Lattice QCD Approach to HVP and Muon g-2

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RIKEN Seminar
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Budapest-Marseille-Wuppertal (BMW) Collab. Refs:

- Phys. Rev. Lett. **121**, no. 2, 022002 (2018).
- Phys. Rev. D **96**, no. 7, 074507 (2017).
- With some updates and preliminary results.

Muon Anomalous Magnetic Moment $a_{\ell=e,\mu,\tau}$

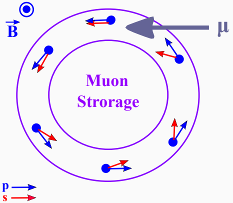
- **Dirac Eq. with B:**

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\boldsymbol{\alpha}\cdot\left(-i\hbar c\nabla - e\mathbf{A}\right) + \beta c^2 m_\ell + eA_0\right]\psi \ ,$$

- **Nonlelativistic Limit, Pauli Eq.:**

$$i\hbar\frac{\partial\phi}{\partial t} = \left[\frac{(-i\hbar c\nabla - e\mathbf{A})^2}{2m_\ell c} - \mathbf{M}_\ell\cdot\mathbf{B} + eA_0\right]\phi \ ,$$

- **Magnetic Moment:** $\mathbf{M}_\ell = g_\ell \frac{e}{2m_\ell c}\frac{\hbar\boldsymbol{\sigma}}{2}$,
- **In Dirac Theory:**
  $g_\ell = 2\ , \quad a_\ell \equiv (g_\ell - 2)/2 = 0\ , \quad \omega_{\rm cyc} = \omega_{\rm prec}$.
- **In QFT (with Loops) for Electron (M.Knecht ,NPPP2015):**
  $a_e^{\rm SM} = 1\ 159\ 652\ 180.07(6)(4)(77)\times 10^{-12} \quad (\mathcal{O}(\alpha^5))\ ,$
  $a_e^{\rm exp} = 1\ 159\ 652\ 180.73(0.28)\times 10^{-12} \quad [0.24 ppb]\ .$



$a_\mu^{exp.} = a_\mu^{\rm SM}?$

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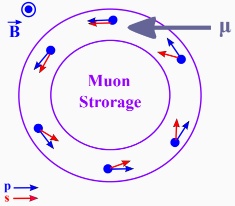


$$a_\mu^{exp.} = a_\mu^{\rm SM}?$$

$a_\mu^{exp.}$ vs. a_μ^{SM}

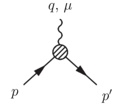
SM contribution	$a_\mu^{\text{contrib.}} \times 10^{10}$	Ref.
QED [5 loops]	11658471.8951 ± 0.0080	[Aoyama et al '12]
HVP-LO (pheno.)	692.6 ± 3.3	[Davier et al '16]
	694.9 ± 4.3	[Hagiwara et al '11]
	681.5 ± 4.2	[Benayoun et al '16]
	688.8 ± 3.4	[Jegerlehner '17]
HVP-NLO (pheno.)	-9.84 ± 0.07	[Hagiwara et al '11]
		[Kurz et al '11]
HVP-NNLO	1.24 ± 0.01	[Kurz et al '11]
HLbyL	10.5 ± 2.6	[Prades et al '09]
Weak (2 loops)	15.36 ± 0.10	[Gnendiger et al '13]
SM tot [0.42 ppm]	11659180.2 ± 4.9	[Davier et al '11]
[0.43 ppm]	11659182.8 ± 5.0	[Hagiwara et al '11]
[0.51 ppm]	11659184.0 ± 5.9	[Aoyama et al '12]
Exp [0.54 ppm]	11659208.9 ± 6.3	[Bennett et al '06]
Exp − SM	28.7 ± 8.0	[Davier et al '11]
	26.1 ± 7.8	[Hagiwara et al '11]
	24.9 ± 8.7	[Aoyama et al '12]

