# Modelling strategies for Nuclear Probabilistic Safety Assessment in case of natural external events

Evelyne FOERSTER, Director of the SEISM Institute, Paris-Saclay University

evelyne.foerster@cea.fr

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## Insight into Probabilistic Safety Assessment for nuclear sites

## On-going developments regarding seismic risk assessment of nuclear sites

- Model reduction techniques to produce virtual charts
- High Performance Computing

## Perspectives

- Modelling strategies for Nuclear Probabilistic Safety Assessment in case of natural external events

# NARSIS (2017-2021)

New Approach to Reactor Safety ImprovementS

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Insight into Probabilistic Safety Assessment for nuclear sites

## Mains objectives:

- **Identifying gaps between practice & needs in existing PSA methodologies** for external multi-hazard events (in particular for low-probability but high-consequences events)
- **Improving parts of these methodologies**, based on & complementing other researches (e.g. European projects, …)
- **Considering 4 main primary hazards** & related secondary effects / combinations: earthquakes, tsunamis, floods, extreme meteo hazards

## Framework of extended PSA

- **Calculates the risk induced by the main sources of radioactivity on the site** (reactor core & spent fuel storages, other sources)
- **Accounts for all plant operating states** for each main source & **all possible relevant accident initiating events** (both internal and external) **affecting one or more nuclear power plants (NPPs) or the environment.**

## A threefold methodology:

- **Theoretical improvements** including progress in **evaluation of uncertainties** and reduction of subjectivity related to expert judgments:
  - Multi-hazard framework with probabilistic modelling of hazards combinations
  - Multi-hazard-harmonized fragility models
  - Multi-risk modelling approach via dynamic Bayesian Belief Networks

- **Verification of the applicability and robustness of the proposed improvements** for the safety assessment (tests on a **virtual PWR NPP**)

- **Application of the outcomes at demonstration level on a real PWR NPP** by providing improved supporting tools for operational and severe accident management purposes.

- **Modelling strategies for Nuclear Probabilistic Safety Assessment in case of natural external events**

**Dr. Pierre-E. Charbonnel**

**Sebastian Rodriguez-Iturra (PhD)**

**Pr. Pierre Ladevèze**

**Pr. David Néron**

**Dr. George Nahas**

## On-going developments regarding seismic risk assessment of nuclear sites

- **Model reduction techniques to produce virtual charts**

⇒ *Verification of the applicability and robustness of the proposed improvements for the safety assessment (tests on a virtual PWR NPP)*

## Motivations for virtual charts

- **Virtual structural testing using "Model Reduction" techniques** to solve **time-dependent nonlinear problems with parameters** (data, design variables)
  - Reduced time costs
  - Possible for a family of structures
  - Works
    - Offline: preparing virtual charts to get outputs of interest (data, design variables)
    - Online: using virtual charts as a decision-making tool, to review design, optimize solutions…

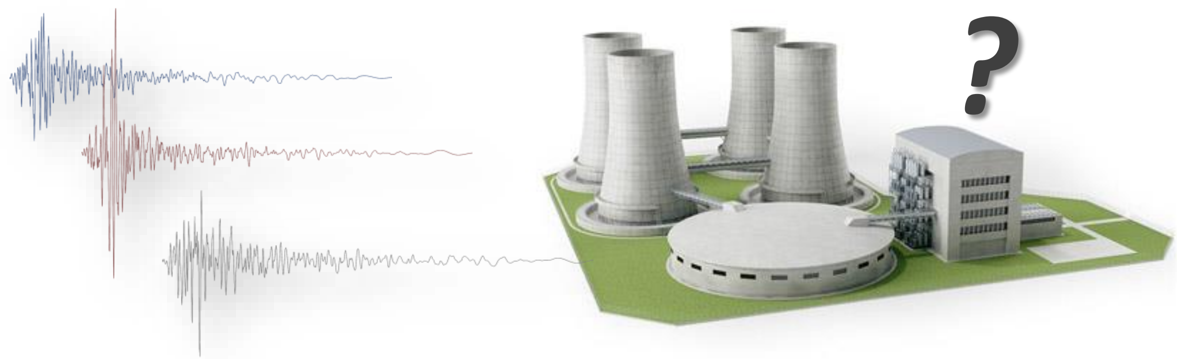
- **Simple illustration** [after Ladeveze]

# Model reduction techniques to produce virtual charts

## Main objectives

***Computing the response of a site with respect to parameters $\gamma \in \Gamma$ defining a seismic scenario $\xi$, to be included in a probabilistic assessment process***

Seismic scenario
$\gamma \in \Gamma$




**Huge uncertainty/variability on the input (loading)**

- *On the parameters $\gamma \in \Gamma$ defining the seismic scenario $\xi$*
  - *Source mechanism, magnitude, distance, velocity structures (propagation), etc.*
- *On equations (e.g. GMPEs) used to derive synthetic signals from $\gamma$*
  - *Infinity of "trajectories" derived from a unique set of parameters (stochastic modelling)*

  ***[Rezaeian & Der Kiureghian,2010] [Zentner et al,2013]***

**Necessity of computing the nonlinear response of a structure (e.g. reactor building) for numerous input signals (time domain)**

- **Several weeks** for a full FEM simulation of a damaging RC structure (in sequential)
- Uncertainties on the constitutive parameters: stiffness, plastic yield/damage thresholds, etc.

