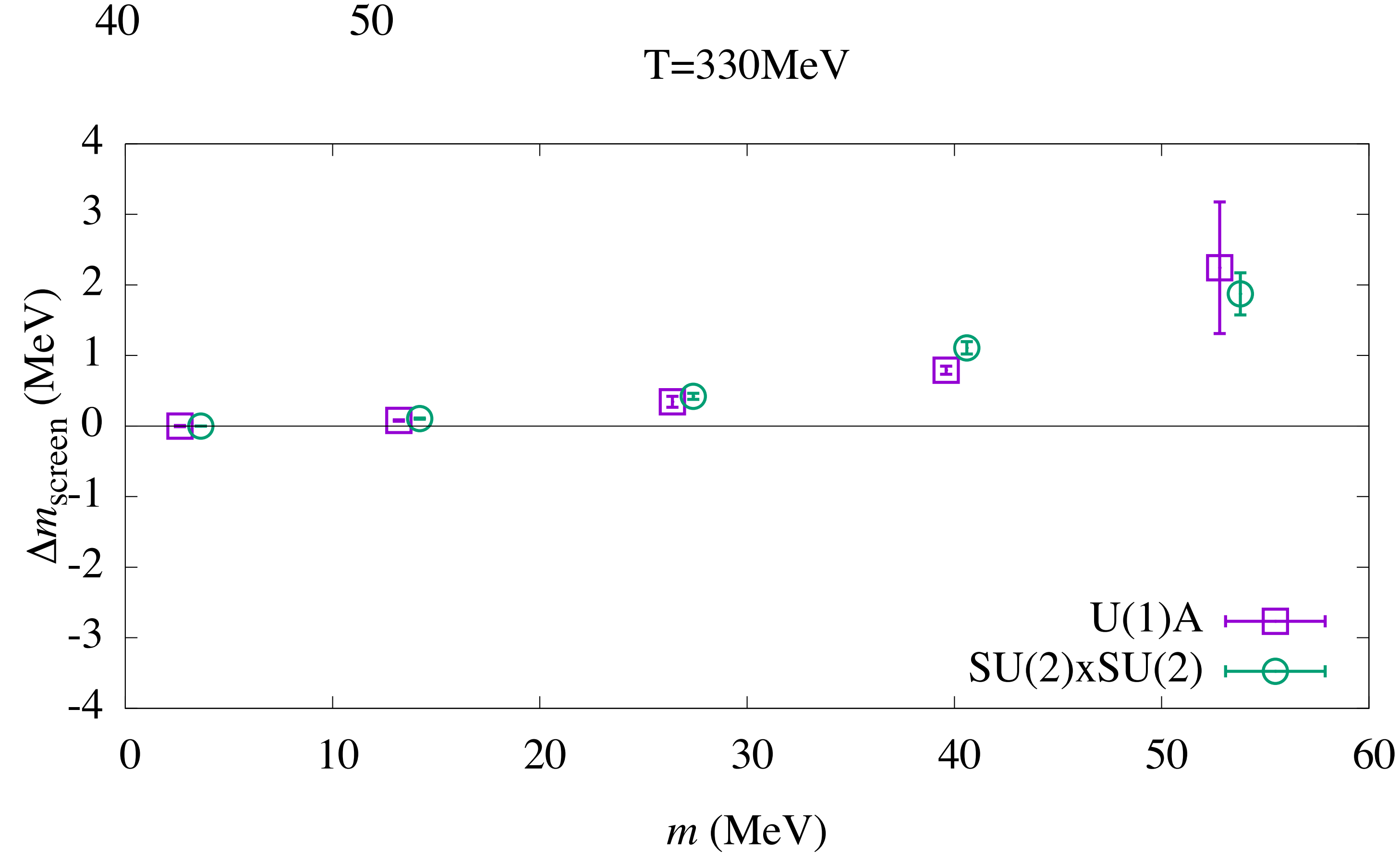
Higher temperatures



Suppression is stronger for higher T.

No big difference from SU(2)xSU(2) restoration.

C. Rohrhofer is now working hard on baryons.



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   - Topology fluctuation is suppressed by ~m^4.
   - U(1) susceptibility goes down to (a few MeV)^2.
   - ~~Meson~~ 2quark correlators show a good U(1)A symmetry.
   - Chiral susceptibility
4. Summary

What is chiral susceptibility?