$a_\mu^{\text{LO-HVP}}|_{NoNewPhys} \times 10^{10} \simeq 720 \pm 7$,

FNAL E989: 0.14-ppm (first data 0.5-ppm: 2019-Dec.?)), J-PARC E34: 0.1-ppm

a_ℓ in QFT

- **QFT Def. for a_ℓ:**

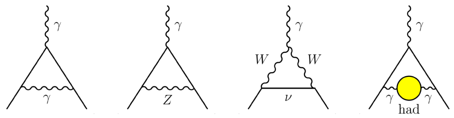


$$= \langle \bar{\ell}^-(p)|\mathcal{J}^\mu|\ell^-(p')\rangle = \bar{u}(p)\Gamma^\mu(p,p')u(p') \quad (1)$$

$$\Gamma^\mu(q=p-p') = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\mu}F_2(q^2) + \cdots\ , \quad (2)$$

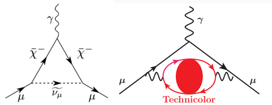
$$F_2(0) = a_\ell = (g_\ell - 2)/2\ . \quad (3)$$

- **Standard Model, Loop Corr.:**



$$a_\ell = \alpha/(2\pi) + \cdots\ .$$

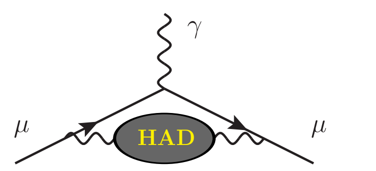
- **BSM = MSSM (Padley et.al.'15) or TC (Kurachi et.al. '13) etc.:**



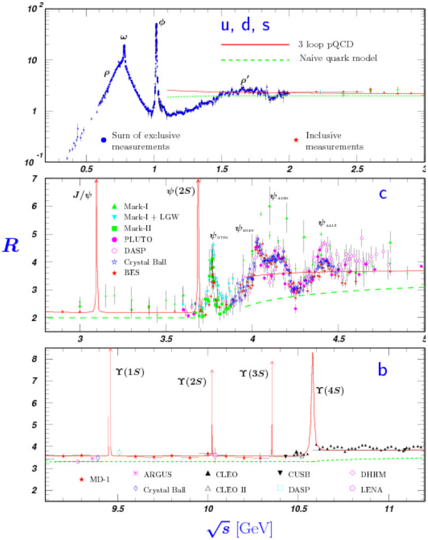
$$\propto (m_\ell/\Lambda_{BSM})^2.$$

Really $a_\mu^{exp.} \neq a_\mu^{SM}$?

The Hadronic Vacuum Polarization (HVP) contributions to a_μ is a bottle-neck to answer for this question.

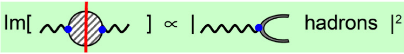


Phenomenology of HVP



(PDG compilation)

Use (Bouchiat et al 61) optical theorem (unitarity)



$$\mathrm{Im}\Pi(s) = -\frac{R(s)}{12\pi}, \qquad R(s) \equiv \frac{\sigma(e^+e^- \to \text{had})}{4\pi\alpha(s)^2/(3s)}$$

and a once subtracted dispersion relation (analyticity)

$$\hat{\Pi}(Q^2) = \int_0^\infty ds\, \frac{Q^2}{s(s+Q^2)} \frac{1}{\pi} \mathrm{Im}\Pi(s) = \frac{Q^2}{12\pi^2} \int_0^\infty ds\, \frac{1}{s(s+Q^2)} R(s)$$

$\Rightarrow \hat{\Pi}(Q^2)$ from data: sum of exclusive $\pi^+\pi^-$ etc. channels from CMD-2&3, SND, BES, KLOE '08,'10&'12, BABAR '09, etc.

Can also use $I(J^{PC}) = 1(1^{--})$ part of $\tau \to \nu_\tau + \text{had}$ and isospin symmetry + corrections

Pion Contributions to a_μ from Experimental Data

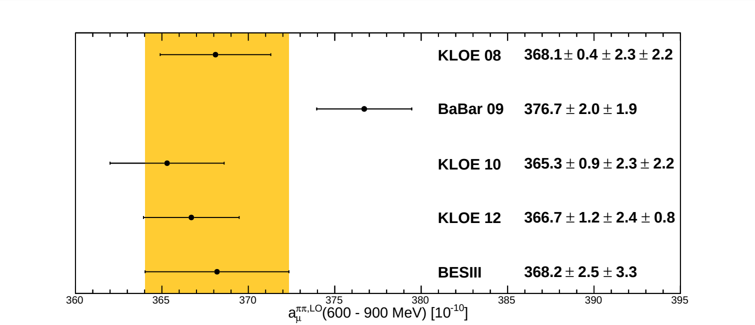


Figure: Borrowed by BESIII, PLB'16: Some tension among experiments on pion contributions to a_μ.