## Proposed approach: LATIN (LArge Time INcrement)/PGD (Proper Generalized Decomposition)

- **Non-incremental method** dedicated to solving nonlinear problems [e.g. Ladeveze 1985, 1999…]

- Using **parametrization strategies for resolution** (so-called **“Model Reduction Techniques”**), even for large number of parameters and/or large number of loading cycles

- **Existing scientific bottlenecks with LATIN/PGD:**
  - Never been applied in Dynamics
  - Loading for large number of cycles described only as sine functions (one frequency)

  ⇒ How to model seismic input signals with a reduced number of deterministic parameters

  ⇒ How to parametrize seismic input signals (large number of cycles + frequencies)

## LATIN/PGD method: principal ingredients

- **Splitting difficulties:**
  - $\boldsymbol{\Gamma}$ **:** topological variety where constitutive material relations are verified
  - $\boldsymbol{A_d}$ **:** affine admissibility space where equilibrium & kinematic equations are verified over the whole time-space domain

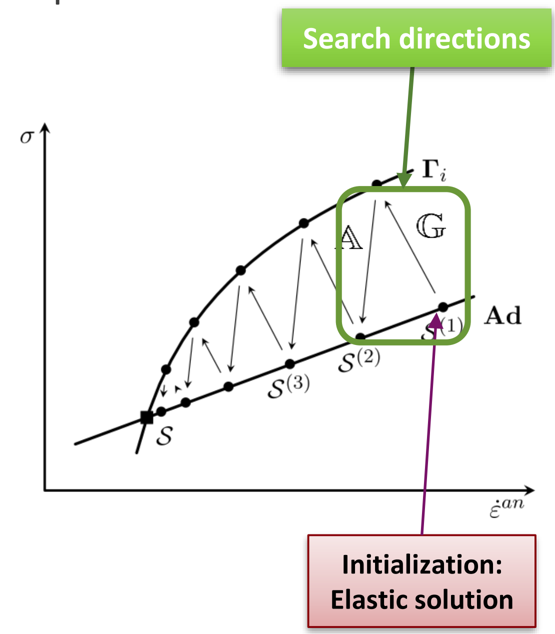
# LATIN/PGD method: principal ingredients

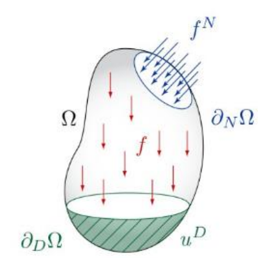
- **Splitting difficulties:**
  - **$\Gamma$ :** topological variety where constitutive material relations are verified
  - **$A_d$ :** affine admissibility space where equilibrium & kinematic equations are verified over the whole time-space domain

- **Iterative resolution by alternating two types of steps and using search directions** ($\mathbb{A}, \mathbb{G}$ operators)**:**
  - **Initialization:** dynamic elastic time-space solution

$$\forall v \in \mathcal{U}^S(\Omega, 0) \otimes \mathcal{U}^T(I)$$

$$\int_{I\times\Omega} \sigma : \varepsilon(v) \, d\Omega dt = -\int_{I\times\Omega} \rho \ddot{u} \cdot v \, d\Omega dt + \int_{I\times\Omega} f \cdot v \, d\Omega dt + \int_{I\times\partial_N\Omega} f^N \cdot v \, dS dt$$



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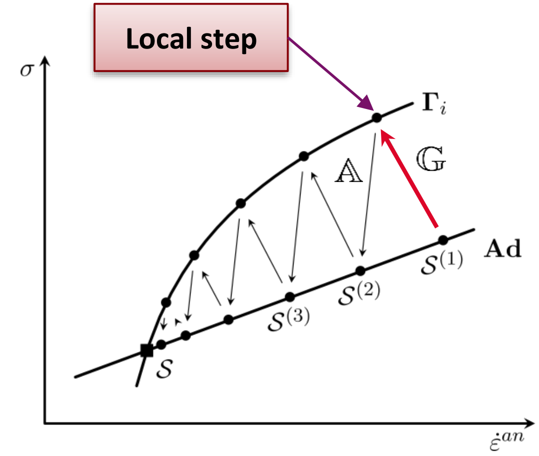
- **Iterative resolution by alternating two types of steps and using search directions** ($\mathbb{A}, \mathbb{G}$ operators):
  - **Initialization**
  - **Nonlinear Local step:** solving constitutive relations on all space integration (Gauss) points, at each time-step

Visco plasticity case: $\dot{\varepsilon}^p = \mathbb{B}(\sigma)$



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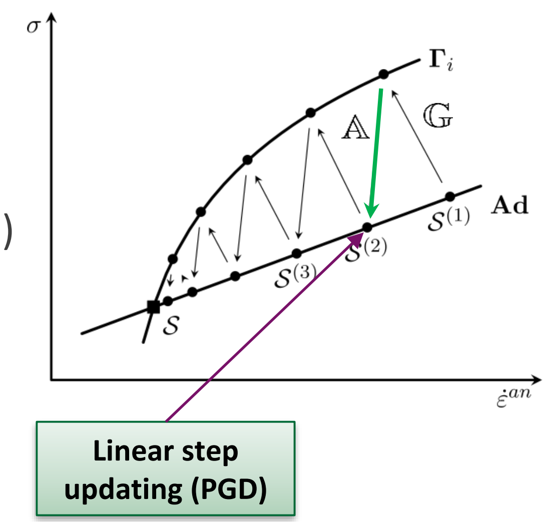
- **Iterative resolution by alternating two types of steps and using search directions** ($\mathbb{A}, \mathbb{G}$ operators)**:**
  - **Initialization**
  - **Nonlinear Local step**
  - **Linear Global step** (on all Gauss points + whole time domain)
    - **Updating admissibility conditions**

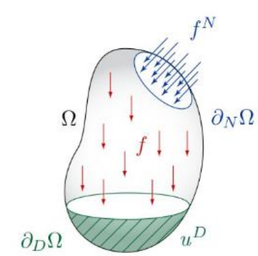



*Weak form - equilibrium equation* $\forall v \in \mathcal{U}^S(\Omega, 0) \otimes \mathcal{U}^T(I)$ $\mathscr{S}(f, f^N)$

$$\int_{I\times\Omega} \sigma : \varepsilon(v)\, d\Omega dt = -\int_{I\times\Omega} \rho \ddot{u} \cdot v \, d\Omega dt + \int_{I\times\Omega} f \cdot v \, d\Omega dt + \int_{I\times\partial_N\Omega} f^N \cdot v \, dS dt$$

*Kinematic admissibility* $\forall \sigma^* \in \mathscr{S}(0)$, $\forall u^* \in \mathcal{U}^S(\Omega, 0) \otimes \mathcal{U}^T(I)$ $\mathscr{E}(u_0, \dot{u}_0, u^D)$

$$-\int_{\Omega\times I} \sigma^* : \varepsilon(u)\, d\Omega dt + \int_{\partial\Omega_D\times I} \sigma^* n . u^D \, dS dt = \int_{\Omega\times I} u^* \cdot \rho \ddot{u} \, d\Omega dt$$