Partition function

$$Z(m) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)} = \int [dA] \prod_\lambda (i\lambda(A)+m)^{N_f} e^{-S_G(A)}$$

chiral condensate

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m) = \frac{1}{V} \left\langle \sum_\lambda \frac{1}{i\lambda(A)+m} \right\rangle ,$$

chiral susceptibility

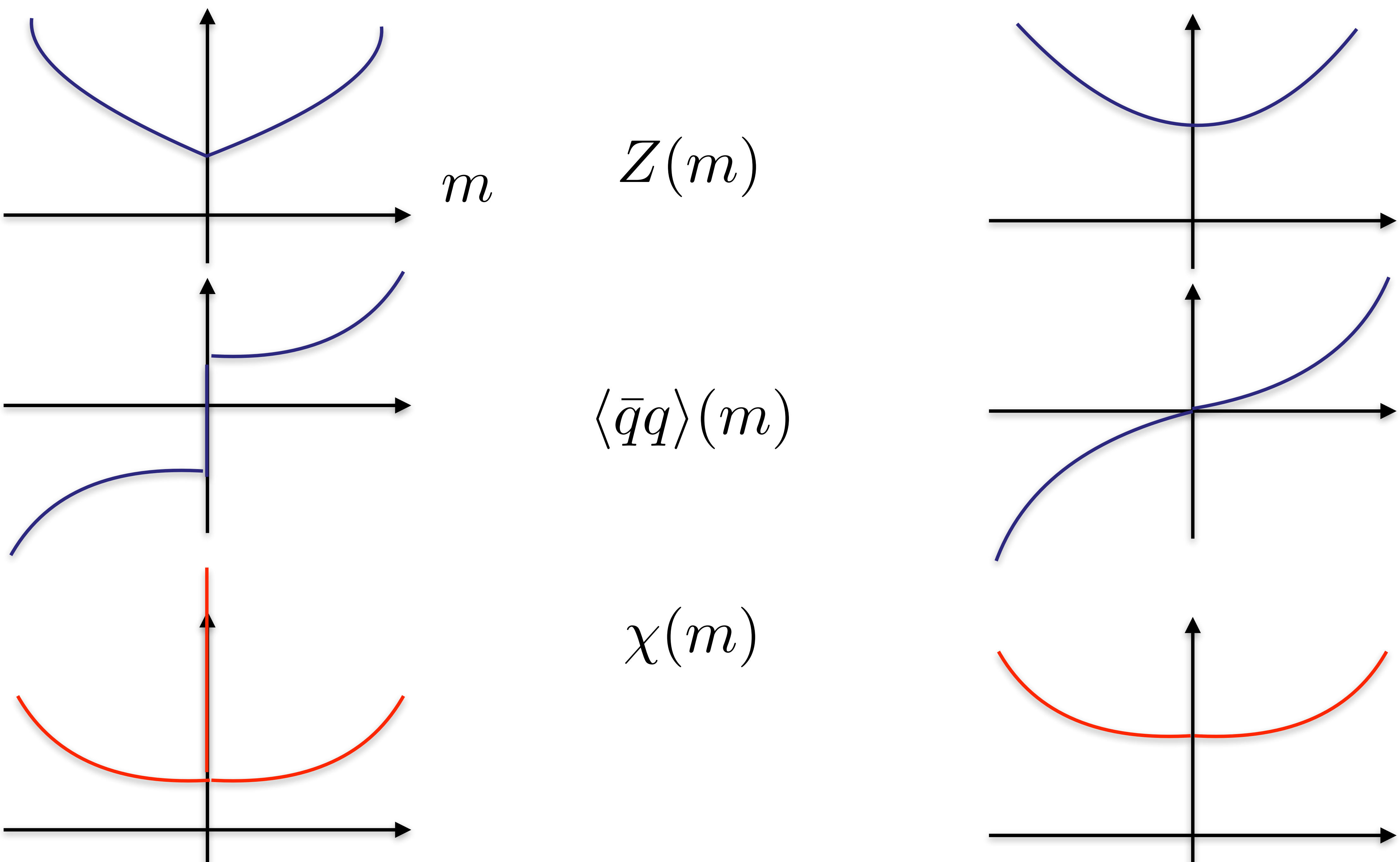
$$\chi(m) = \frac{1}{N_f V} \frac{\partial^2}{\partial m^2} \ln Z(m) = \chi^{con.}(m) + \chi^{dis.}(m),$$

empirically small

What is chiral susceptibility?