THIS TALK

Lattice QCD for Muon $g-2$

- First Principle Crosschecks of the dispersive results.
- First Principle Predictions for assessing SM with measurements by FermiLab/J-PARC experiments (0.1-ppm).

THIS TALK:

- Report BMW-Collab. results for muon $g-2$.
- Compare/Discuss various results from lattice QCD as well as dispersive method.

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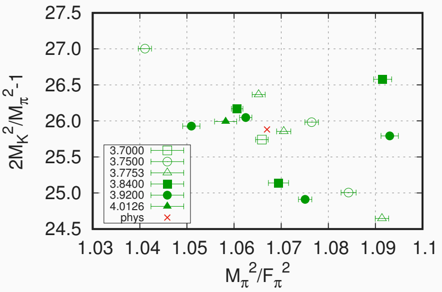
Simulation Setup (BMWc. PRD-2017 and PRL-2018)

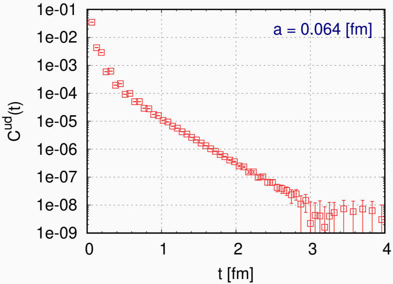
BMW Ensemble PRD2017 and PRL2018

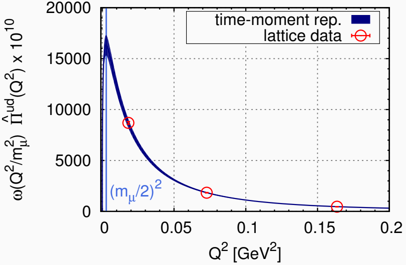
- 6-β, 15 simulation with all physical masses.
- Nf=(2+1+1) staggered quarks.
- Large Volume: $(L, T) \sim (6, 9-12) fm$.
- AMA with 6000-9000 random-source meas. for disconnected.



β	a[fm]	N_t	N_s	#traj.	M_π[MeV]	M_K[MeV]	#SRC (l,s,c,d)
3.7000	0.134	64	48	10000	~ 131	~ 479	$(768, 64, 64, 9000)$
3.7500	0.118	96	56	15000	~ 132	~ 483	$(768, 64, 64, 6000)$
3.7753	0.111	84	56	15000	~ 133	~ 483	$(768, 64, 64, 6144)$
3.8400	0.095	96	64	25000	~ 133	~ 488	$(768, 64, 64, 3600)$
3.9200	0.078	128	80	35000	~ 133	~ 488	$(768, 64, 64, 6144)$
4.0126	0.064	144	96	04500	~ 133	~ 490	$(768, 64, 64, -)$

Observables and Objectives





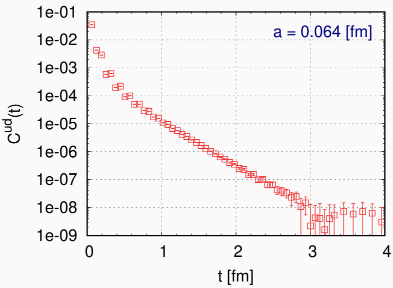
$$\Pi_{\mu\nu}(Q) = (Q_\mu Q_\nu - \delta_{\mu\nu}Q^2)\Pi(Q^2) = \int d^4x\, e^{iQx}\langle j_\mu(x) j_\nu(0)\rangle\ , \tag{4}$$