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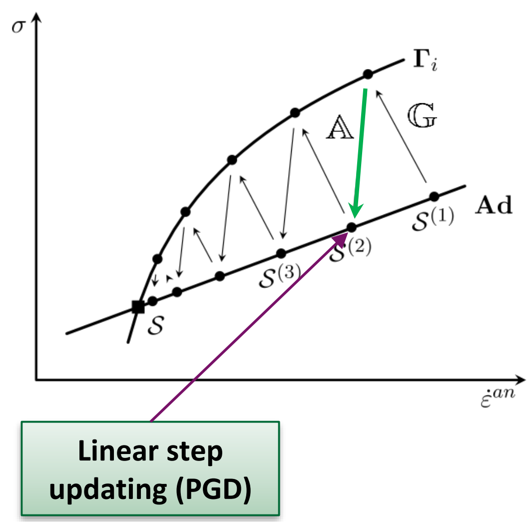
- **Iterative resolution by alternating two types of steps and using search directions** ($\mathbb{A}, \mathbb{G}$ operators)**:**
  - **Initialization**
  - **Nonlinear Local step**
  - **Linear Global step**
    - **Updating admissibility conditions**
    - **Seeking an approximate global solution $\boldsymbol{S(t,x)}$ of the form:**




$$\mathcal{S}^{(n)}(t,x) = \sum_{m=1}^{M} \alpha_m(t)\ \phi_m(x)$$ *with* $(\alpha, \phi)$ *= PGD "modes"*

**Viscoplasticity case:**

$$\sigma(x,t) = \sum_{m=1}^{M-1} \alpha_m(t)\ C_m(x)$$

$$\dot{\varepsilon}^{an}(x,t) = \sum_{m=1}^{M-1} \dot{\alpha}_m(t)\ E_m^{an}(x)$$

## LATIN/PGD method: principal ingredients

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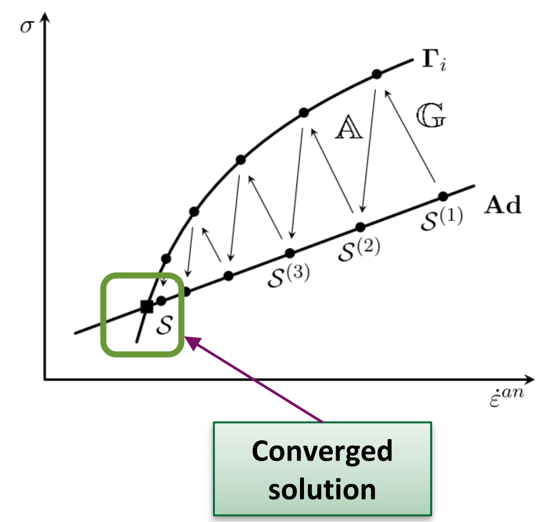
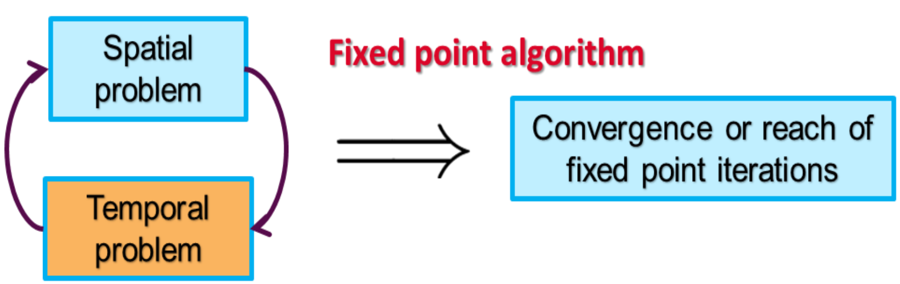
    - **Converged time-space solution:** $\boldsymbol{S = A_d \cap \Gamma}$



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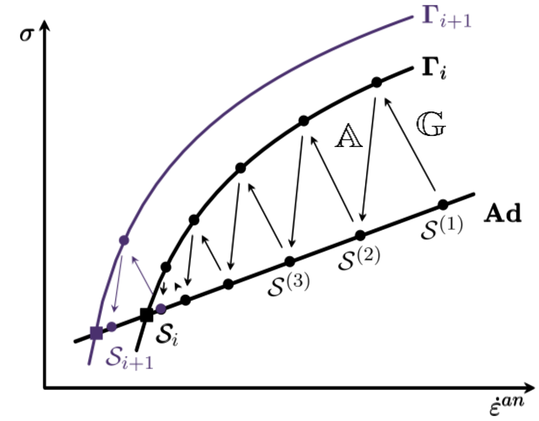



- **Well fitted for solving a parametrized problems**

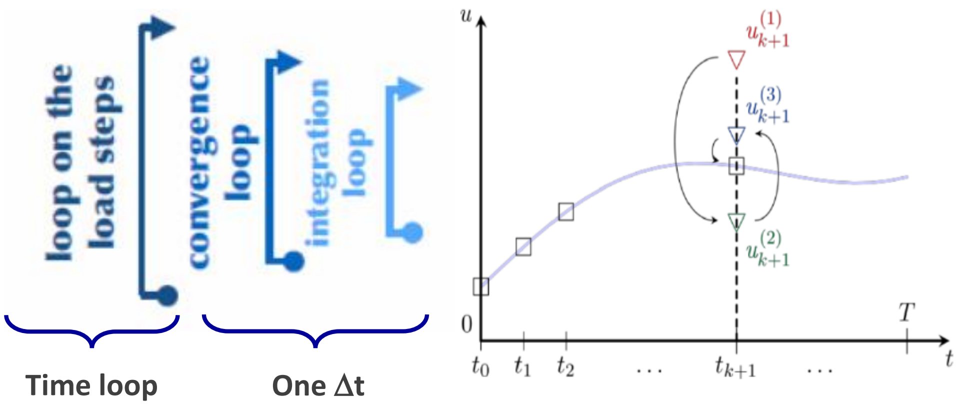
# LATIN/PGD method vs. standard step-by-step methods

- **Newton-Raphson scheme (incremental)**

⇒Time loop then convergence loop

⇒Minimizing the energy residual in the convergence loop only

$$u_{k+1} = \arg \min_{\substack{u \in CA0 \\ v \in CA0}} \mathcal{R}(u, v; \mathcal{S}_k)$$