Broken phase

Symmetric phase



SU(2) or U(1)?

Chiral condensate and chiral susceptibility is used for the probe of SU(2)xSU(2) chiral symmetry breaking.
But they also break axial U(1).

In this work we show that chiral susceptibility is dominated by axial U(1) anomaly, rather than SU(2)xSU(2).

Dirac eigenmode decomposition

$$Z(m) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)} = \int [dA] \prod_\lambda (i\lambda(A)+m)^{N_f} e^{-S_G(A)}$$

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m) = \frac{1}{V} \left\langle \sum_\lambda \frac{1}{i\lambda(A)+m} \right\rangle,$$

$$\chi(m) = \frac{1}{N_f V} \frac{\partial^2}{\partial m^2} \ln Z(m) = \chi^{con.}(m) + \chi^{dis.}(m),$$

$$\chi^{con.}(m) = -\frac{\partial}{\partial m_{valence}} \langle \bar{q}q \rangle \Big|_{m_{valence}=m}$$

$$\chi^{dis.}(m) = -\frac{\partial}{\partial m_{sea}} \langle \bar{q}q \rangle \Big|_{m_{sea}=m}$$

Connected part

$$\chi^{con.}(m) = -\frac{1}{V}\left\langle \sum_\lambda \frac{1}{(i\lambda(A)+m)^2} \right\rangle$$

$$= -\frac{1}{V}\left\langle \sum_\lambda \frac{2m^2}{(\lambda(A)^2+m^2)^2} \right\rangle + \frac{1}{m}\left[\frac{1}{V}\left\langle \sum_\lambda \frac{m}{\lambda(A)^2+m^2} \right\rangle\right]$$

$$= -\Delta(m) + \frac{-\langle \bar{q}q \rangle}{m},$$

axial U(1) susceptibility!

Namely, connected part includes pure U(1) anomaly effect.

Disconnected part

$$\chi^{dis.}(m) = \frac{N_f}{V}\left[\frac{\langle N_0^2\rangle - \langle N_0\rangle^2}{m^2} + \frac{2}{m}\left(\left\langle N_0 \sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right\rangle - \langle N_0\rangle\left\langle\sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right\rangle\right)\right.$$
$$\left. + \left\langle\left(\sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right)^2\right\rangle - \left\langle\sum_{\lambda>0}\frac{2m}{\lambda(A)^2+m^2}\right\rangle^2\right].$$

$$N_0 = n_+ + n_- : \text{number of zero modes}$$

It is interesting to compare with

topological susceptibility

$$\frac{N_f}{m^2}\chi_t = \frac{N_f}{m^2}\frac{\langle Q^2\rangle - \langle Q\rangle^2}{V} \qquad Q = n_+ - n_-$$

UV divergence

The chiral condensate has quadratic divergence at most.

Λ : cut off

$$\langle \bar{q}q \rangle = \mathrm{sgn}(m) \left(\Sigma + \alpha |m| \Lambda^2 + \beta |m|^2 \Lambda + \gamma |m|^3 + \cdots \right)$$

$$-\frac{d}{dm}\langle \bar{q}q \rangle = \chi^{con.}(m) + \chi^{dis.}(m) = 2\delta(m)\Sigma - \alpha\Lambda^2 - 2\beta|m|\Lambda - 3\gamma m^2 + \cdots .$$

quadratic divergence appears only in connected part.

Disconnected part is logarighmically divergent.