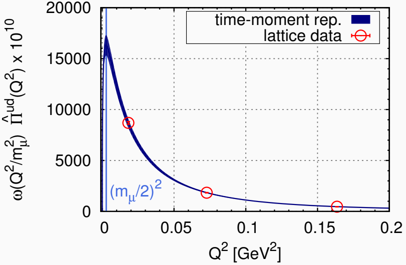
$$j_\mu = (2/3)\bar{u}\gamma_\mu u - (1/3)\bar{d}\gamma_\mu d - (1/3)\bar{s}\gamma_\mu s + (2/3)\bar{c}\gamma_\mu c + \cdots\ , \tag{5}$$

$$\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0) = \sum_t t^2\left[1 - \left(\frac{\sin[Qt/2]}{Qt/2}\right)^2\right]\frac{1}{3}\sum_{i=1}^{3}\langle j_i(t) j_i(0)\rangle\ . \tag{6}$$

$$a_{\ell=e,\mu,\tau}^{\text{LO-HVP}} = \frac{\alpha^2}{\pi^2}\int_0^\infty dQ^2\ \omega\left(\frac{Q^2}{m_{\ell=e,\mu,\tau}^2}\right)\hat{\Pi}(Q^2)\ . \tag{7}$$

Observables and Objectives





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$$a^{\text{LO-HVP}}_{\ell=e,\mu,\tau} = \frac{\alpha^2}{\pi^2} \int_0^\infty dQ^2 \;\omega\!\left(\frac{Q^2}{m^2_{\ell=e,\mu,\tau}}\right) \hat{\Pi}(Q^2) \,. \tag{7}$$

Bounding [BMW PRD2017 and PRL2018]

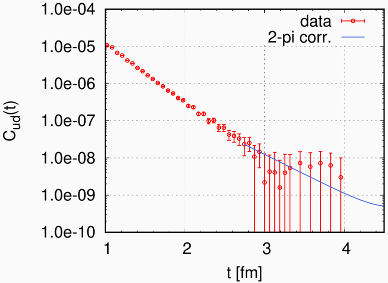


Figure shows

$$C^{ud}(t) = \frac{5}{9}\sum_{\vec{x}}\frac{1}{3}\sum_{i=1}^{3}\langle j_i^{ud}(\vec{x},t)j_i^{ud}(0)\rangle ,$$

by BMW Ensemble with $a = 0.064$ [fm] used in PRD2017/PRL2018.

- The connected-light correlator $C^{ud}(t)$ loses signal for $t > 3fm$. To control statistical error, consider $C^{ud}(t > t_c) \to C^{ud}_{\rm up/low}(t, t_c)$, where

  $$C^{ud}_{\rm up}(t, t_c) = C^{ud}(t_c)\ \varphi(t)/\varphi(t_c),$$
  $$C^{ud}_{\rm low}(t, t_c) = 0.0,$$

  with $\varphi(t) = \cosh[E_{2\pi}(T/2 - t)]$, and $E_{2\pi} = 2(M_\pi^2 + (2\pi/L)^2)^{1/2}$.

- Similarly, $C^{disc}(t) \to C^{disc}_{\rm up/low}(t, t_c)$,

  $$-C^{disc}_{\rm up}(t > t_c) = 0.1\,C^{ud}(t_c)\ \varphi(t)/\varphi(t_c),$$
  $$-C^{disc}_{\rm low}(t > t_c) = 0.0.$$

- By construction, $C^{ud,disc}_{low}(t, t_c) \le C^{ud,disc}(t) \le C^{ud,disc}_{up}(t, t_c)$.

Bounding [BMW PRL2018]