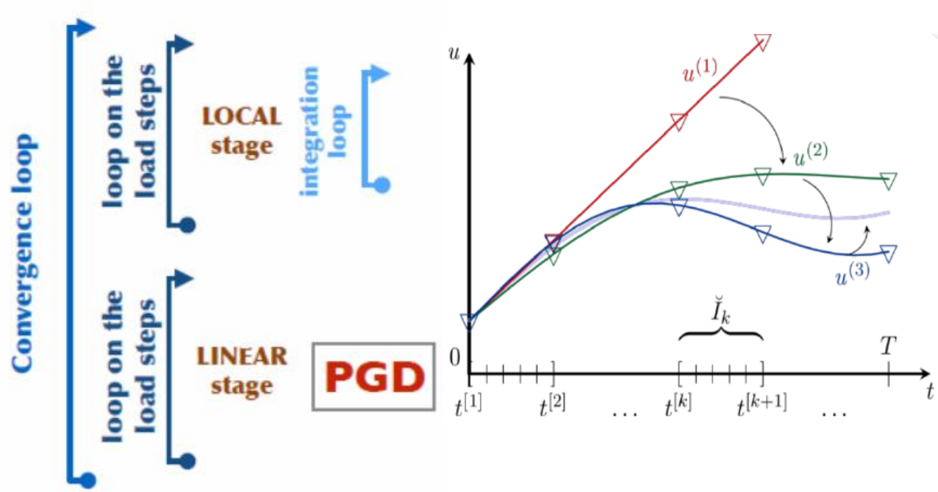



- **LATIN method (non-incremental): ~Newton-Raphson on processes**

⇒ Time & convergence loops inverted

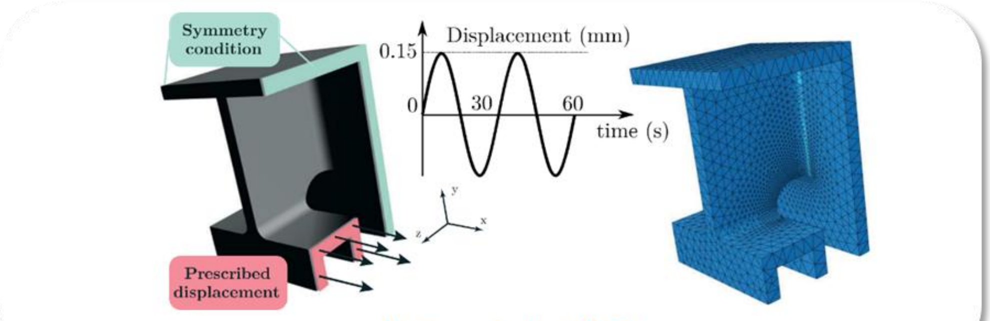
⇒ Solution and residual minimization performed over the whole time-space domain

$$\mathcal{S}^{(n+1)} = \arg \min_{\mathcal{S} \in \mathrm{Ad}} \underbrace{\left\| \mathcal{S} - \mathcal{S}^{(n)} + \Delta \right\|_{\Omega, T}}_{\mathcal{R}}$$



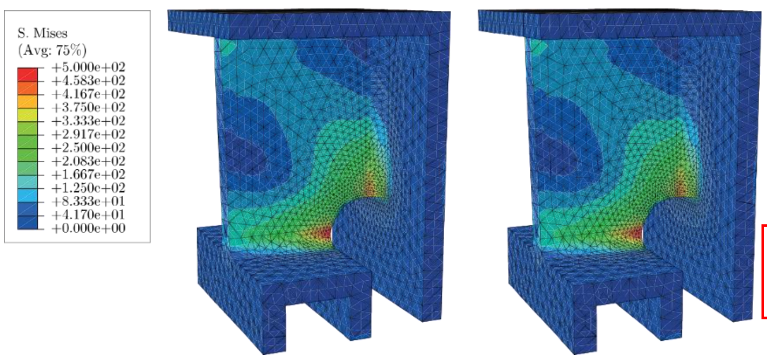
## Test example for LATIN/PGD

- Simple 3D nonlinear parametrized problem (viscoelastoplasticity, quasi-static loading)




[Néron et al., 2015]

142 KDofs
60 $\Delta$t
12 Intel cores

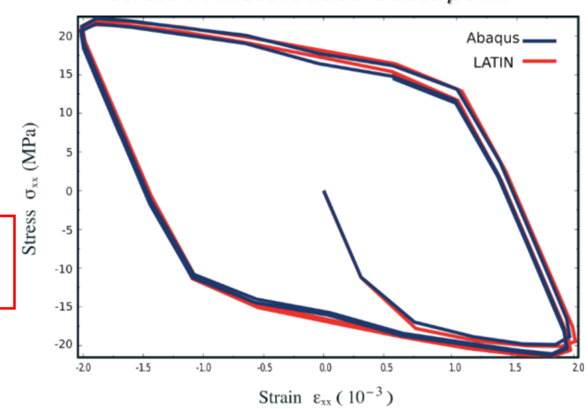



*LATIN* *Abaqus*

Comparison with Abaqus (Zmat module)

30% gain on a single simulation run

*Stress at most loaded Gauss point*



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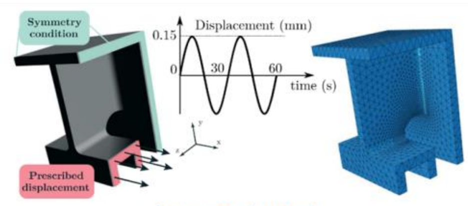

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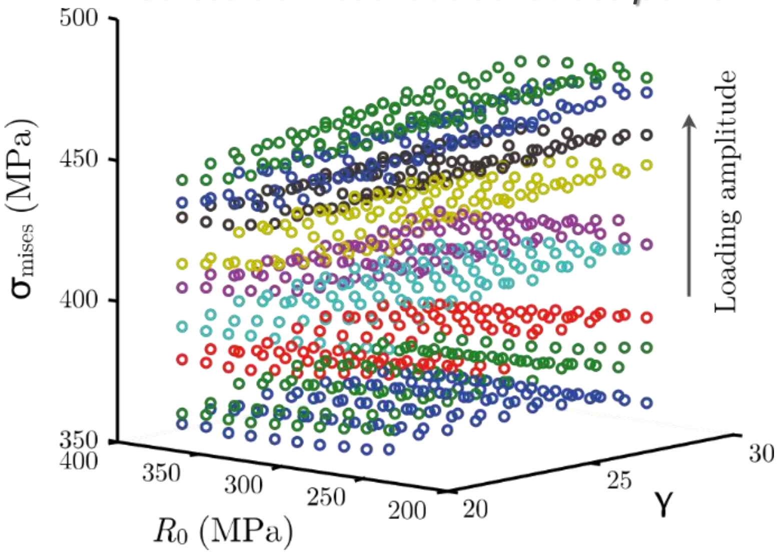