Lattice formulas

With the overlap Dirac operator, we have

$$\chi^{con.lat}(m) = -\Delta^{lat}(m) + \frac{-\langle\bar{q}q\rangle^{lat}}{m},$$

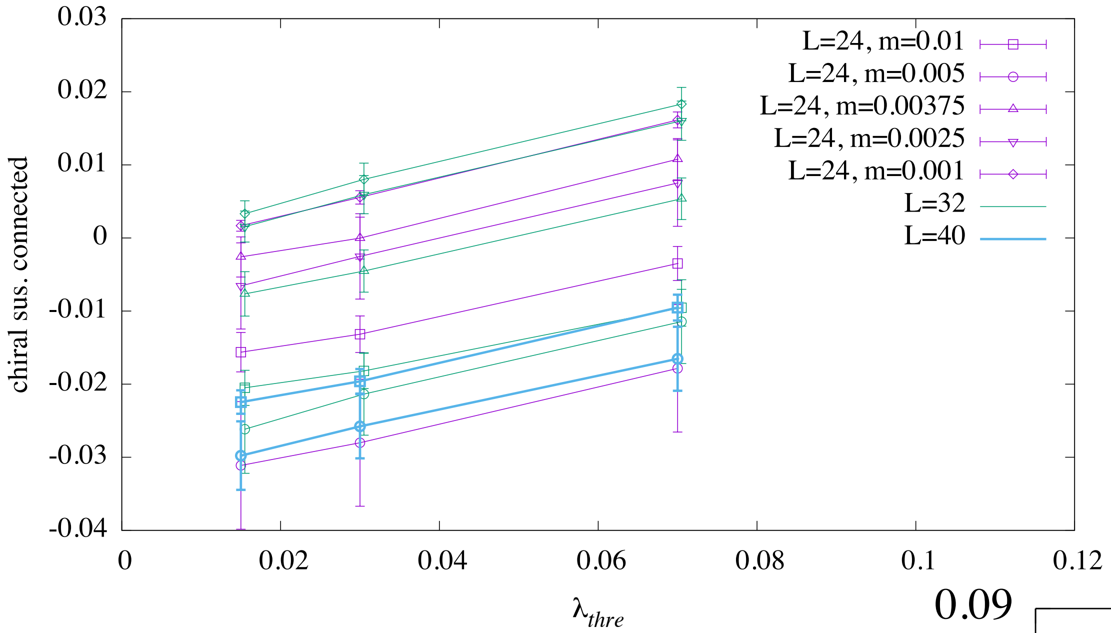
$$\Delta^{lat}(m) = \frac{1}{V(1-m^2)^2}\left\langle \sum_{\mathrm{all}\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

$$-\langle\bar{q}q\rangle^{lat} = \frac{1}{V(1-m^2)}\left\langle \sum_{\mathrm{all}\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right\rangle.$$

$$\chi^{dis.lat}(m) = \frac{N_f}{V}\left[\frac{1}{(1-m^2)^2}\left\langle \left(\sum_{\mathrm{all}\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2}\right)^2 \right\rangle - |\langle\bar{q}q\rangle^{lat}|^2 V^2\right].$$