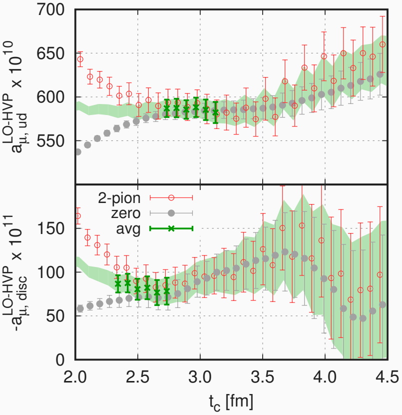
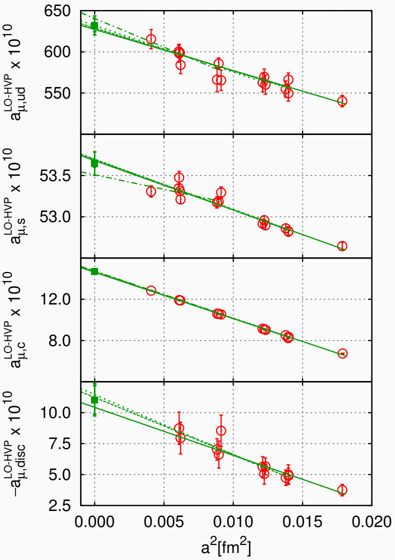


Figure: BMW, PRL2018.

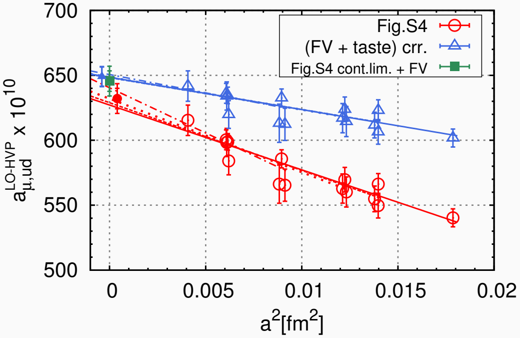
- Corresponding to $C^{ud,disc}_{up/low}(t_c)$, we obtain upper/lower bounds for muon g-2: $a^{ud,disc}_{\mu,up/low}(t_c)$.
- Two bounds meet around $t_c = 3fm$. Consider the average of bounds: $\bar{a}^{ud,disc}_{\mu}(t_c) = 0.5(a^{ud,disc}_{\mu,up} + a^{ud,disc}_{\mu,low})(t_c)$, which is stable around $t_c = 3fm$.
- We pick up such averages $\bar{a}^{ud,disc}_{\mu}(t_c)$ with $4-6$ kinds of t_c around $3fm$. The average of average is adopted as $a^{\text{LO-HVP}}_{\mu,\text{ud/disc}}$ to be analysed, and a fluctuation over selected t_c gives systematic error.
- A similar method is proposed by C.Lehner in Lattice2016 and used in RBC/UKQCD-PRL2018. Improved bounding method with GEVP: [A. Meyer/C. Lehner, 27 Fri Hadron Structure].

Controlled Continuum Extrap. [BMW PRL2018]



- With 6 $\beta' s = 15$ $a^2[fm^2]$ simulations, allowing full control over continuum limit.
- Get systematic uncertainty from various cuttings: no-cut, or cutting $a \geq 0.134,\ 0.111,$ or 0.095.
- Get good χ^2/dof with extrapolation linear in a^2, and interpolation linear in M_K^2 (strange) or M_π^2 and $M_{\eta c}$ (charm).
- Strong a^2 dependences for $a^{\text{LO-HVP}}_{\mu,ud/disc}$ due to taste violations, and for $a^{\text{LO-HVP}}_{\mu,c}$ due to large m_c.

Crosscheck of Continuum Extrapolation [BMW PRL2018]



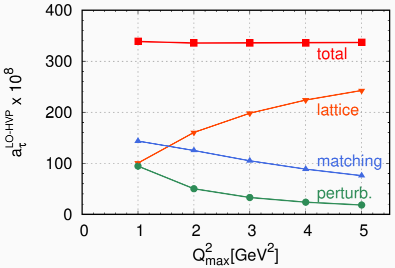
1. Red open-circles are raw lattice data and continuum-extrapolated (red filled-circle). Then finite-volume correction using XPT is added to get the green-square point.
2. Similarly to HPQCD-PRD2017, raw data (red-circles) are first corrected with finite-volume and taste-partner effects to get blue open-triangles, which are continuum-extrapolated to get blue filled-triangle.

Various Corrections

- **High Q^2 Control:**
  The lattice data have enough overlap to perturbative regime even in tau case.
  $a_{\ell,f}^{\text{LO-HVP}} = a_{\ell,f}^{\text{LO-HVP}}(Q \leq Q_{max}) + (\gamma_\ell \hat{\Pi}^f)(Q_{max}) + \Delta^{pert} a_{\ell,f}^{\text{LO-HVP}}(Q > Q_{max})$.

