Finding successful strategies for social dilemma using K computer

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iterated Prisoner's Dilemma

player B

player A

	cooperation	defection
cooperation	(3,3)	(0,5)
defection	(5,0)	(1,1)

implementation error occurs with probability e

long-term payoff

$$f_i \equiv \lim_{e \to 0} \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} F_i^{(t)}$$

Strategies for IPD game

Tit-For-Tat (TFT)

It is guaranteed that your payoff is no less than the co-players'.

Cooperation is fragile against an error.

c c d c d c

c c c d c d

Win-Stay-Lose-Shift (WSLS)

Cooperation is tolerant against an error.

Repeatedly exploited against AllD.

c d c d

d d d d

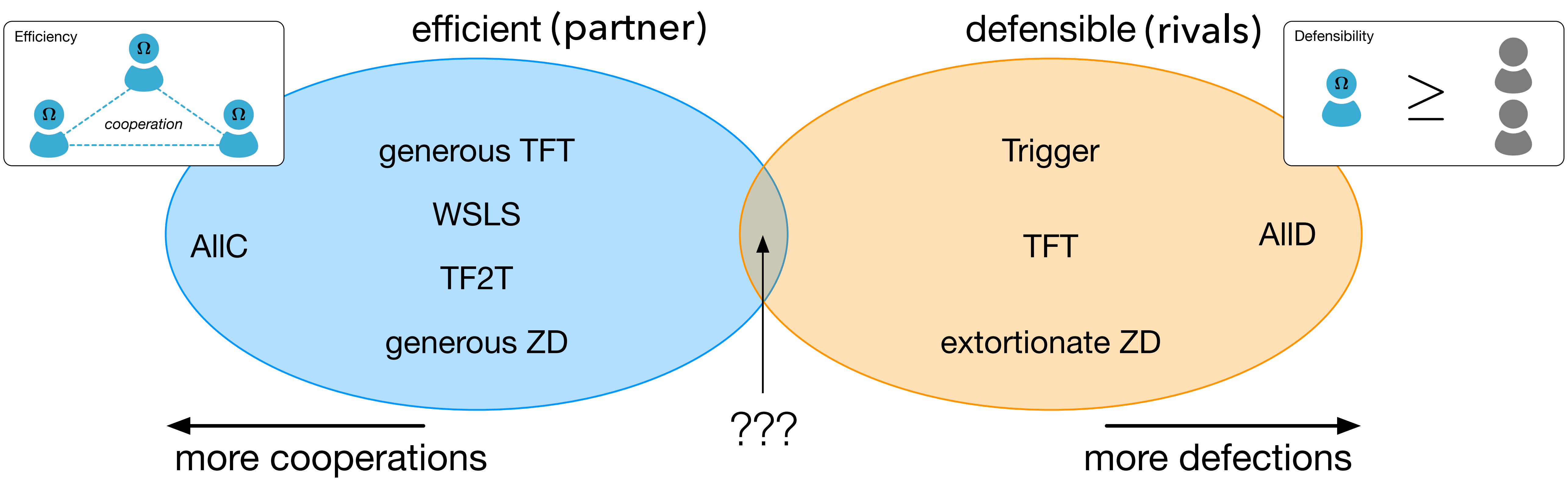
Other strategies (**TF2T, ZD strategies, generous TFT…**) have positives and negatives.

partners or rivals?


nature human behaviour
REVIEW ARTICLE
https://doi.org/10.1038/s41562-018-0320-9
Corrected: Publisher Correction

Partners and rivals in direct reciprocity

Christian Hilbe[1,2]*, Krishnendu Chatterjee[2] and Martin A. Nowak[1,3]



It would be great if a **single strategy** satisfies these advantages simultaneously.
=> **Nash equilibrium** with a guarantee that you'll never lose.