Virtual charts
$u_0$ : loading amplitude
$\gamma$ : kinematic hardening coeff.
$R_0$ : yield stress threshold

$\Rightarrow$ Variation of each parameter (10 values): 1,000 sets of parameters

**CPU times for the 1,000 nonlinear sets:**
- **25 days** with Abaqus
- **17h** for LATIN/PGD (multiple runs algorithm with erratic exploration of the design space)

## Modelling strategies for Nuclear Probabilistic Safety Assessment in case of natural external events


Dr. Afeef M. Badri (postdoc)

Dr. Evelyne Foerster

Dr. Giuseppe Rastiello


### On-going developments regarding seismic risk assessment of nuclear sites

- **High Performance Computing**

⇒ *Verification of the applicability and robustness of the proposed improvements for the safety assessment (tests on a virtual PWR NPP)*

## Our main goals:

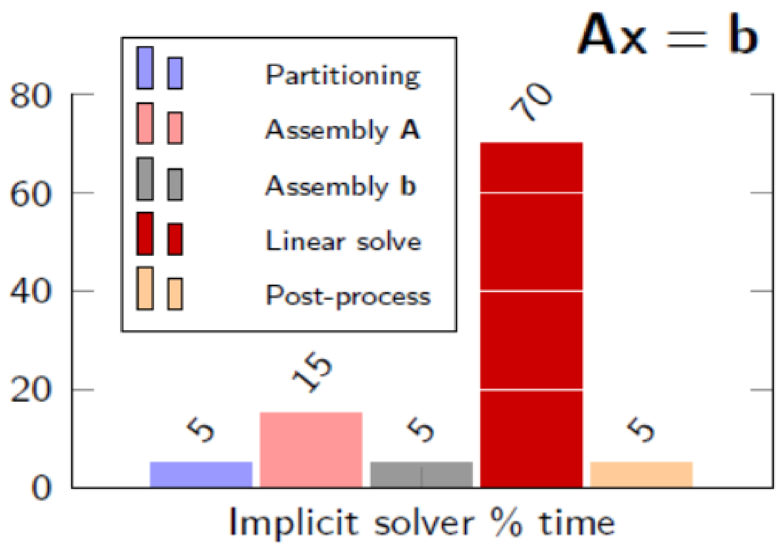
- To **achieve full FEM "best-estimate" and/or "high-fidelity" 3D modeling**
  e.g. for seismic PSA of nuclear sites including interactions (soils, structures, components) and detailed material behaviors (damage, …), variabilities and uncertainties

- To have a **full parallel perspective** for **computing** but also for pre- & post-processing (meshing, visualization, …)

- To work either on **Exascale** parallel or **multi-core** computing **architectures** (even on the "every-day" laptops and PC's)

## On-going developments:

- **Linear/nonlinear implicit iterative solvers** based on domain decomposition, for **damage mechanics and dynamics**
- **Tailored Algebraic Multi-Grid preconditioner** to improve the solver performances and reach quasi-linear scaling
- **Vectorial FEM approach**
- **Fully parallel process:** unstructured meshing, partitioning, assembling, solving & post-processing




The solving phase is more critical in nonlinear dynamics (numerous time steps and repeated updating of A needed)

# Linear solver spectra

## Direct Solver

**Examples:**

- LU
- Cholesky
- QR
- MUMPS

**Pros:**

Robust

**Cons:**

Parallelism

Memory

- 2D $10^6$ dofs
- 3D $10^5$ dofs

## Hybrid Solver

- **Domain decomposition**
- **Multigrid methods**

**Pros:**

**Flexible**

**Naturally ||**

## Iterative Solver

**Examples:**

- Jacobi
- Gauss-Seidel
- Krylov methods (GMRES, CG, …)

**Pros:**

Parallelism

Memory

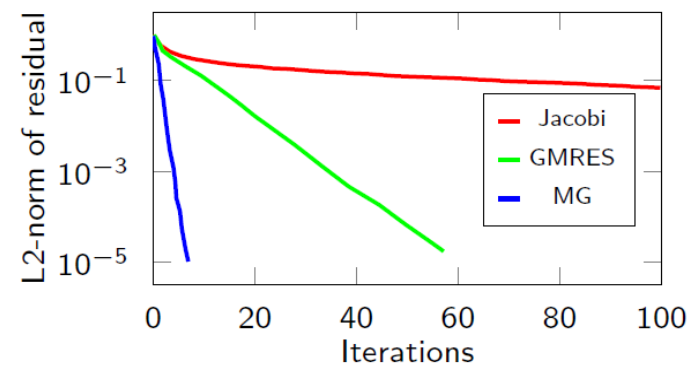
**Cons:**

Robust

# Preconditioning:

- **What:**
  - Means to faster solution $\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$
  - Means to decrease number of iterations
- **Why:**
  - Ill-conditioned problems
  - Strongly coupled
  - Efficient parallel algorithm

**Example** [1]

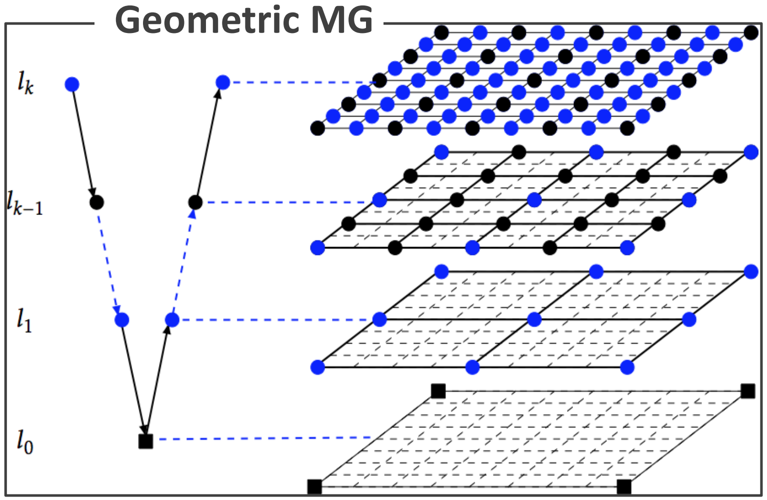



*Jacobi extremely slow, GMRES and MG better"*

1. *M. Seaid et al., J. of Computational and Applied Mathematics, v. 170 (2004).*