- **Isospin/QED Collections:**
  Model estimates amounts to 1.1% corrections (table thanks to F.Jegerlehner (& M. Benayoun)).

- **FV Collections:**
  The dominant FV in $I = 1$, $\pi^+\pi^-$ loop channel is estimated by XPT (Aubin et al '16): $(a_{\mu,I=1}^{\text{LO-HVP}}(\infty) - a_{\mu,I=1}^{\text{LO-HVP}}(6fm))|_{\text{XPT}} = 13.42(13.42) \times 10^{-10}$, (1.9%) .



Effect	$\delta a_\mu^{\text{LO-HVP}} \times 10^{10}$
$\rho-\omega$ mix.	2.71 ± 1.36
FSR	4.22 ± 2.11
$M_\pi \to M_{\pi\pm}$	-4.47 ± 4.47
$\pi^0\gamma$	4.64 ± 0.04
$\eta\gamma$	0.65 ± 0.01
Total	$\mathbf{7.8 \pm 5.1}$

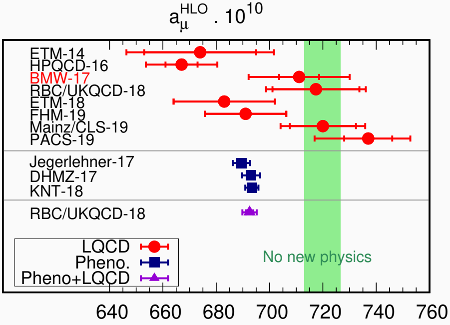
Summary on $a_\mu^{\text{LO-HVP}}$ PRL2018

$a_\mu^{\text{LO-HVP}}$ BMWc

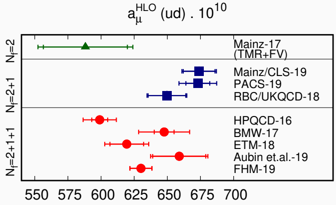
$I = 1$	$582.9(6.7)_{st}(7.2)_{acut}(0.1)_{tcut}(0.0)_{qcut}(4.5)_{da}(13.5)_{fv}$
$I = 0$	$120.5(3.4)_{st}(3.5)_{acut}(0.2)_{tcut}(0.0)_{qcut}(1.0)_{da}$
total	$711.1(7.5)_{st}(8.0)_{acut}(0.2)_{tcut}(0.0)_{qcut}(5.5)_{da}(13.5)_{fv}(5.1)_{iso}$

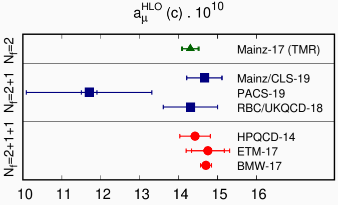
Remarks

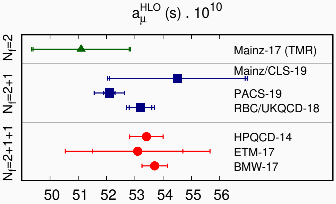
- Our Lattice QCD results are consistent with both "No New Physics" and Dispersive Method.
- Total error in our LQCD result is 2.6%, dominated FV effects.

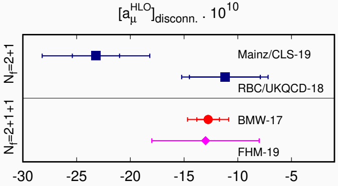


$a_\mu^{\text{LO-HVP}}$: flavor by flavor comparison









- The results do not yet converge in all flavors...
- "Disagreement" is particularly on $a_{\mu,\,ud}^{\text{LO-HVP}}$

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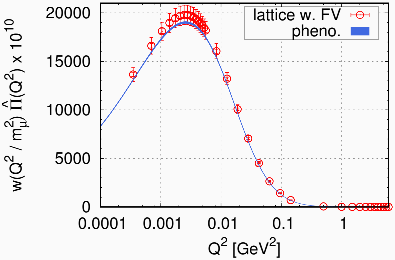
$\hat{\Pi}^{lat}(Q^2)$ vs $\hat{\Pi}^{pheno}(Q^2)$ for Various Q^2 Preliminary