A solution for Prisoner's Dilemma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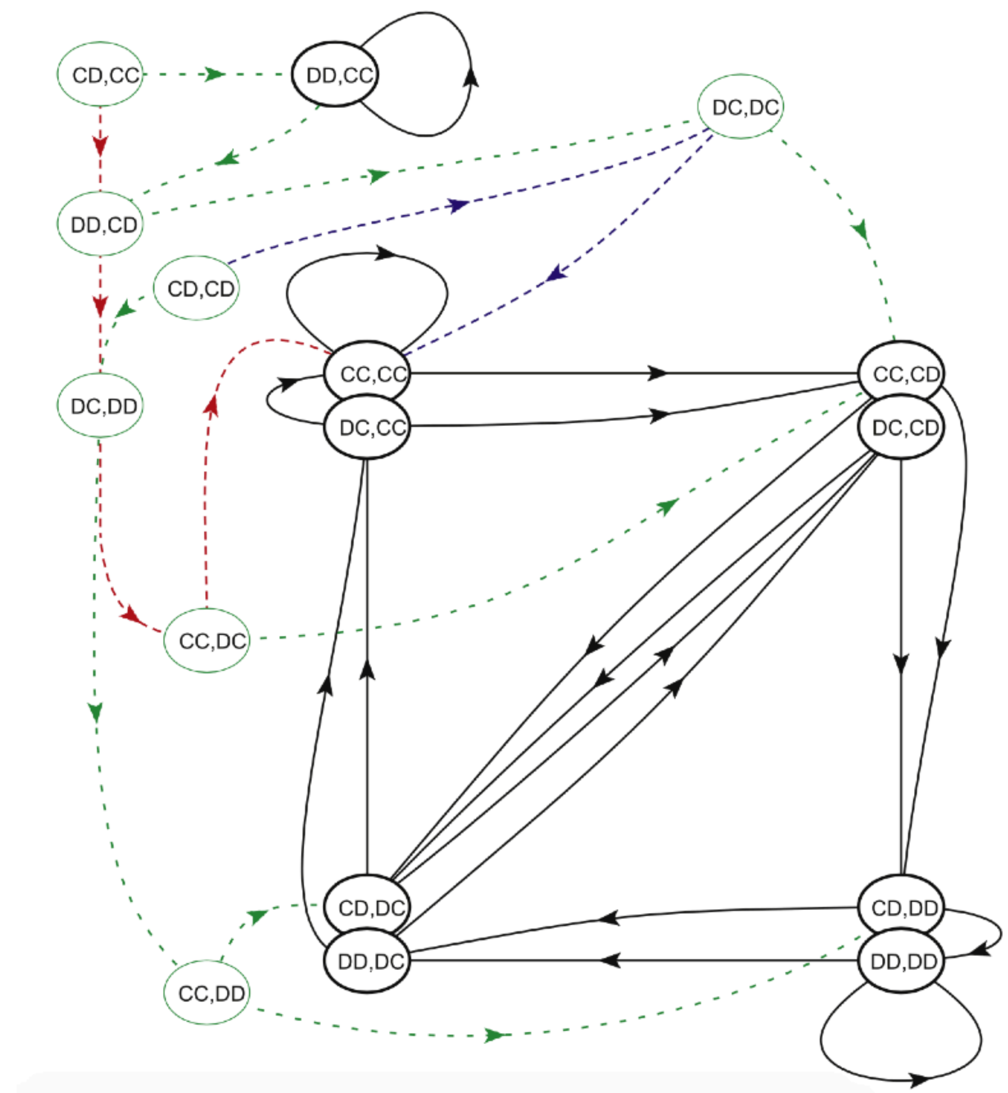
Combination with anti-tit-for-tat remedies problems of tit-for-tat

Su Do Yi[a], Seung Ki Baek[b,*], Jung-Kyoo Choi[c,*]

CrossMark

Yi et al., J. Theor. Biol. (2017)



comprehensive search in Memory-2 strategies

4 out of 65,536 strategies satisfies the conditions

TFT-ATFT

It is indeed possible to realize cooperation
without exposing themselves to the risk of being exploited.

Murase & Baek, J.Theor.Biol. (2018)

What about n-person public-goods game?

n=3

the number of defecting co-players

$$M \equiv \left(\begin{array}{c|ccc} & 0 & 1 & 2 \\ c & \rho & \frac{2}{3}\rho & \frac{1}{3}\rho \\ d & 1+\frac{2}{3}\rho & 1+\frac{1}{3}\rho & 1 \end{array} \right)$$

TFT-ATFT does not work for n=3.
Does a solution exist for n=3?