- **How:**
  - Use the Krylov subspace method (PETSc) on modified system such as:
    - Left preconditioned system: $M^{-1}\boldsymbol{A}\boldsymbol{x} = M^{-1}\boldsymbol{b}$
    - Right preconditioned system: $\boldsymbol{A}M^{-1}y = \boldsymbol{b}$ with $\boldsymbol{x} = M^{-1}y$
  - One level: CG Jacobi / Block Jacobi (BJacobi)
  - **Multi-level?**

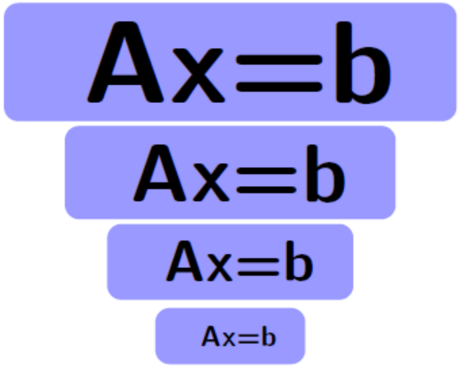
## Multigrid Preconditioning:




- **What:**
  - Use of hierarchy discretization
  - Restrict and interpolate cycle
- **Cons:**
  - Additional meshes
  - Non trivial for unstructured meshes

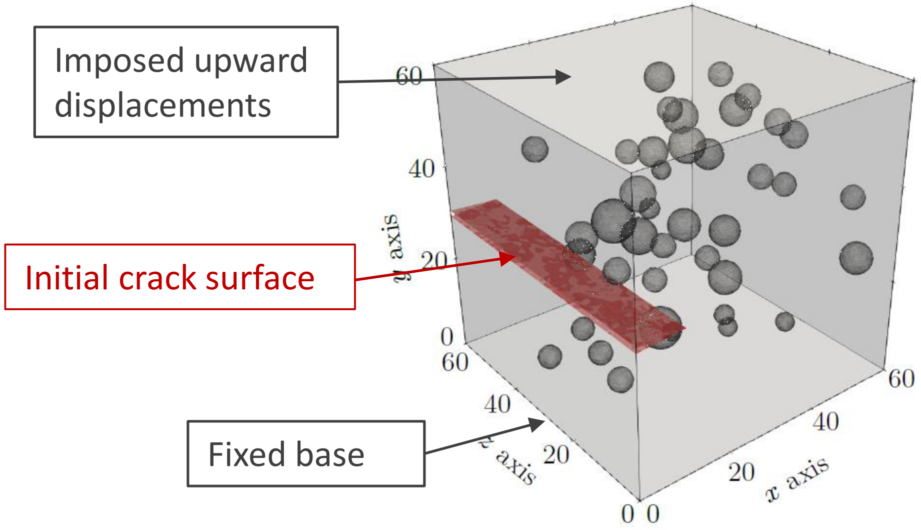
- **Alternative: Algebraic Multigrid (AMG)**
  - Construct a hierarchy of independent coarser operators (i.e. subsets of indices of the unknowns) from the refined grid (operator $A$)
  - Coarsen until LU or SVD

  - **Cons:** difficult to implement and tune (on a case-basis)
  
    ⇒Threshold parameter (coarsening rate)
  - **Pros:** reduced computing costs and high scalability

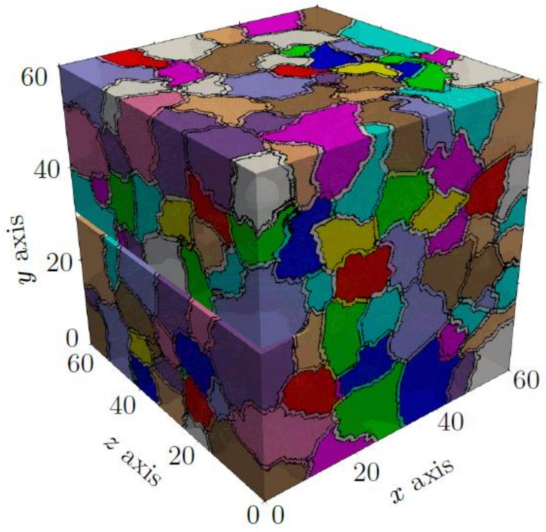


# Applications:

- **3D brittle cracking in randomly perforated medium (quasi-static), using hybrid phase-field formulation (Ambati et al., 2014):**
  - **In-house monolithic Vectorial FEM** fracture mechanic **solver** (Badri et al. (submitted))
  - Crack propagation needs extremely refined meshing:
    - 81 Mdofs
    - Unstructured mesh (tetrahedral elements)
  - MPI-based domain decomposition method
  - 1,008 cores (Intel nodes, Inti supercomputer at CEA/TGCC, France)