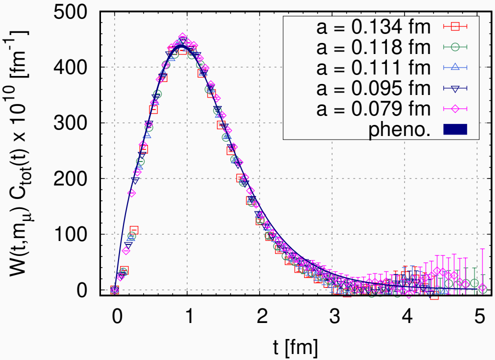
$$\hat{\Pi}^{lat}(\omega^2) = \lim_{a\to 0} \sum_{t=0}^{T/2} t^2 \left[1 - \mathrm{sinc}^2[\omega t/2]\right] C(t) \,,$$

$$\hat{\Pi}^{pheno}(Q^2) = \frac{Q^2}{12\pi^2} \int_0^\infty ds \frac{R_{had}(s)}{s(s+Q^2)} \,.$$

Lat (BMWc) vs Pheno (alphaQEDc17 by Jegerlehner) for $\omega(Q^2/m_\mu^2)\hat{\Pi}(Q^2)$

- The contributions at $Q^2 \sim (m_\mu/2)^2$ are dominant, and the lattice and phemenology are consistent within the error-bars there.
- However, the lattice error gets larger at $Q^2 \sim (m_\mu/2)^2$. More precise estimates are demanded and in progress.

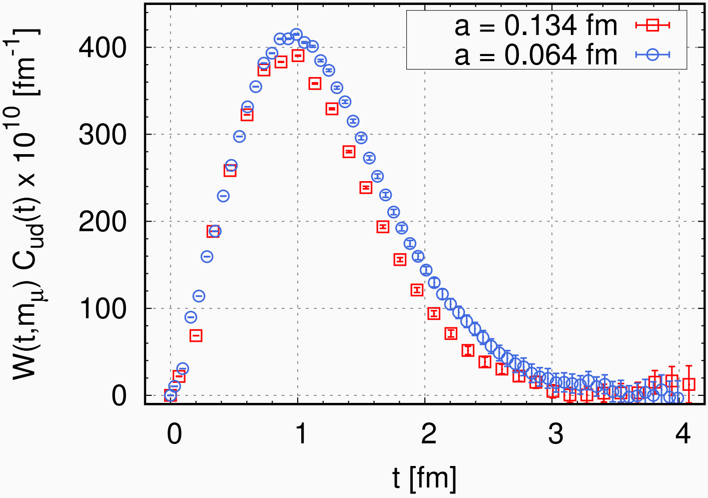
Integrand of $a_{\mu,ud}^{\text{LO-HVP}}$ I



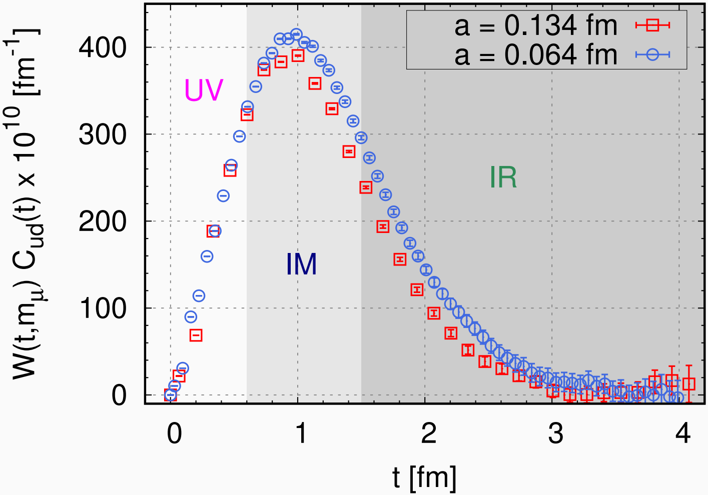
$$a_{\mu,ud}^{\text{LO-HVP}} = \sum_t W(t, m_\mu) C_{tot}(t) \,, \tag{8}$$

$$c.f. \; C_{tot}^{pheno}(t) = \int_0^\infty ds \sqrt{s} R_{had}(s) e^{-\sqrt{s}|t|} \,. \tag{9}$$