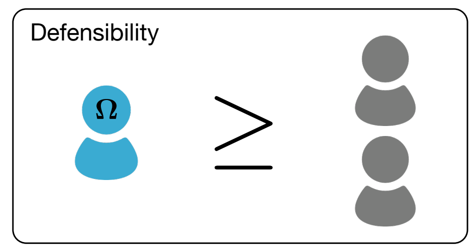
Enumeration using K-computer

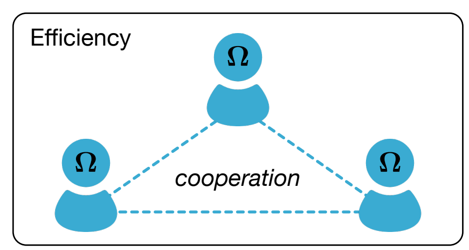
$$2^{2^{nm}}$$

n=3, m=1 : 256
n=3, m=2 : 1,099,511,627,776 (2^40)

Enumeration of strategies

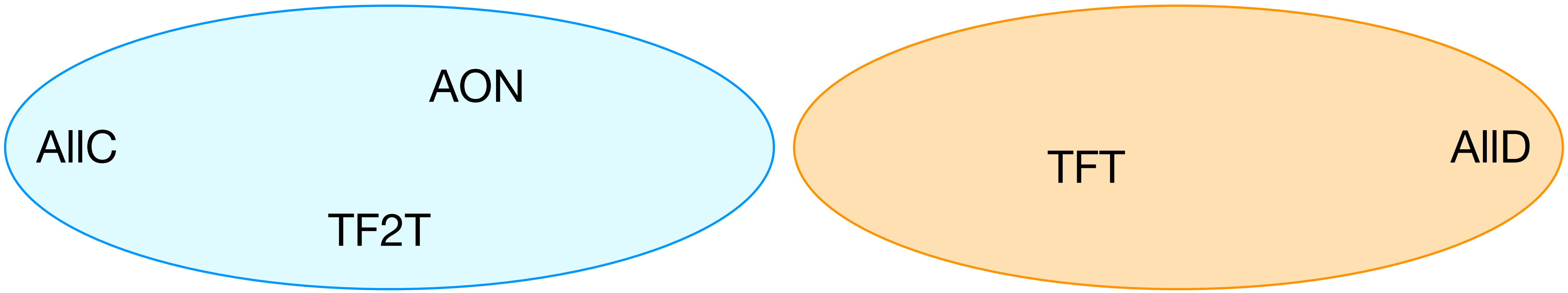
# of m=2 strategies	1,099,511,627,776
Defensibility against AllD	805,306,368
Defensibility	3,483,008
Efficiency	**0**





Impossibility for m=2

memory-***2*** strategies



no successful strategy exists in memory-2 strategy space.

0 / 1,099,511,627,776

No solution for n=3 game? or The solution exists when m>2?

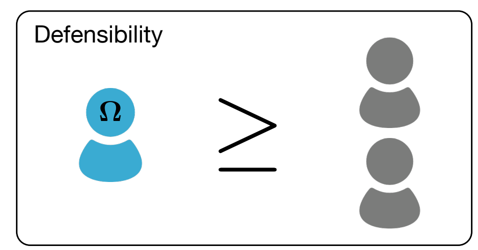
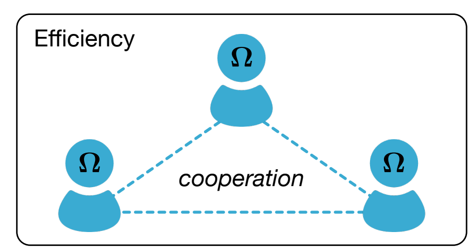
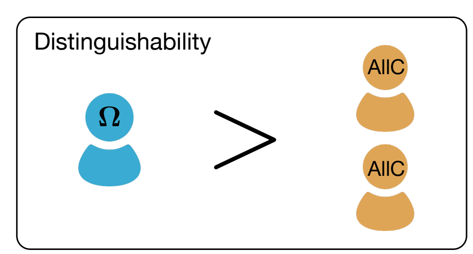
Memory-3 strategies

$$2^{288} = 497323236409786642155382248146\,820840100456150797347717440463\,976893159497012533375533056$$