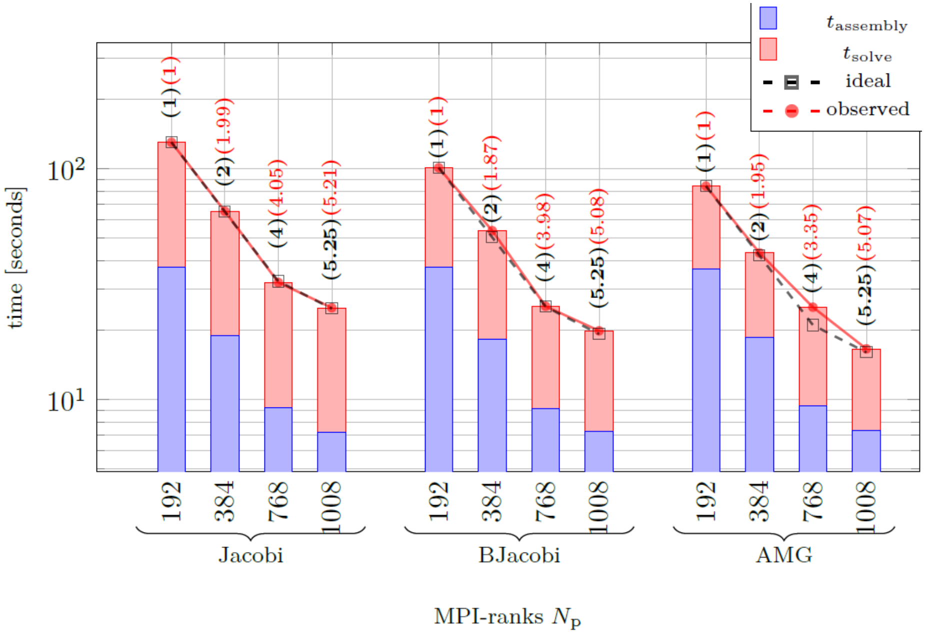

(a) domain $\Omega$.

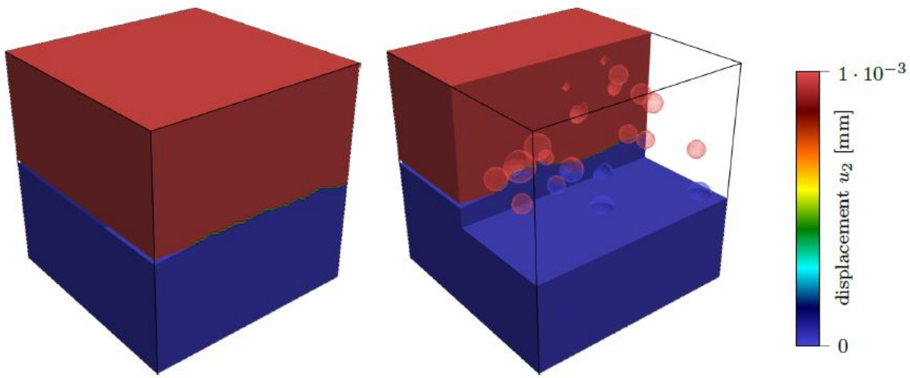



(b) partitioned mesh $\{\Omega_i^h\}_{i=1}^{1008}$.

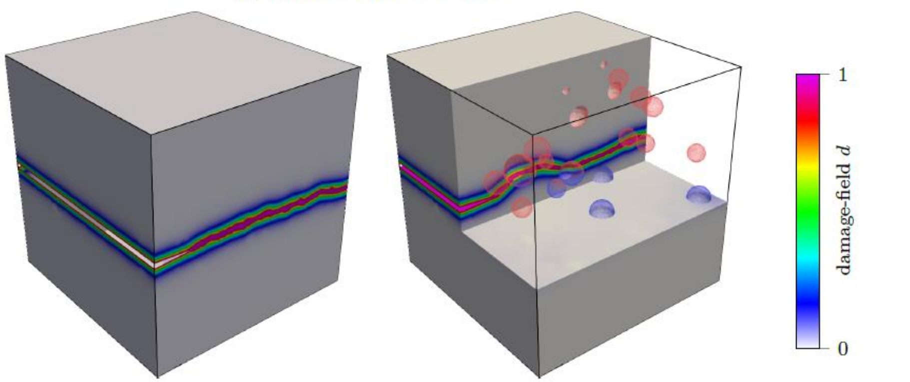
# Applications:

- **3D brittle cracking in randomly perforated medium (quasi-static):**
  - Performing **865 solving steps** (phase-field) performed in **less than ~145 min**, using preconditioning (CG Jacobi, CG BJacobi or CG AMG) instead of ~101 days (sequential)




(a) vertical displacement field $u_2$.

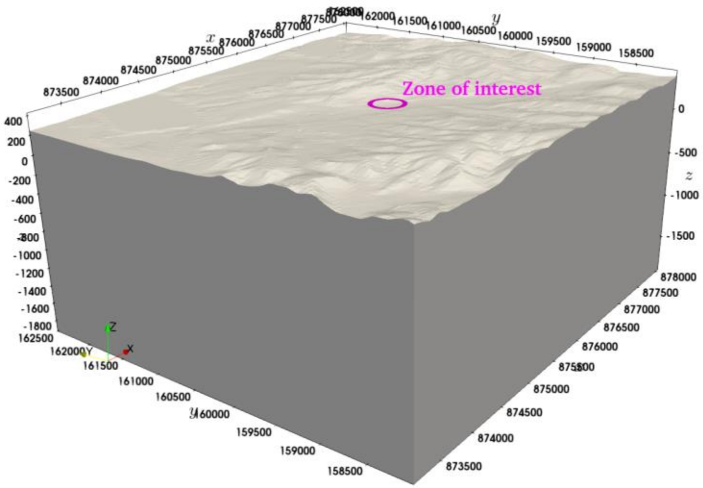


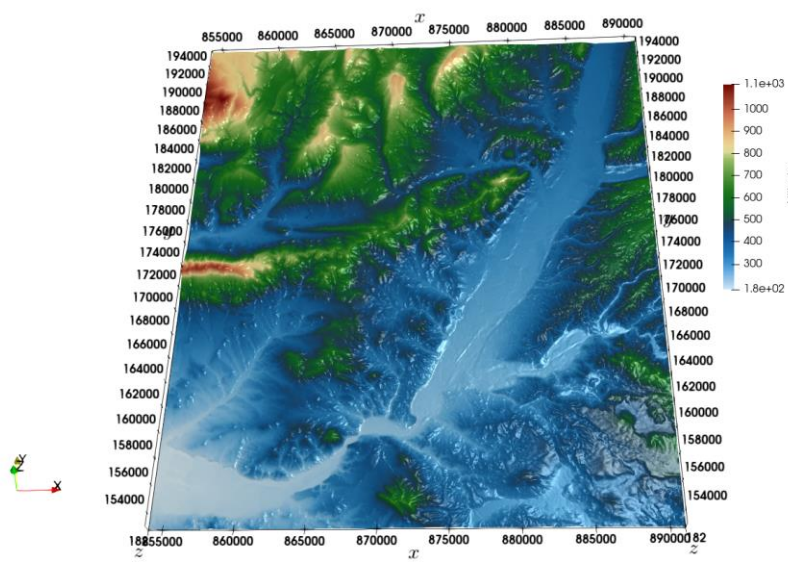

(b) damage-field $d$.