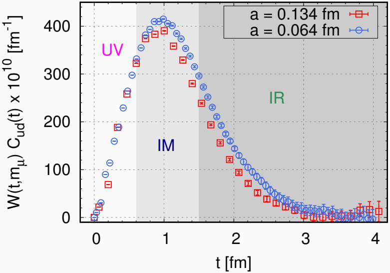
Integrand of $a_{\mu,ud}^{\text{LO-HVP}}$ II

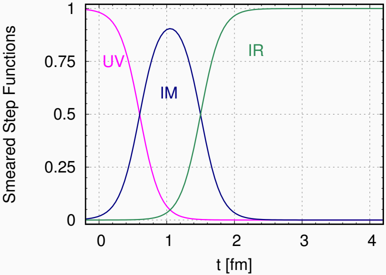


Integrand of $a_{\mu,ud}^{\text{LO-HVP}}$ III



Window Method





$$\text{UV: } S_{UV}(t) = 1.0 - (1.0 + \tanh[(t - t_0)/\Delta])/2\,, \tag{10}$$

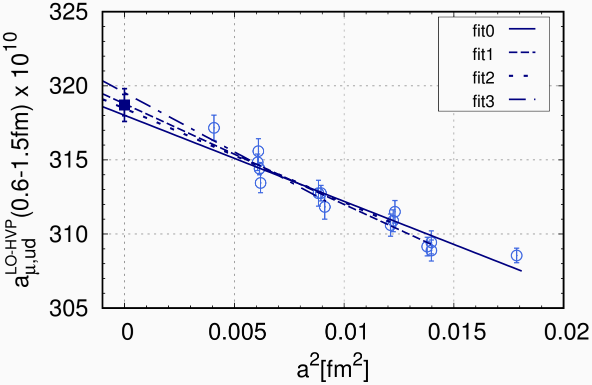
$$\text{IM: } S_{IM}(t) = \frac{1}{2}\left(\tanh[(t - t_0)/\Delta] - \tanh[(t - t_1)/\Delta]\right)\,, \tag{11}$$

$$\text{IR: } S_{IR}(t) = (1.0 + \tanh[(t - t_1)/\Delta])/2\,, \tag{12}$$

$$\text{We shall adopt } t_0 = 0.6fm\,, \quad t_1 = 1.5fm\,, \quad \Delta = 0.3fm\,. \tag{13}$$

c.f. RBC-UKQCD (PRL2018), Aubin et.al. (1905.09307)

Continuum Extrapolation in Dominant Window Preliminary



For the most important window (0.6 – 1.5 fm), the lattice QCD provides very precise data with per-mil level precision.

Table of Contents

Summary and Perspective

- We have obtained $a_\mu^{\text{LO-HVP}}$ directly at physical point masses: $a_\mu^{\text{LO-HVP}} = 711.1(7.5)(17.4) \times 10^{-10}$.
- Full controlled continuum extrapolation and matching to perturbation theory. Model assumptions are put on only for small corrections from FV/QED/isospin breaking. Total error is 2.6%, dominated by FV.
- Our Lattice QCD results are consistent with "No New Physics" as well as Phenomenological Dispersive Methods with a conservative systematic errors.
- Lat-Pheno. comparisons are made for HVP: consistent at small Q^2, but lattice tends to be larger, leading to larger $a_{\mu,lat}^{\text{LO-HVP}}$.
- Need $\sim$ 0.2% precision to match Fermilab/J-PARC experiments!!
  1. lat-pheno combined analyses: window method (on going, per-mil level precision at present statistics).
  2. QED/SIB based on lattice QCD (on going, correction to Dashen's theorem as an exercise).
  3. control FV effects directly based on the first-principle.