comparable to the number of protons in the universe

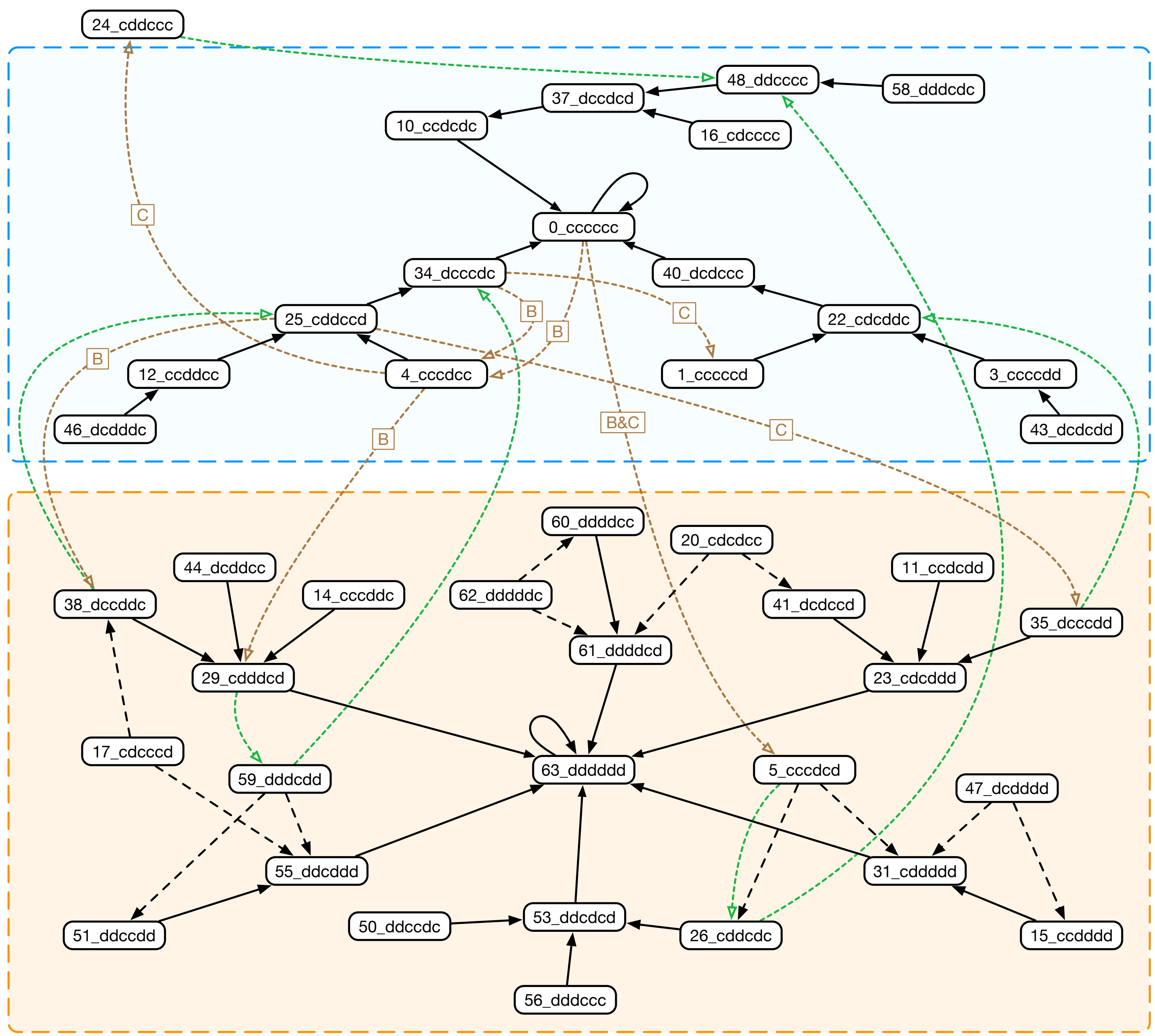
Although direct enumeration is impossible,
we found there are at least 256 solutions!