| $N_p$ = 1008 (64 MDofs) | Scaling (%) | $t_{solve}$ (sec) |
|---|---|---|
| CG Jacobi | 99 | 17.7 |
| CG BJacobi | 97 | 12.6 |
| CG AMG | 96 | **9.2** |

# Applications:

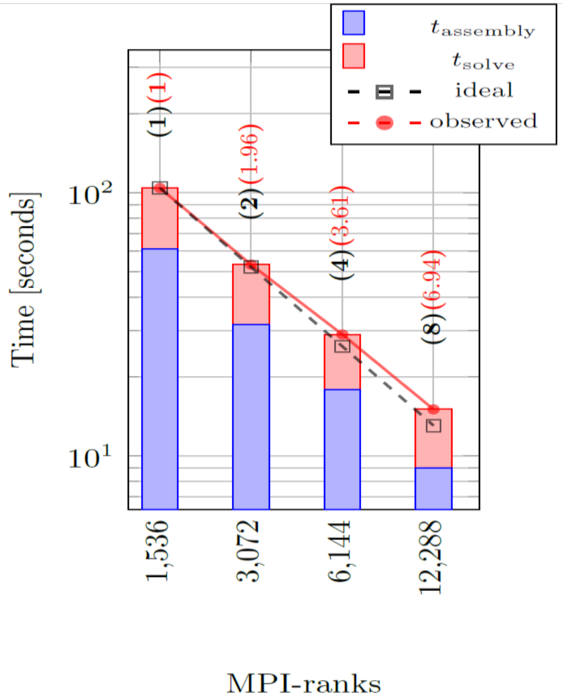
- **3D seismic wave propagation on a real nuclear site:**
  - **In-house Vectorial FEM** dynamic solver (linear)
  - Basin domain: 5 x 4.5 x 2 km$^3$
  - Unstructured mesh (tetrahedral elements) with ~1.9 Bdofs
  - Use of paraxial elements (order 0): input motion + absorbing boundary conditions
  - Max frequency > 40 Hz (required for equipment analysis)
  - MPI-based domain decomposition method
  - 12 Kcores (Skylake nodes, Irene Joliot Curie supercomputer at CEA/TGCC, France)

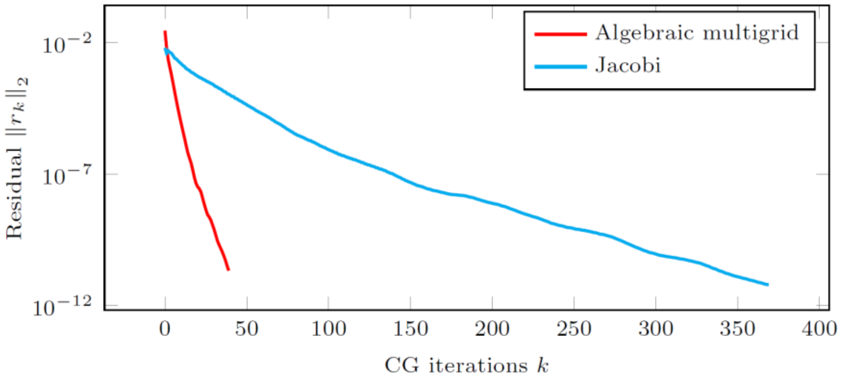


# Applications:

- **3D seismic wave propagation on a real nuclear site:**
  - Monolithic residual drop for AMG
  - Quasi-linear scaling (> 86%)



# Perspectives

## HPC

- **Testing nonlinear dynamic solving strategies** with different constitutive models (soils, structures, components)

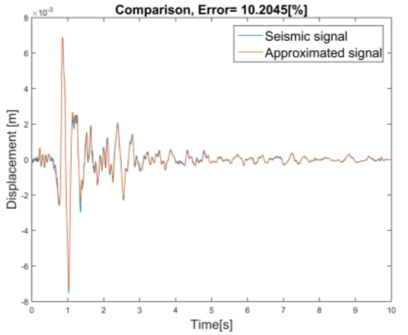
- **Testing on various supercomputers and architectures:**
  - TGCC Joliot Curie IRENE (CEA Bruyères-le-Châtel, France):
    - Bull Sequana X1000 (SKL/KNL, 9.4 PFlops, ~136 Kcores)
    - AMD Rome (11.75PFlops, ~293Kcores)
  - CINES OCCIGEN supercomputer (Atos-Bull B720, Bull Sequana X800, 3.5PFlops)
  - R-CCS supercomputer (on-going RIKEN-CEA collaboration)?

- **Applying to real sites (on-going)**:
  - Increased domain sizes to include seismic sources
  - Increased number of seismic scenarios for probabilistic assessment
  - Soil-structure-components interactions (to be tested with the virtual reactor building from NARSIS project)

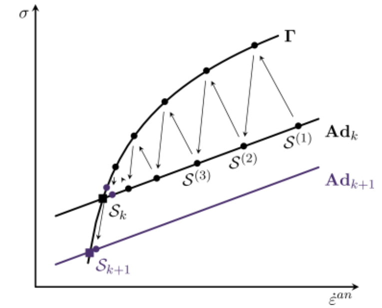
- Going towards **full digital twins of nuclear plants** for safety assessment purposes (among others) and **hybrid testing** (real-time assimilation of physical data and simulations)

## Multi-scale (in time) LATIN/PGD for nonlinear dynamics

- **Parametrization of seismic signals (on-going)**: modeling the time-frequency content e.g. with a sum of simple sine functions [Ladeveze, 2018]
- **Simulations (FEM kernel) with input parametrized signals (natural or synthetics):**
  - Using **Big Data strategies** for data clustering combined with **damage indicators** for structures, systems or components (SSCs)
  - Defining a new strategy to **produce virtual charts for SSCs**
- **Combining LATIN/PGD model reduction technique with optimized parallel solving strategies for PSA of nuclear sites**




*Example of parametrization with 7 modes, different macro discretization (sines)*



# Thank you for your attention!