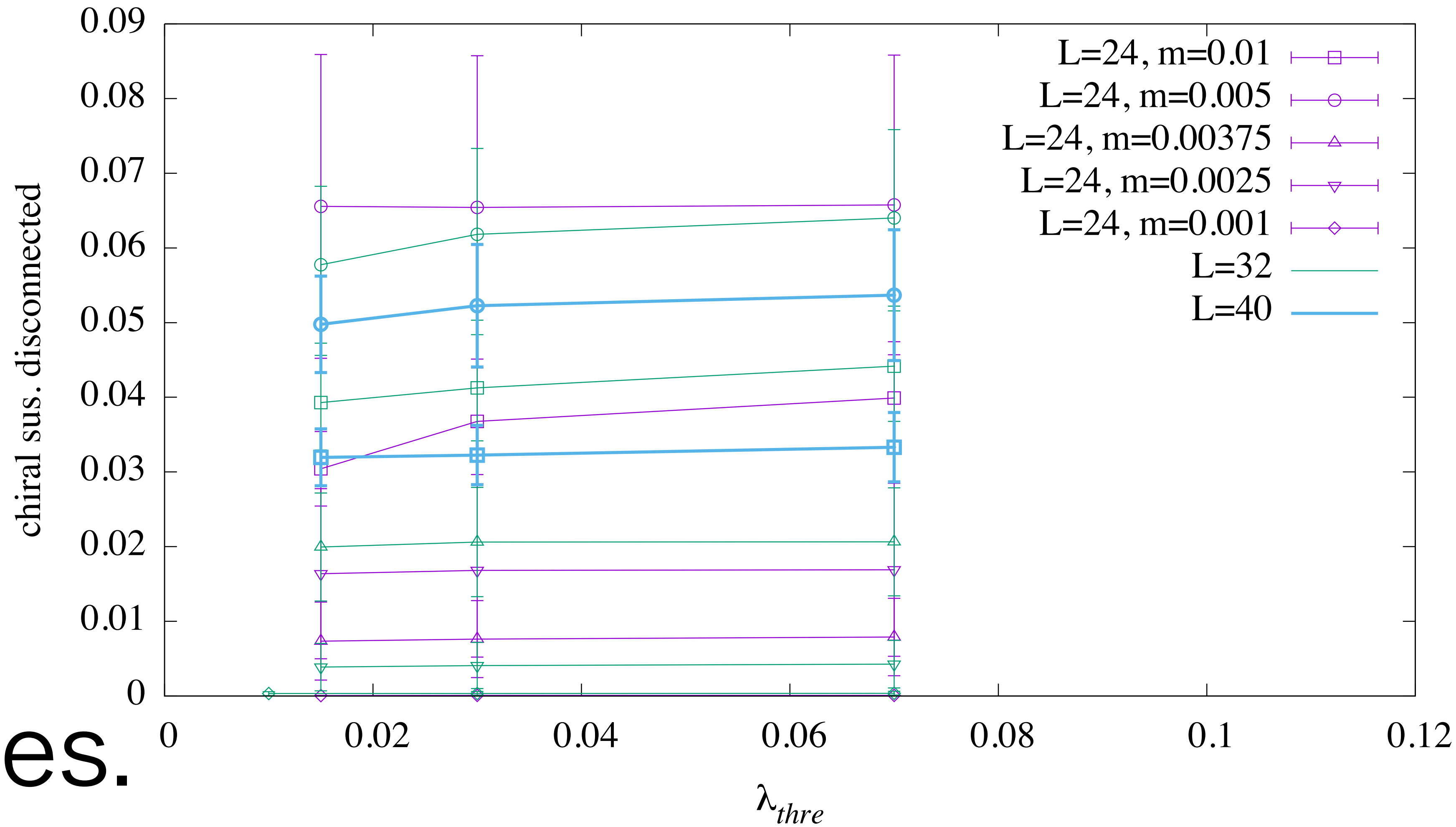
where $\lambda_m = \text{eigenvalues of } H_m = \gamma_5[(1-m)D_{ov} + m]$

Low mode approximation



connected

disconnected part is well described by 40 lowest modes.