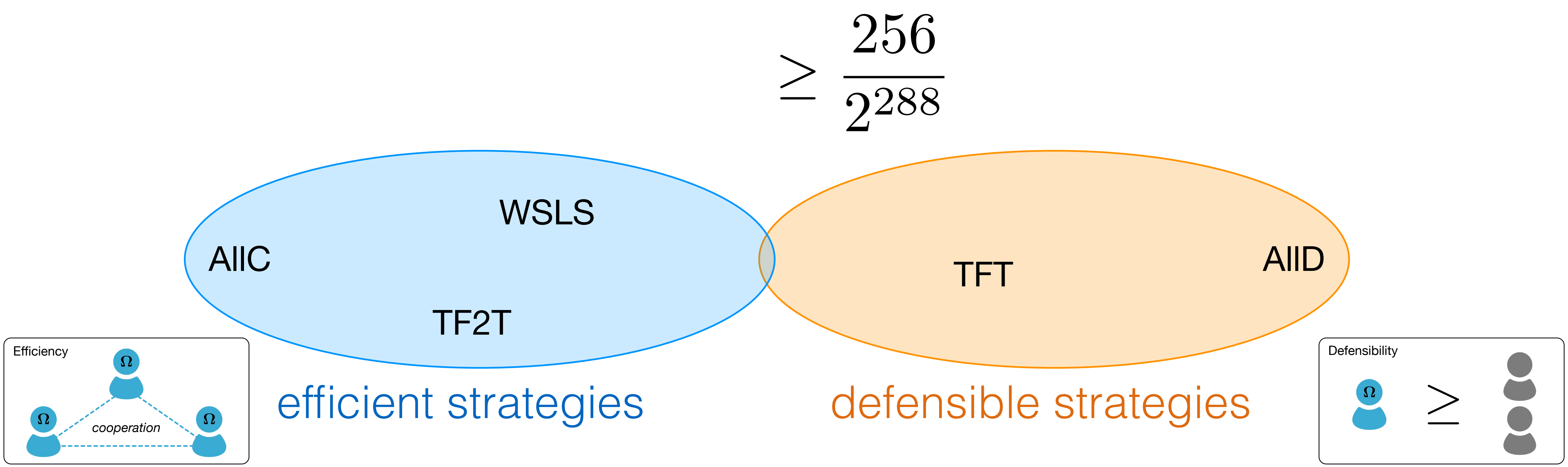
Enumeration of strategies

# of m=2 strategies		1,099,511,627,776
Defensibility against AllD		805,306,368
Defensibility		3,483,008
"Partial" Efficiency ($p_{cooperation} > 0$)		544
Distinguishability		**256**

***"Partially"* Successful Strategies (PS2)**

elevating m=2 PS2 to m=3 successful strategies





Successful strategies indeed exists when m=3.
Full cooperation is achieved
while keeping the defensibility and the distinguishability.

Table 4
One of successful memory-3 strategies. We have picked up the strategy having the largest number of c. The left column shows the state of Bob and Charlie, whereas Alice's state is shown on the right.

$B_{t-3}B_{t-2}B_{t-1}C_{t-3}C_{t-2}C_{t-1}$	$A_{t-3}A_{t-2}A_{t-1}$							
	ccc	ccd	cdc	cdd	dcc	dcd	ddc	ddd
cccccc	c	c	d	c	c	c	d	c
cccccd/ccdccc	d	c	c	c	d	c	c	c
ccccdc/cdcccc	c	d	c	d	c	d	c	d
ccccdd/cddccc	d	d	d	d	d	d	d	d
cccdcc/dccccc	c	c	d	c	c	c	d	c
cccdcd/dcdccc	d	c	c	c	d	c	c	c
cccddc/ddcccc	c	d	c	d	c	d	c	d
cccddd/dddccc	d	d	d	d	d	d	d	d
ccdccd	d	c	c	c	d	c	c	d
ccdcdc/cdcccd	c	c	c	c	d	c	d	c
ccdcdd/cddccd	d	d	d	d	d	d	d	d
ccddcc/dccccd	d	c	c	c	d	c	c	c
ccddcd/dcdccd	d	c	c	d	d	c	c	d
ccdddc/ddcccd	d	c	d	c	d	c	d	c
ccdddd/dddccd	d	d	d	d	d	d	d	d
cdccdc	c	d	c	d	c	c	c	d
cdccdd/cddcdc	d	d	c	d	d	d	c	d
cdcdcc/dcccdc	c	d	c	c	c	d	c	d
cdcdcd/dcdcdc	d	c	d	c	d	c	d	c
cdcddc/ddccdc	c	d	c	d	c	d	c	d
cdcddd/dddcdc	d	d	c	d	d	d	c	d
cddcdd	d	d	c	d	d	d	c	d
cdddcc/dcccdd	d	d	d	d	d	d	d	d
cdddcd/dcdcdd	d	d	d	d	d	d	d	d
cddddc/ddccdd	d	d	c	c	d	d	c	d
cddddd/dddcdd	d	d	c	d	d	d	c	d
dccdcc	c	c	d	c	c	c	d	c
dccdcd/dcddcc	d	c	c	c	d	c	c	c
dccddc/ddcdcc	c	d	c	d	c	d	c	d
dccddd/ddddcc	d	d	d	d	d	d	d	d
dcddcd	d	c	c	d	d	c	c	d
dcdddc/ddcdcd	d	c	d	c	d	c	d	c
dcdddd/ddddcd	d	d	d	d	d	d	d	d
ddcddc	c	d	c	d	c	d	c	d
ddcddd/dddddc	d	d	c	d	d	d	c	d
dddddd	d	d	c	d	d	d	c	d