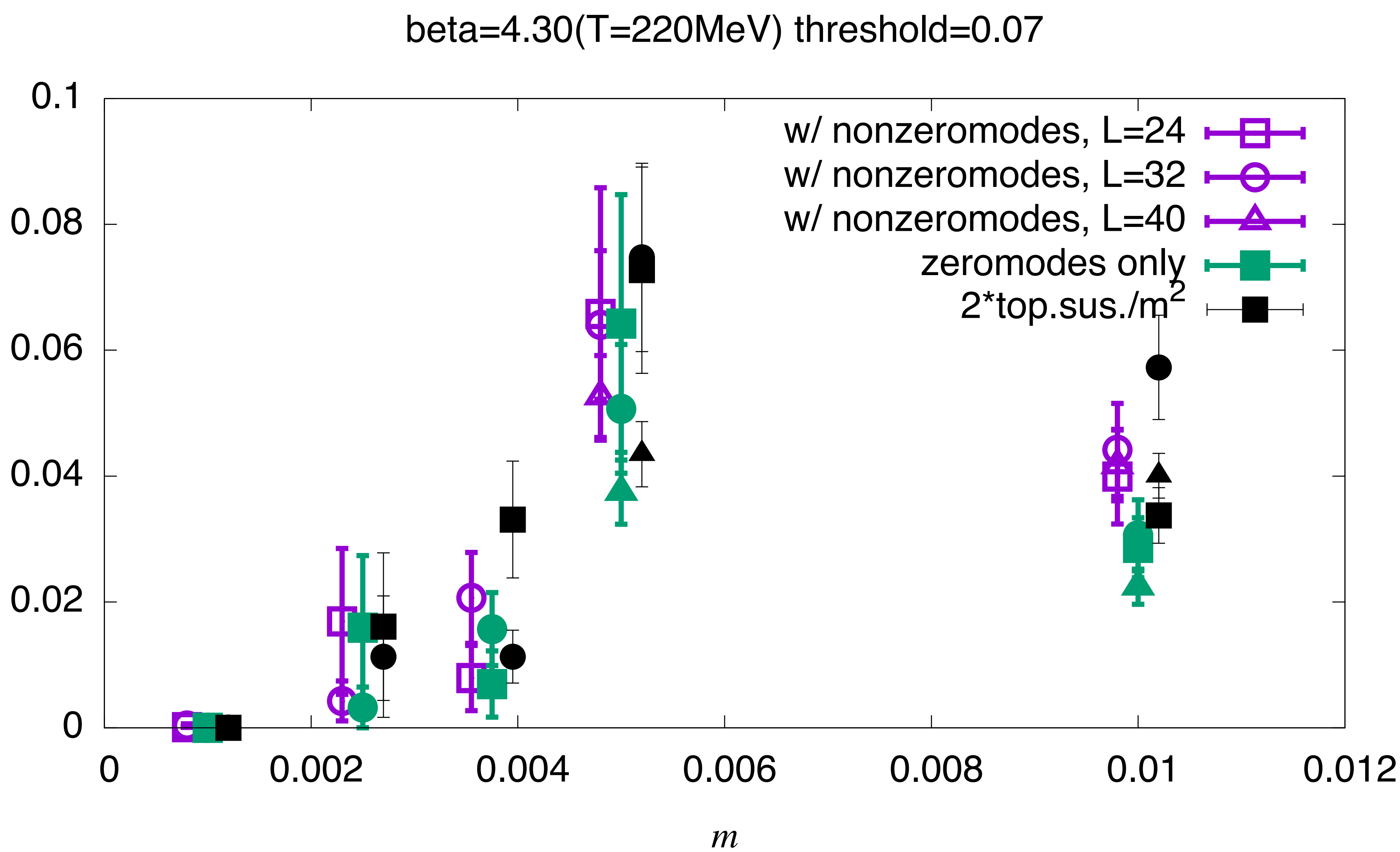
Disconnected part

$$\chi^{dis.}(m) = \frac{N_f}{V}\left[\left\langle\left(\sum_\lambda \frac{m}{\lambda(A)^2+m^2}\right)^2\right\rangle - \left\langle\sum_\lambda \frac{m}{\lambda(A)^2+m^2}\right\rangle^2\right].$$

$$\sim \frac{N_f}{V}\frac{\langle N_0^2\rangle - \langle N_0\rangle^2}{m^2} \quad \sim \frac{N_f}{V}\frac{\langle Q^2\rangle - \langle Q\rangle^2}{m^2}$$

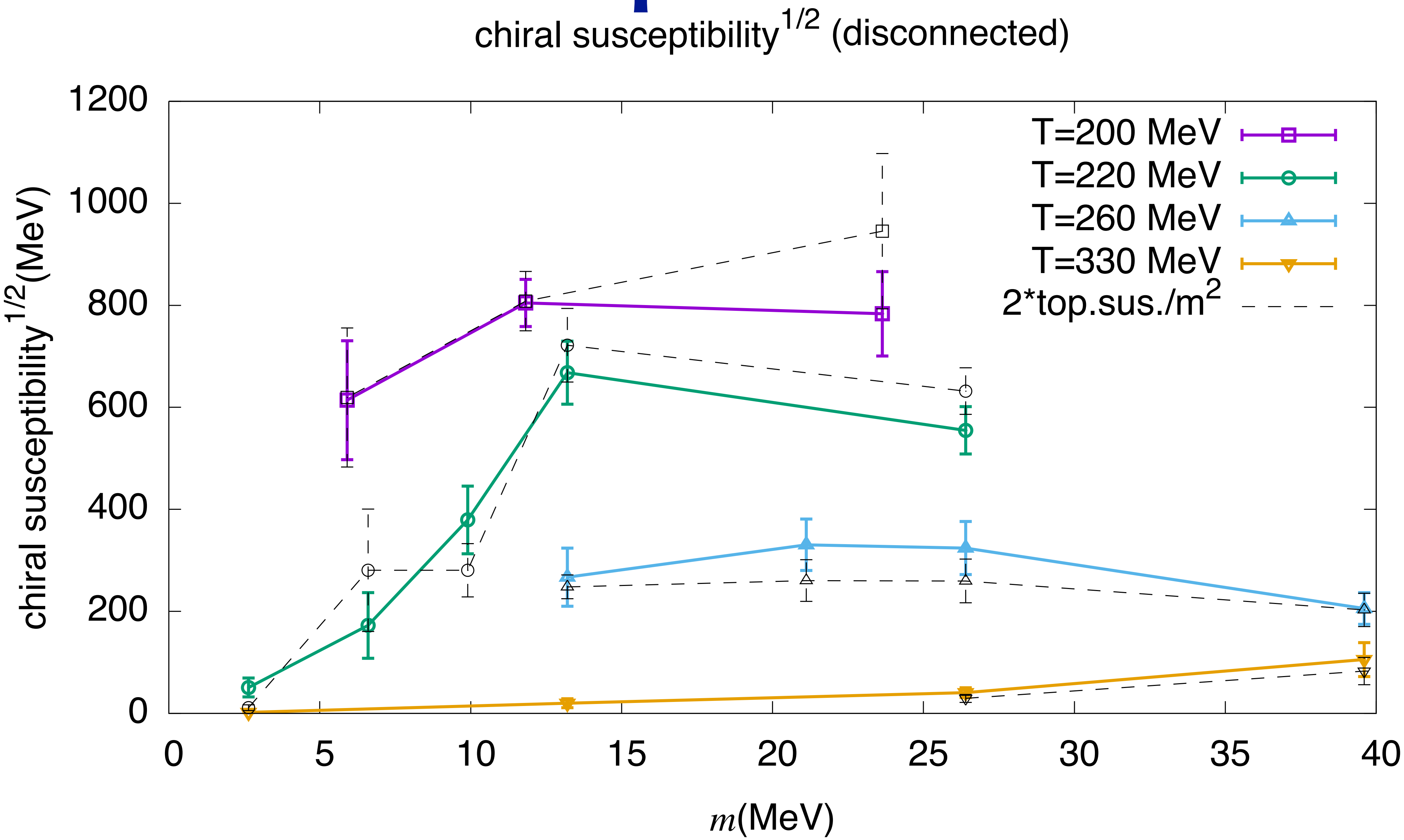
N_0 : number of zero modes

Q: topological charge



dominated by topological susceptibility. (vol. dependence is small.)

Disconnected part at different T



The topological susceptibility (or U(1) anomaly) dominance is seen at 4 different temperatures.

Contents

✔ 1. Introduction

We study Nf=2 QCD with chiral fermions at ~mphys, focusing on U(1) anomaly.

✔ 2. Lattice setup

Nf=2 QCD w/ MDWF and rewegihting overlap. at T=190-330MeV near physical m~4MeV.

✔ 3. Numerical results

- Dirac spectrum has a peak but vanishes in the m→0 limit.
- Topology fluctuation is suppressed by $\sim m^4$.
- U(1) susceptibility goes down to (a few MeV)^2.
- ~~Meson~~ 2quark correlators show a good U(1)A symmetry.
- Chiral susceptibility is dominated by axial U(1) anomaly.

4. Summary

Summary

We study Nf=2 QCD with Mobius domain-wall and rewegihting overlap fermions at T=190-330MeV near physical m~4MeV. We observe

disappearance of axial U(1) anomaly (~ a few MeV)

- Dirac spectrum has a peak but vanishes in the $m \to 0$ limit.
- Topology fluctuation is suppressed by $\sim m^4$.
- U(1) susceptibility goes down to (a few MeV)^2.
- ~~Meson~~ 2quark correlators show a good U(1)A symmetry (m_{screen} difference ~ a few MeV).
- Chiral susceptibility is dominated by axial U(1) anomaly (anomaly controls the phase transition?).

Outlook

Enhancement of symmetry to SU(4)?

Polyakov loop

Comparison with pQCD + instantons

Axion dark matter

Nf=2+1 QCD started!

beta=4.17, L=32, Lt=12, T=204MeV

m=0.002(almost physical),0.0035,0.007,0.012

ms =0.04(almost physical)