An example of successful strategies.

memory length and # of players

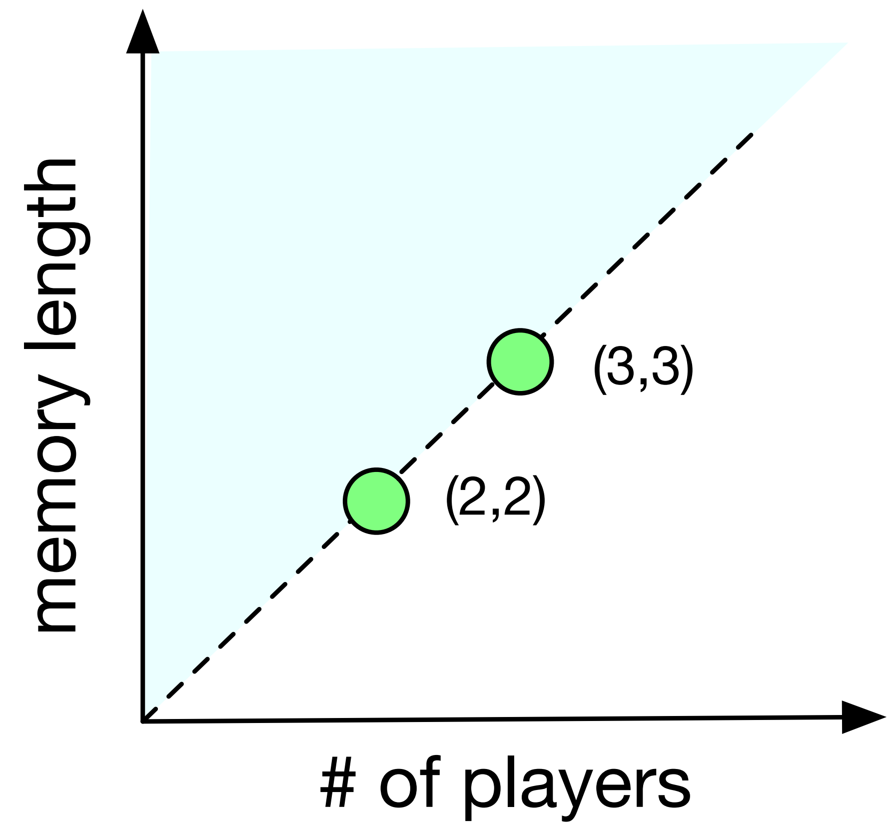
Murase & Baek, J.Theor.Biol. (2018)

n=2 : TFT-ATFT (m=2)
n=3 : m=3 Successful Strategies
...

for general n (>=3), we show

$$m \geq n$$

must be satisfied.



There is a critical memory length above which
a fundamentally new class of strategies may exist.

Conclusions

- A new class of Nash equilibrium strategies was found with the aid of the K computer.
- Well-computed strategies can solve social dilemma without appealing to our moral.

Exploring a broader strategy space does make a difference.

- public-goods game for n>3
- perception error
- multiple choice of actions
- indirect reciprocity
- network reciprocity

富岳

References

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